CS 880: Advanced Complexity Theory 2/24/2006

Final Project

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Goal

The final project consists of an in-depth study of a specific topic in computational complexity. Below is a list of suggestions that are all related to the course material. Feel free to come up with a project of your own that is not on the list. The project should have the potential of turning into a longer-term research program. Thus, your interest in the matter is very important.

You can work on your own or in small groups. For each of the projects below, a suggested group size is indicated.

Time Line

The project involves the following steps.

1. Read the blurbs below and do some preliminary research to find a project topic. You may also want to browse recent theory conference proceedings (say from 2000 or later) to get an idea of current research in computational complexity. Talk to me about ideas, suggestions, questions you have. Talk to your fellow students to see who shares your interest and is willing to work with you.

   You should decide on your topic and your collaborators and discuss your choice with me before Spring break.

2. Read the relevant papers, do the research, and write a report at the level of your fellow students in CS 880. Try to present the material in a unified way and focus on the underlying ideas. The expected quantity depends on the size of your group but in general I care more about quality.

   You should write your report in LaTeX using the template we will make available from the web page. I should receive LaTeX source and a PostScript version of your paper at the latest 48 hours before the final oral presentation.

3. Present your work and answer questions. The oral presentation should be about one hour long, and each member of your team should participate. I will schedule a 90-minute meeting with each group during exam week but we can also do it earlier or have multiple meetings.

At any time during the project, feel free to make an appointment and discuss your progress, ask for additional references, directions, etc. Good luck!

Suggested Topics

1. Spectral Gap of Gabber-Galil and its Cousins [2 people].

   The usual way to establish the vertex expansion of a graph goes through a lower bound on the spectral gap. The original proof of the vertex expansion of the Gabber-Galil graph does not seem to follow that pattern. One goal of this project is to investigate that matter – does the proof directly lower bound the spectral gap? And what is the best known lower bound?

   The Gabber-Galil graph can be viewed as a so-called Cayley graph of a group. There has been some recent progress on lower bounds for the spectral gap of a wide class of Cayley graphs. Another goal of this project is to survey those developments.

2. Expanders of Low Complexity [2 people].

   The Gabber-Galil construction from class is very explicit. Given a start vertex ρ_0 and a sequence of labels s, you can compute the vertex ρ_s at the end of the walk from ρ along s in logarithmic space, i.e., in space $O(\log(|\rho_0| + |s|))$. The goal of this project is to investigate whether the latter process can be computed in NC^1, i.e., by circuits of logarithmic depth, either for the Gabber-Galil construction or for one of the other constructions. The repercussions for some of the applications like pseudorandom generators would be a next step.

   This project involves studying some results on the power of logarithmic space and NC^1. There is also a connection with the project on pseudorandom generators. One of the unsubstantiated claims in the paper "Pseudorandomness for Network Algorithms" by Impagliazzo, Nisan, and Wigderson is that an appropriate choice of expanders makes their pseudorandom generators computable in simultaneous polynomial time and polylogarithmic space.

3. Pseudorandom Generators [3 people].

   The goal of this project is to investigate the relationship of the pseudorandom generator construction from class with other known constructions, in particular with Nisan's pseudorandom generator for space-bounded computation. Compare these constructions with respect to the applications we discussed in class as well as some other applications developed in the literature.

   The most relevant papers for this project are "Pseudorandomness for Network Algorithms" by Impagliazzo, Nisan, and Wigderson, and "On Recycling the Randomness of the States in Bounded Space Computation" by Raz and Reingold. The former paper is a bit sketchy at some points; working out those points would be a good start of the project.

4. Connections between Expanders, Dispersers, and Extractors [2 people].

   We indicated some of the relationships between expanders, dispersers, and extractors in class. The goal of this project is to survey the known connections.

A good start would be the paper "Linear-Degree Extractors and the Inapproximability of Max Clique and Chromatic Number" by Zuckerman. Another relevant paper is "Randomness Conductors and Constant-Degree Lossless Expanders," in which Capalbo, Reingold, Vadhan, and Wigderson suggest conductors as a generalization that captures all three notions.

5. Expanders in Coding Theory [1 person].

   The goal of this project is to study recent results on error correcting codes based on expanders, in particular their list decoding properties. The survey paper "Error-Correcting Codes and Expander Graphs" by Guruswami would be a good guide for this project. You are welcome to make excursions into new error correcting codes that are inspired by but expander codes but are not really based on expanders, such as turbo codes. Or you may study applications of coding theory in complexity theory. For the latter, see the survey paper "Some Applications of Coding Theory in Computational Complexity" by Trevisan.

6. Unique Neighbor Expanders [2 people].

   A graph is (K, c) unique-neighbor expanding if for every subset S of vertices of size at most K there are at least $c|S|$ vertices in $\overline{S}$ that have a unique neighbor in S. Graphs with such properties play a role in some applications of expanders, such as error correcting codes.

   Such strong expansion property doesn't seem to follow from the usual analysis based on the spectral gap. The goal of this project is to survey the known constructions of unique-neighbor expanders and their applications. A good starting point would be the paper "Explicit Unique-Neighbor Expanders" by Alon and Capalbo for the constructions and the paper "Randomness Conductors and Constant-Degree Lossless Expanders," by Capalbo, Reingold, Vadhan, and Wigderson for the applications.

7. Further Research on the Logspace Algorithm for Undirected Connectivity [1 person].

   In class we will cover the recent logspace algorithm for undirected connectivity, which has spurred a lot of further research. The goal of this project is to survey the work that followed after the algorithm was published.

   One direction involves other applications of the underlying idea as in the paper "Pseudorandom Walks on Regular Digraphs and the RL vs. L Problem" by Reingold, Trevisan, and Vadhan. Another direction aims at further simplifying the algorithm. An example of the latter is "Derandomizing Squaring of Graphs" by Rozenman and Vadhan.

8. Expanders and Metric Embeddings [2 people].

   Any finite metric space can be embedded into Euclidean space but with some distortion of the distances. The graph metric of an expander turns out to be the hardest to embed in the sense that of all finite metric spaces on a given number of points, expanders require the largest distortion.

The goal of this project is to survey the expander embeddability results and their applications in approximation algorithms. A good starting point is the paper "Expander Flows, Geometric Embeddings, and Graph Partitionings" by Arora, Rao, and Vazirani.

9. Expanders in Quantum Information Processing [2 people].

   The goal of this project is to survey the known applications of expanders in quantum information processing and to think of possible new ones. The only application I'm aware of at this point is in the paper "A Randomness-Efficient Sampler for Matrix-valued Functions and Applications" by Wigderson and Xiao. In that sense, this project is the most risky one on the list. If need be, the project can move more in the direction of randomness-efficient sampling techniques.