

Homework 8

CS 547

Due Friday, Apr. 27, in class

You Got to Know When to Hold 'Em, Know When to Fold 'Em

Abe and Bart are stereotypical Old West gamblers playing a coin-flipping game. If the coin comes up heads, Abe wins a dollar and Bart loses one. If the coin comes up tails, Abe loses a dollar and Bart wins one. At the start of the game, Abe has A dollars and Bart has B dollars. The game ends when one of the two gamblers wins all of the money and the other one is broke.

Draw out a Markov chain where each state tracks the amount of money Abe currently holds. Label the arcs in your drawing with the appropriate transition probabilities. Once the chain reaches either state 0 or state $A + B$ it will remain in that state permanently.

Is your model aperiodic? Is it irreducible?

Suppose $A + B = 5$. Write down the transition probability matrix P for the game, assuming it's played with a fair coin.

Now, evaluate P^n for a large value of n . Does the matrix converge? Are all of the rows identical? How do you interpret the entries of the matrix?

An Inventory Model

KEIA is an distributor of medium-quality Finnish furniture and home goods. One of their moderately popular products is the Sampo salt shaker.

KEIA keeps at most three salt shakers in stock at any particular time. There is a 20% chance of selling a single shaker in any given week, and KEIA never sells more than one a week.

A truck delivers new items to the store weekly. If there are fewer than three shakers currently in stock, there is a 25% chance that a week's delivery will contain a salt shaker.

Draw out a Markov chain for modeling the number of salt shakers in stock during each week. Your model should have four states, corresponding to having zero, one, two, or three shakers in inventory. Note that it's possible to stay in state one or state two – there can be both a delivery and a sale in the same week, or no delivery and no sale. It isn't possible to make a sale if there are zero shakers in stock, nor is it possible to have a delivery if there are already three shakers.

Write down the transition probability matrix and solve for the limiting probabilities. How often will KEIA be out of salt shakers?

M/M/1 with Discouraged Arrivals

Consider an M/M/1 queue where the arrival rate depends on the number of customers. When there are k customers in the system, the arrival rate is $\frac{\lambda}{k+1}$. The exponential service rate is always μ , regardless of the number of customers.

Draw out the Markov chain model for this system and label the transition arcs with the appropriate arrival and service rates. Solve the local balance equations for your model to find an expression for π_k , the long-run probability of having k customers in the queue.