Zeros of Monosplines

The monospline M_I has a zero at ξ if and only if the quadrature rule I is exact for the truncated power $(\cdot - \xi)_+^{k-1}$. This makes it important to count the number of zeros of a monospline. Here is the basic result (due to Micchelli72).

Theorem. *The number of zeros of the monospline M of order k on the open interval $(a \,..\, b)$ with knot sequence $(x_i^{[\mu_i]} : i = 1, \ldots, n)$ cannot exceed*

$$\sum_{i=1}^{n} 2\lfloor(\mu_i + 1)/2\rfloor + S^-(D^j M(a) : j = 0, \ldots, k) - S^+(D^j M(b) : j = 0, \ldots, k).$$

Here, $2\lfloor(\mu + 1)/2\rfloor$ is the smallest ewordeven integer $\geq$ the integer μ.

Since the Bernoulli monospline is 1-periodic and has simple knots, at the integers, it can, by this theorem, have at most 2 zeros in the half-open interval $[0 \,..\, 1]$, hence it provides equality in the theorem's bound.

Source Chapter 7 of BojanovHakopianSahakian93. Unfortunately, the display there, just prior to Theorem 7.1 , has 'even' and 'odd' interchanged, making Theorem 7.1 there (which corresponds to the above Theorem) incorrect.

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