Error in (univariate) polynomial Hermite interpolation

Shadrin95a proves the following: Let H_Θ be the Hermite projector based on the n-sequence Θ and set

$$\omega_\Theta := \prod_{\theta\in\Theta} (\cdot - \theta).$$

Then

$$\|D^j(f - H_\Theta)f\|_\infty \le \frac{\|D^j\omega_\Theta\|_\infty}{n!} \|D^n f\|_\infty, \quad j = 0, \ldots, n,$$

with $\|g\|_\infty := \|g\|_\infty(I)$ for an arbitrary finite interval I. Moreover, this inequality is sharp, being equality for any $f \in \Pi_n$.

In the same paper, he provides a sharp pointwise error bound.

Many people have worked very hard to establish this result in special cases. E.g., BirkhoffPriver67 proves this for $n = 2, 4, 6$, just for the case that I is the smallest interval containing Θ, and there are many partial results in the recent book AgarwalWong93 on 'Error inequalities in Polynomial Interpolation and their Applications'.

Waldron9xa proves the following pointwise bound in terms of the L_q-norm of $D^n f$,

$$|(f - H_\Theta f)(x)| \le \frac{n^{1/q}}{n!} \frac{|\omega_\Theta|}{\operatorname{diam}\{x, \Theta\}^{1/q}} \|D^n f\|_{L_q(\operatorname{conv}\{x,\Theta\})}.$$

However, the sharpness has not yet been established in full generality.

This, too, covers, or improves on, all corresponding results in the prior literature. The latter is often phrased in terms of Beesack's inequality and Wirtinger's inequality.

Rolle's theorem may be used to derive bounds on the error in derivatives, but the resulting bounds will not be sharp in general.

Waldron is planning to write a book on this.

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