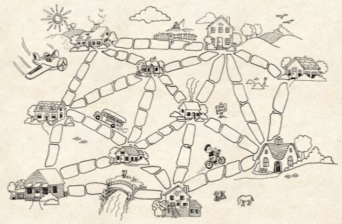
UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

CS 202
Introduction to Computation

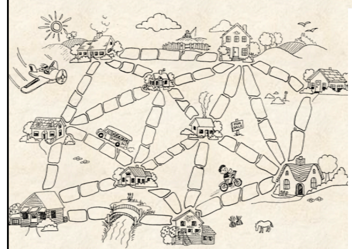
Professor Andrea Arpaci-Dusseau
Fall 2010

Lecture 41: What problems stretch the limits of computation?

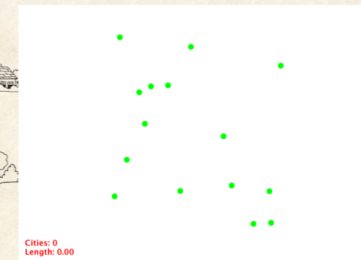


Handout

Create the minimal spanning tree that connects all of the houses



Construct the shortest route for a Traveling Salesperson



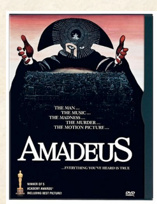
Discussion

Is there an inherent difference between

being brilliant

and

being able to appreciate brilliance?



What is Brilliance?

Ability to find "needle in a haystack"

- Mozart found "satisfying assignments" to our neural circuits for music appreciation
- Relatively easy to identify the fact needle has been found

What is a computational analogue of this phenomenon?

Many hard computation problems require solutions involving "finding a needle in a haystack"

Compare 4 Algorithms

- Path between nodes of graph
- Minimal spanning tree
- Monkey puzzle
- Travelling salesperson

Which ones are easy and which are hard to solve?

Problem 1: Path?

Social network or graph

- Each node represents student
- Two nodes connected by edge if those students are friends

Scenario:

- Kate starts a rumor
- Will it reach Ronak?
- Is there a path or connection between two?

What algorithm could you use?

Problem 1: Path?

Social network or graph

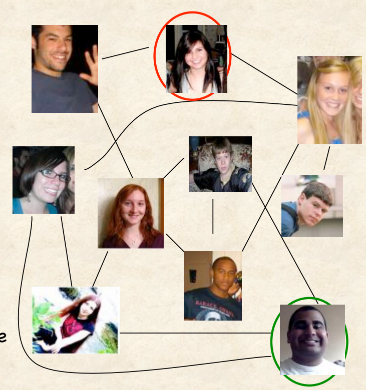
- Each node represents student
- Two nodes connected by edge if those students are friends

Scenario:

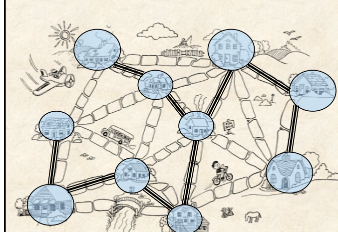
- Julia starts a rumor
- Will it reach Ronak?
- Is there a path or connection between two?

How does running time depend on network size (number of edges, E)?

- Never need to visit an edge more than once
- At most O(E)



Problem 2: Spanning Trees



Goal: Connect all houses (nodes) with shortest path (edges)

- Uses: roads and utilities
- Uses: Wiring chips on circuit boards

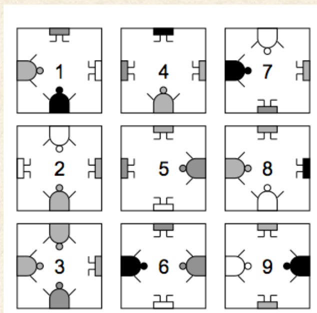
Algorithm?

- Greedy: make step-by-step decisions that work best for current situation
- Begin: Connect closest pair of nodes
- Each step: Connect to next closest
- Don't need to look at different combinations

Problem 3: Monkey Puzzle

M=3, N=9

M=3



Given:

- Set of N square cards with top and bottom halves of colored monkeys
- $N = M^2$
- Cannot rotate cards

Problem:

- Is there an arrangement of cards such that each pair of adjacent cards completes monkey?

Algorithm?

Monkey Puzzle Algorithm

Try every combination of cards
and see if it works

- Try every card for 1st box
- Try each of remaining cards in 2nd box
- Try each of remaining cards in 3rd box…
- Etc…

1st	4th	7th
2nd	5th	8th
3rd	6th	9th

Does a greedy algorithm work?

How many combinations possible?

- 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1
  = 9 Factorial = 9!

Analysis of Monkey Puzzle

For N cards, number of arrangements to examine is N!

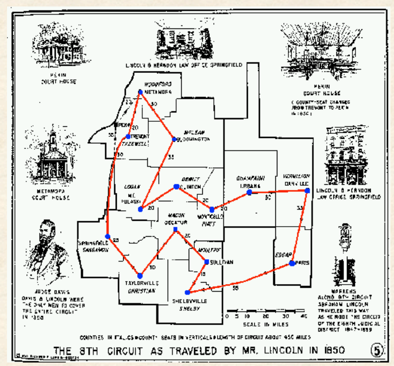
Assume can analyze one arrangement in 1 microsecond

How long to solve for N=9, 16, 25?

N	Time to analyze	
9		
16		
25		

Requires brilliance to solve quickly!

Problem 4: Travelling Salesperson (TSP)



Politicians

Visiting all ball parks in US

Collecting coins from meters

Delivering mail

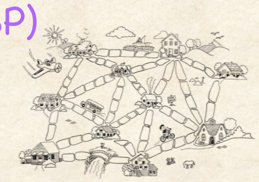
Star imagery

DNA sequencing

Computer networks

Power cables

Problem 4: Traveling Salesperson (TSP)

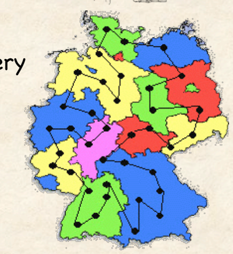


Given:

- Weighted graph of nodes for cities and edges for paths (weight is length)

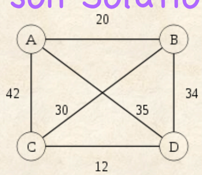
Problem:

- Is there a route thru every city (and back to start) with cost < K?
  - Can't revisit same cities



Algorithm?

Traveling Salesperson Solution



Approach

- Compute cost of every route

Worst-case

- Path connecting every city

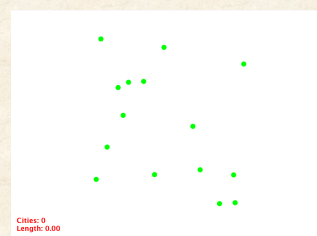
Build every route

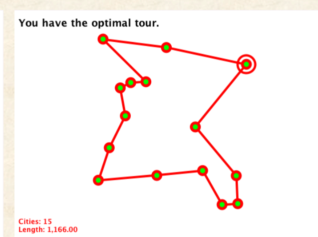
- Pick starting city
- Pick next city (N-1 choices)
- Pick 3rd city (N-2) choices

Number of routes?

- N! (factorial)
- Greedy algorithm will not work here!

Try It Yourself





30 cities is fun!

Common Solution for Problems Requiring "Brilliance"

Exhaustive Search

Naïve algorithms for many "needle in a haystack" tasks involve checking all possible answers

- Combinatorial Explosion
- Exponential running time

Common in many interesting problems

Can we design smarter algorithms?

P vs NP Question

P: Problems for which solutions exist in polynomial time

- cN^k : c and k are fixed integers; N is input size
- $O(1)$, $O(\log N)$, $O(N)$, $O(N \log N)$, $O(N^2)$, $O(N^3)$
- Example: Searching, sorting, Path, Spanning Tree
- Reasonable, tractable

NP: Problems where solution can be ***checked*** in polynomial time

- Examples: Monkey Puzzle, Traveling Salesman
- Current solutions require super-polynomial-time
  - $O(2^N)$, $O(N^N)$, $O(N!)$
- Unreasonable, intractable

Question: Is P = NP?

- "Can we automate brilliance?"
- Computer scientists have not yet proved equal or not equal

NP-complete Problems

Problems in NP that are "the hardest"

- If they are in P then so is every NP problem
- All NP-complete problems essentially equivalent

How do we handle NP-Complete Problems?

1. Heuristics
   - Algorithms that produce reasonable solutions in practice
2. Approximation algorithms
   - Compute provably near-optimal solutions

Today's Summary

P vs NP

- P problems can be solved in polynomial time
  - Example: Minimal spanning tree uses a greedy algorithm to find shortest path connecting all nodes
- NP problems can only be checked in polynomial time
  - Unknown if polynomial-time solutions exist
  - Naïve solutions exhaustively examine all possibilities

Announcements

- Sign up for Project 2 demo if you haven't
  - Must be this week! (Mostly tomorrow: Thursday)
- Final: Dec 22 (Wed) at 10:05 – 12:05 in Psych 113
  - All Multiple Choice, Not cumulative
    - Sorting (Selection, Insertion, Merge, Quicksort) – Basic algorithm and complexity
    - Web services (Networking, Google, cryptography, lying components)
    - Complexity of problems