

nonnegative splines

¶UL¶ While a spline is nonnegative in case its B-spline coefficients are nonnegative, the converse fails to hold for splines of order greater than 2.

However, it is always possible for given nonnegative spline $f = \sum a_i B_{i,k,t}$ to obtain a refined knot sequence t' so that $f =: \sum_i a'_i B_{i,k,t'}$ has nonnegative B-coefficients. The only issue here is the possibility that f is not strictly positive. In that case, for every maximal zero interval $[z_l . . z_r]$ for f , make sure that both endpoints appear in t' at least $k - 1$ times. Any other negative coefficients are bound to disappear after finitely many knot insertions nearby.

Early work on generalized convex splines (i.e., splines whose j th derivative is nonnegative for some j) can be found in

Burchard, H.; Extremal positive splines with applications to interpolation and approximation by generalized convex functions; ; 79; 1974; 959–963;