multivariate B-splines

The **simplex spline** was the first multivariate B-spline to be studied. A bivariate, quadratic simplex spline appeared in a letter of Schoenberg to P. Davis. A formal definition was first given in the B-spline survey as the function

$$M_\sigma \mathbb{R}^d \to \mathbb{R} : x \mapsto \mathrm{vol}_{n-d}(\sigma \cap P^{-1}(x))/\mathrm{vol}_n(\sigma),$$

with σ a simplex in $\mathbb{R}^n$, and $P : \mathbb{R}^n \to \mathbb{R}^d : x \mapsto (x_1, \ldots, x_d, 0, \ldots, 0)$.

Micchelli used instead the much more effective definition of M_σ as the *distribution*

$$M_\sigma : f \mapsto \int_\sigma f \circ P/\mathrm{vol}_n(\sigma),$$

and used it very effectively to established recurrence relations and other properties of M_σ.

Following this lead, de Boor and Höllig defined the distribution

$$M_B : f \mapsto \int_B f \circ P$$

for an arbitrary polyhedral body in $\mathbb{R}^n$ and an arbitrary affine map $P : \mathbb{R}^n \to \mathbb{R}^d$. See "A formula for M_B"(qv) for a precise definition of $\int_B$ as well as a careful derivation of a formula for M_B as a function on $\flat(PB)$.