CS 547 Lecture 18: Open Queueing Networks

Daniel Myers

Many systems can be represented as networks of M/G/1 queues with probabilistic routing. These models are *open*, because they receive a stream of arrivals (typically Poisson) from the outside world. Departing jobs exit the system to the outside world and do not return.

To solve an open queueing network:

- determine $\overline{s}$, c_s^2, and the scheduling discipline at each resource
- measure or calculate the expected visit count, $\overline{V}$, at each resource
- determine the overall system arrival rate λ
- use the forced-flow law to calculate the thoughput at each resource
- calculate the utilization of each resource
- calculate the residence time at each resource
- the total residence time is the sum of the resource residence times, weighted by their visit counts

Example

Consider a network with a CPU, a disk, and an SSD that receives Poisson arrivals at rate λ. Each request visits the CPU, makes one I/O request, then exits the system. A fraction p of I/O requests go to the disk, and $1 - p$ go to the SSD.

Using the forced-flow law, the arrival rates through each resource are

- $\lambda_{CPU} = \lambda$
- $\lambda_{disk} = p\lambda$
- $\lambda_{SSD} = (1 - p)\lambda$

The utilizations at each resource are

- $U_{CPU} = \overline{s}_{CPU}\lambda_{CPU}$
- $U_{disk} = \overline{s}_{disk}\lambda_{disk}$
- $U_{SSD} = \overline{s}_{SSD}\lambda_{SSD}$

If we assume each resource can be modeled as an M/G/1 queue, we have the following residence times

$$\overline{R}_{CPU} = \overline{s}_{CPU} + \frac{\overline{s}_{CPU} U_{CPU} (1 + c_{CPU}^2)}{2(1 - U_{CPU})}$$
$$\overline{R}_{disk} = \overline{s}_{disk} + \frac{\overline{s}_{disk} U_{disk} (1 + c_{disk}^2)}{2(1 - U_{disk})}$$
$$\overline{R}_{SSD} = \overline{s}_{SSD} + \frac{\overline{s}_{SSD} U_{SSD} (1 + c_{SSD}^2)}{2(1 - U_{SSD})}$$

The total residence time through the network is the weighted combination of the residence times at each indvidual resource.

$$\overline{R} = \overline{R}_{CPU} + p\overline{R}_{disk} + (1 - p)\overline{R}_{SSD}$$

Other examples with full calculations were shown in class.