CS 547 Lecture 25: Bulk Arrivals

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Some systems produce arrivals in "bulks", where multiple customers arrive simultaneously at each arrival instant. Anlayzing bulk arrivals is a straightforward variation on our basic tagged customer analysis, and serves as a good introduction to customized modeling.

Bulks of Two Customers

Consider an M/M/1 queue where customers always arrive in bulks of 2. Bulk arrivals still follow a Poisson process with rate λ and the average time to serve an individual customer is $\overline{s}$.

Consider a tagged customer arriving in one of these bulks of two. There are three sources of delay the customer can encounter.

- the waiting time due to customers already in the queue (including the one in service)
- the tagged customer's own service time
- the waiting time for customers that arrive in the same bulk as the tagged customer, but enter service before the tagged customer

If customers always arrive in groups of two, then the tagged customer will be second in its group with probability $\frac{1}{2}$. Therefore, the tagged customer can expect to wait $\frac{1}{2}\overline{s}$ due to other customers in its own bulk arrival.

Little's law still applies to this system. If bulks arrive at rate λ, then the total throughput is 2λ customers per unit time. The expected number of customers in the queue is thus

$$\overline{Q} = 2\lambda\overline{R}$$

Combining all of these results, we derive the residence time equation

$$\overline{R} = \overline{Q}\overline{s} + \overline{s} + \frac{\overline{s}}{2}$$

To obtain the final equation, simply apply Little's result to $\overline{Q}$ and solve for $\overline{R}$.

Bulks of k Customers

Now consider the exact same system, but with bulks of k customers at each arrival instant.

If a customer arrives in a bulk of size k, there will be $k-1$ other customers in the bulk. On average, we expect the tagged customer to wait for half of those other customers, so the expected waiting time due to other customers in the same bulk arrival is $\frac{k-1}{2}\overline{s}$.

The residence time equation is now

$$\overline{R} = \overline{Q}\overline{s} + \overline{s} + \frac{k-1}{2}\overline{s}$$

where $\overline{Q} = k\lambda\overline{R}$.

Randomly Sized Bulks

Suppose the size of each bulk is geometrically distributed with average size b and parameter $\frac{1}{b}$.

$$P[\text{a bulk contains } k \text{ customers}] = \left(1 - \frac{1}{b}\right)^{k-1} \frac{1}{b}$$

We can still apply the tagged customer analysis from the previous sections. All we need to do is derive the expected waiting time due to other customers in the bulk. Let $\overline{W}_{bulk}$ denote this value.

A general strategy for problems of this type is to condition on the number of customers in the bulk, determine the average residence time in the conditional case, then use total probability to determine the unconditional average result.

$$\overline{W}_{bulk} = \sum_{k=1}^{\infty} E[\overline{W}_{bulk} \mid \text{arriving in a bulk of size } k] \;\; P[\text{arriving in a bulk of size } k]$$

Suppose we know that a customer arrived in a bulk of size k. Then the expected waiting time due to other customers in the bulk is simply

$$E[\overline{W}_{bulk} \mid \text{arriving in a bulk of size } k] = \frac{k-1}{2} \overline{s}$$

An individual customer is more likely to arrive in a larger bulk, even if large bulks are a small fraction of total arrivals. Therefore, we should use the *mass-weighted distribution* to determine the fraction of arrivals that occur in bulks of size k.[1]

$$P[\text{arriving in a bulk of size } k] = \frac{k}{b} \left(1 - \frac{1}{b}\right)^{k-1} \frac{1}{b}$$

Now, using total probability to combine all the conditional cases

$$\overline{W}_{bulk} = \frac{\overline{s}}{2\,b^2} \sum_{k=1}^{\infty} (k-1)\,k \left(1 - \frac{1}{b}\right)^{k-1}$$

[1]This is exactly the approach we used when reasoning about M/G/1 residual service times. Review those notes for more discussion on the mass-weighted distribution.