Homework 3

$\mathrm{CS}~547$

Due Friday, Feb. 17, in class

Plotting Something

Use the inverse CDF method to generate 100000 random values from an exponential distribution with $\lambda = 10$.

Calculate the mean and variance of your generated data. How can you verify that your data is exponentially distributed using only these two numbers?

Plot the CDF of your randomly generated values.

Note: you can use any software tool for this problem as long as it produces nice, legible plots. If you don't have a preferred tool, look into MATLAB or R.

Mr. Fish's Variable

A Poisson¹ random variable is often used to represent the occurence of random events over time. Its distribution is discrete and has a single parameter, $\lambda > 0$, which is called the *rate*. Very shortly, we'll study the *Poisson process*, which is closely related to the Poisson distribution and has several nice properties that are useful in analyzing queueing systems.

Let X be a random variable denoting the number of events that occur in a time period of fixed length. If $X \sim \text{Pois}(\lambda)$, the probability of obtaining exactly k events in the period is given by

$$P[X = k] = \frac{e^{-\lambda}(\lambda)^k}{k!}.$$

Show that the expected number of events in the period is simply λ .

Hint: it may be helpful to know that

$$\sum_{m=0}^{\infty} \frac{x^m}{m!} = e^x.$$

Lightbulbs

We've been given a bin filled with lightbulbs. There are three different kinds of bulbs, labeled A, B, and C.

The probability that a type A lightbulb lasts for at least 1000 hours is .7. The probabilities that type B and C lightbulbs last at least 1000 hours are .4 and .3, respectively.

If 20% of the lightbulbs in the bin are type A, 30% are type B, and 50% are type C, what is the probability that a randomly chosen lightbulb lasts for at least 1000 hours?

 $^{^{1}}Poisson$ is the French word for fish.

Suppose we pick two lightbulbs independently and at random. What is the probability that both lightbulbs last for at least 1000 hours?

Suppose we pick 10 bulbs independently and at random. What is the probability that *at least one bulb* lasts for at least 1000 hours?

The Gamma Function

The gamma function, $\Gamma(\alpha)$, is defined by

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha - 1} \, dx$$

Show that the gamma function satisfies the following two properties:

- $\Gamma(\alpha) = (\alpha 1)\Gamma(\alpha 1)$
- $\Gamma(1) = 1$

Together, these two results imply that $\Gamma(n) = (n-1)!$ for positive integral values of n.