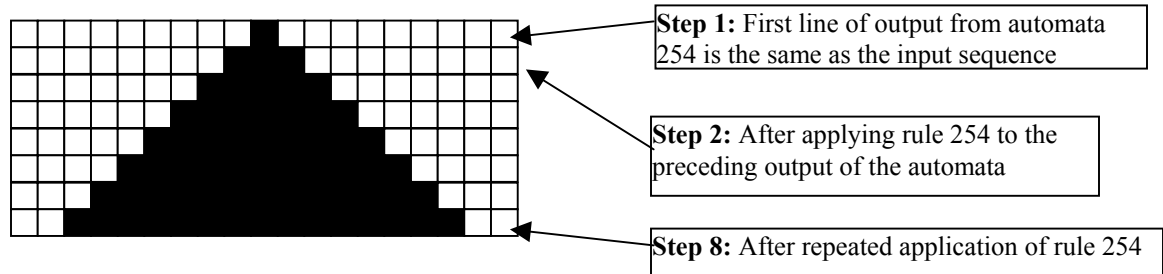# Problem 8: Boundaries on "A New Kind of Science"

Stephen Wolfram in "A New Kind of Science" describes a special kind of cellular automata. They consist of rows of blocks, where blocks are either filled or not filled depending on the previous row. To generate a new row of blocks, the automata looks at the preceding row and then follows a pre-set "rule" to either color or not color a square on the output row.

The diagram below illustrates the "output" from one of these special kinds of cellular automata.

Step 1: First line of output from automata 254 is the same as the input sequence

Step 2: After applying rule 254 to the preceding output of the automata

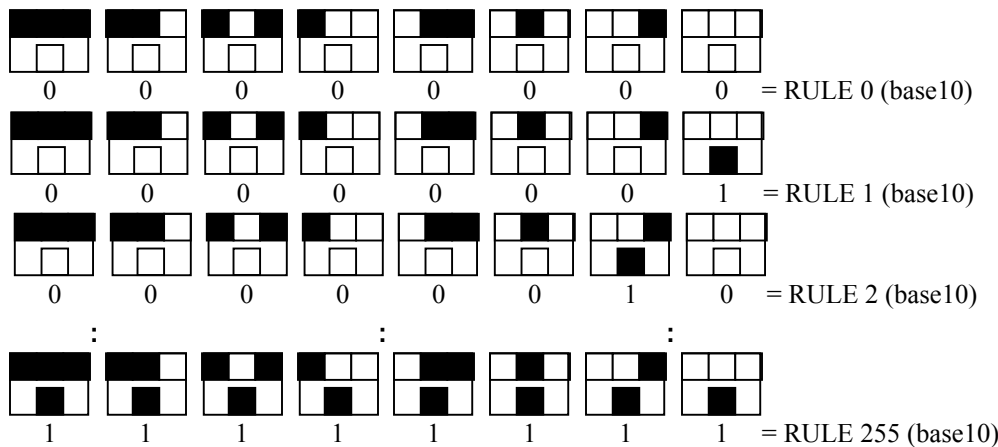**Step 8:** After repeated application of rule 254

This was generated by repeated application of "Rule 254". The automaton was initialized with an input line that consisted of a single black square. Repeated application of rule 254 to the output line from the preceding step generated the black triangle shown above. Each "rule" can be described graphically using a set of eight input/output patterns, as illustrated below for rule 254.

1   1   1   1   1   1   1   0   = RULE 254 (base 10)
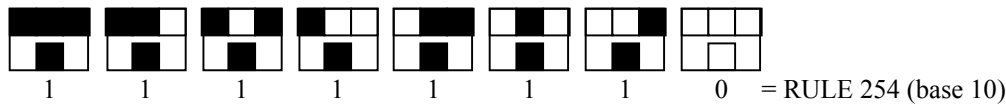
The top row in each pattern gives one of eight possible color combinations for three adjacent cells (the middle cell and its left and right neighbors). The bottom row in a pattern specifies the color that the middle cell should be given on the next step for each of the 8 possible cases.

Given this arrangement, there are 256 generating rules or automata numbered as follows:

0   0   0   0   0   0   0   0   = RULE 0 (base10)

0   0   0   0   0   0   0   1   = RULE 1 (base10)

0   0   0   0   0   0   1   0   = RULE 2 (base10)

:   :   :

1   1   1   1   1   1   1   1   = RULE 255 (base10)

The automata for this problem operate in a bounded space. That is, each line output by an automaton consists of exactly n squares, and the first square and the last square of the output are always white, regardless of the rule being used for the automata. While an automaton can examine the first and last square of its input line when computing the next line, it cannot change the first or last square when it outputs the next line. These two squares must always remain white.

Bounded automata will always start life on a line with an odd number of squares, and all of the squares except the middle square are white. The middle square for step 1 is always black.

For every input case, your program must determine which, if any, of the 256 possible automata could have generated the particular line given for that input case. If none of the automata can generate the specified line by the given maximum step number (also given in the input), you must display "NONE". If one or more automata can generate the specified line, then you must identify them (as described below).

Input

The input will contain multiple cases, with the input for each case occupying a single input line. For each case the input line will contain a positive integer which specifies the maximum step number for the automata to run. After one or more whitespace (blank or tab) characters there will appear an odd-length string of characters which represents the "squares" on the line. The character "W" represents a white square and "B" represents a black square. If this string contains characters other than "W" or "B", or if the string is invalid (not a properly bounded string as previously described), then the string cannot be generated by any of the automata, and the output will be "NONE" (as illustrated below). The string may possibly be followed by one or more whitespace characters before the end of line. The input line for the last case will be followed by a line containing only the integer –1.

Output

For each input case, display the case number (1, 2, …) followed by pairs of numbers (*rule*, *step*), where *rule* is the rule number of the automata (0 through 255) that generated a particular output and *step* is the first step in the sequence of outputs at which the automata generated the desired output. If more than one rule generated the desired output sequence before or on the maximum step number, list a pair for each rule number that generated the desired output, in order, from the lowest automata number to the highest automata number, with a single space between each output pair. If none of the automata generate the particular line from the input file before or on the maximum step number, then display the case number and the word NONE. Your output should closely resemble that shown in the sample output provided.

Sample Input

```
3 WBWBWBWBW
1000 WBWBWBWBBBW
5235 WBWWBWWBBBBWWBBWWWWWWBBWWWBBWWWWWWBBWWBBBBWWBWWBW
5 WBWBDCWBW
–1
```

Output for the Sample Input

```
Case 1: (91,3)
Case 2: (15,8)(158,11)(159,14)(243,8)
Case 3: (129,84)(161,84)
Case 4: NONE
```