UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

CS 202
Introduction to Computation

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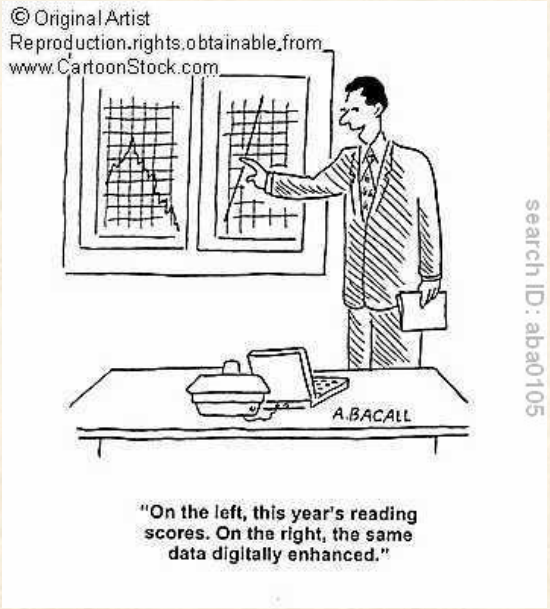
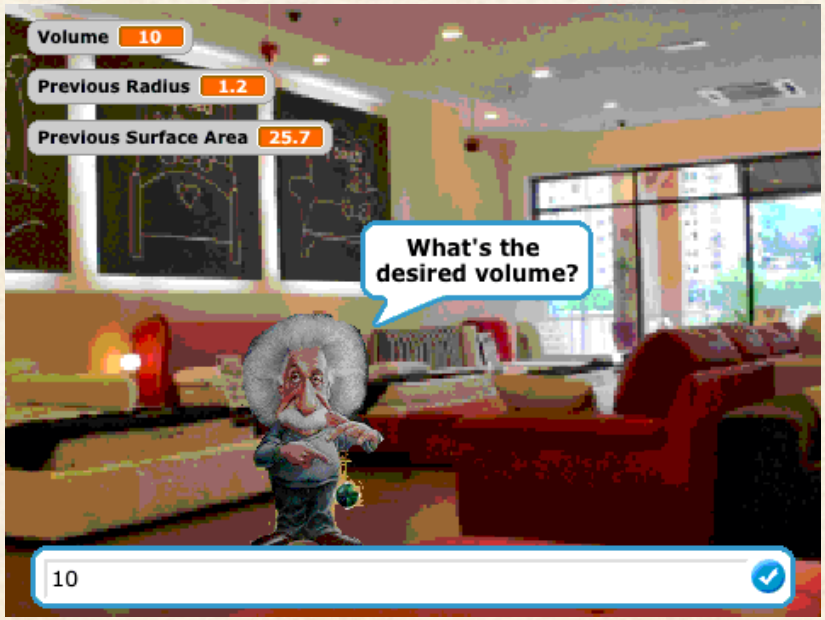
Fall 2010

Lecture 21:
How can computation…
pick best data values?
Or, … turn math into searching?





Data is Important

Every field values their data

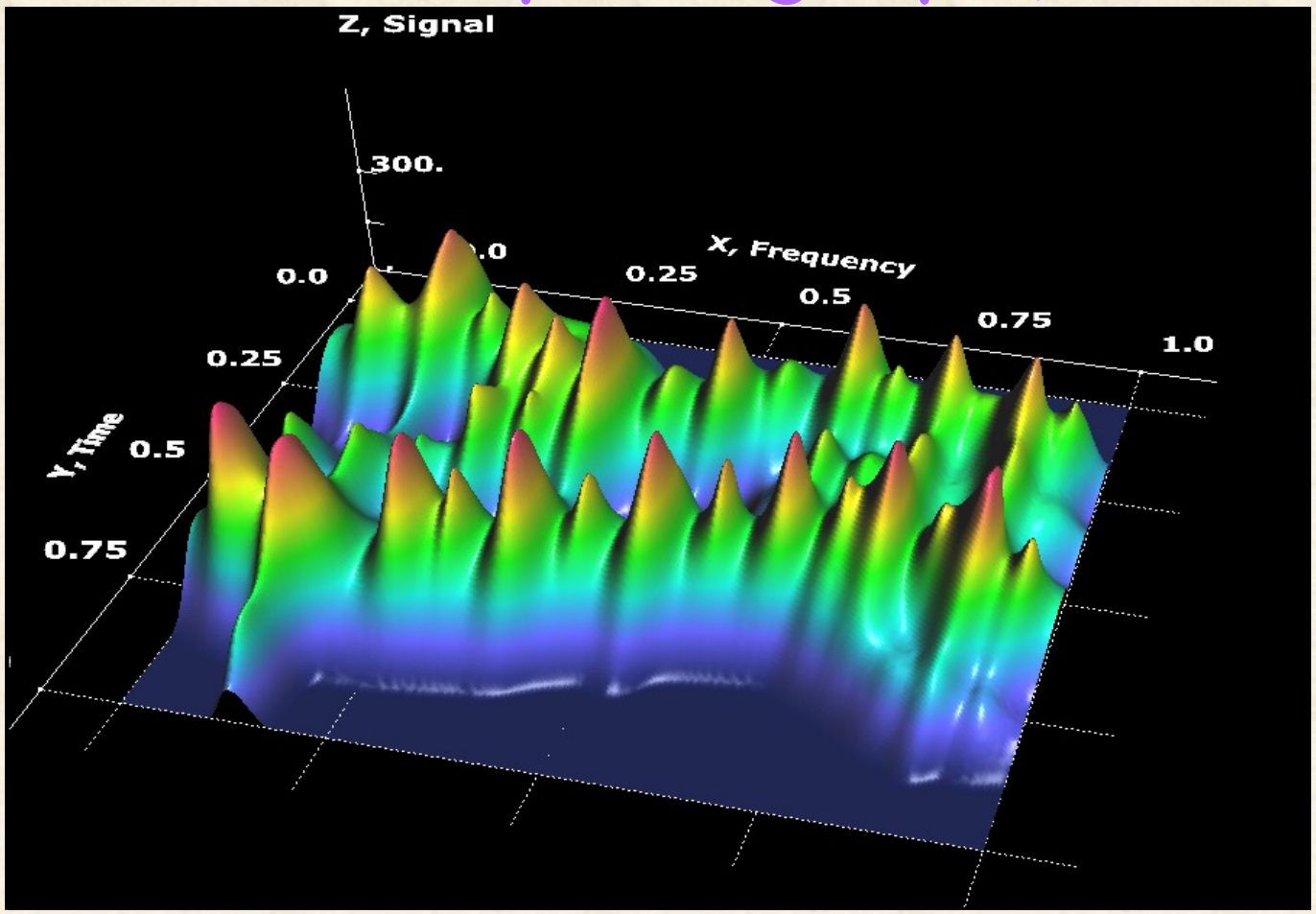
- Science, engineering, medical, business, sports
- Data source: Measurements, simulations, mathematical functions

What does one often want to do with data?

- Pick set of input parameters that lead to best result
  - Which materials at which temps are strongest?
  - Which medical procedure saves most lives at least cost?
  - What stock portfolio gives best profits while minimizing risk?
  - Which pitchers most likely to get next batters out?

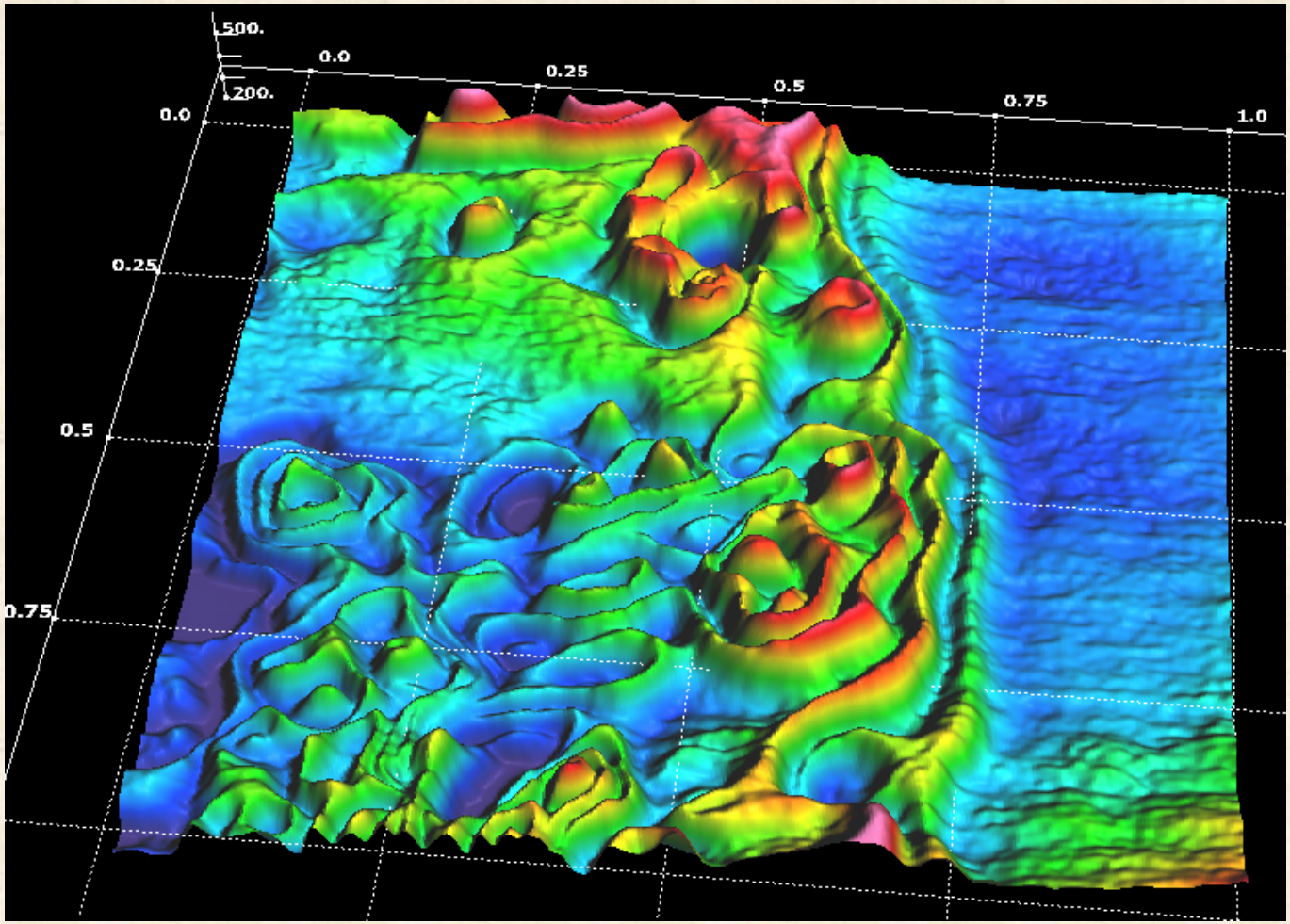
General Question: Which parameter values optimize (maximize or minimize) desired function while meeting some constraints?

Data Example: Electroencephalography (EEG)



Assume two inputs: Time and frequency; Assume Output: Spectral signal
How to find (x, y) that maximizes z??

Data Example: Quantum Hall effect



2D electrons: Topographic map of random potential by SPM microscopy
How to find (x, y) that maximizes z?

Much simplified Scenario: Business Owner

Consider single input variable:

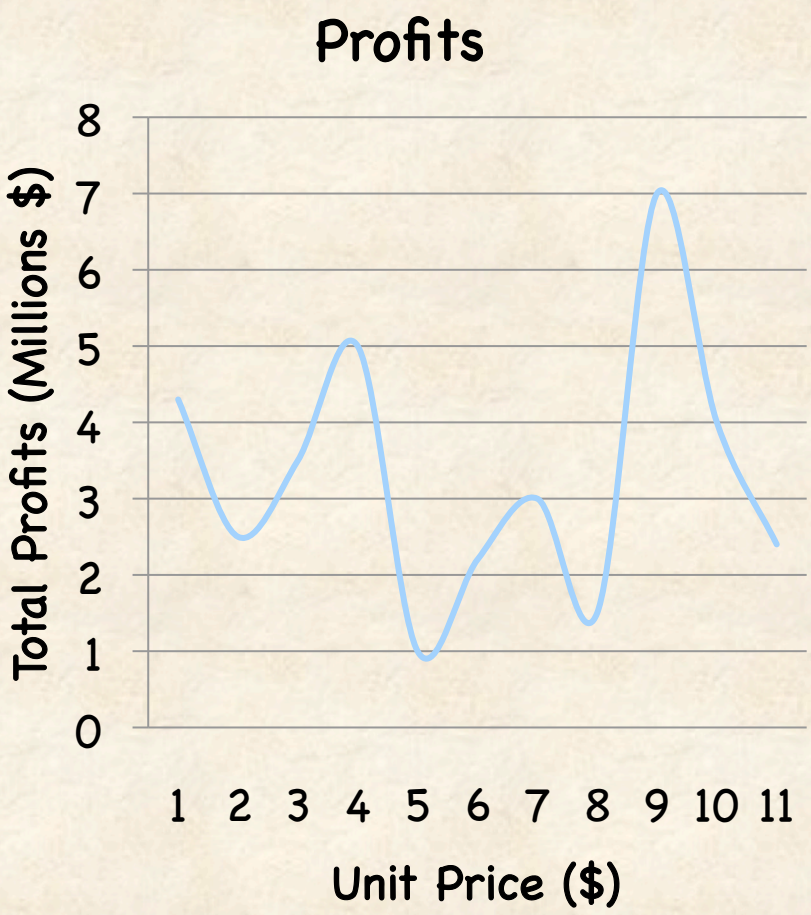
- Unit Price

Imagine: Profit = f(unit price)

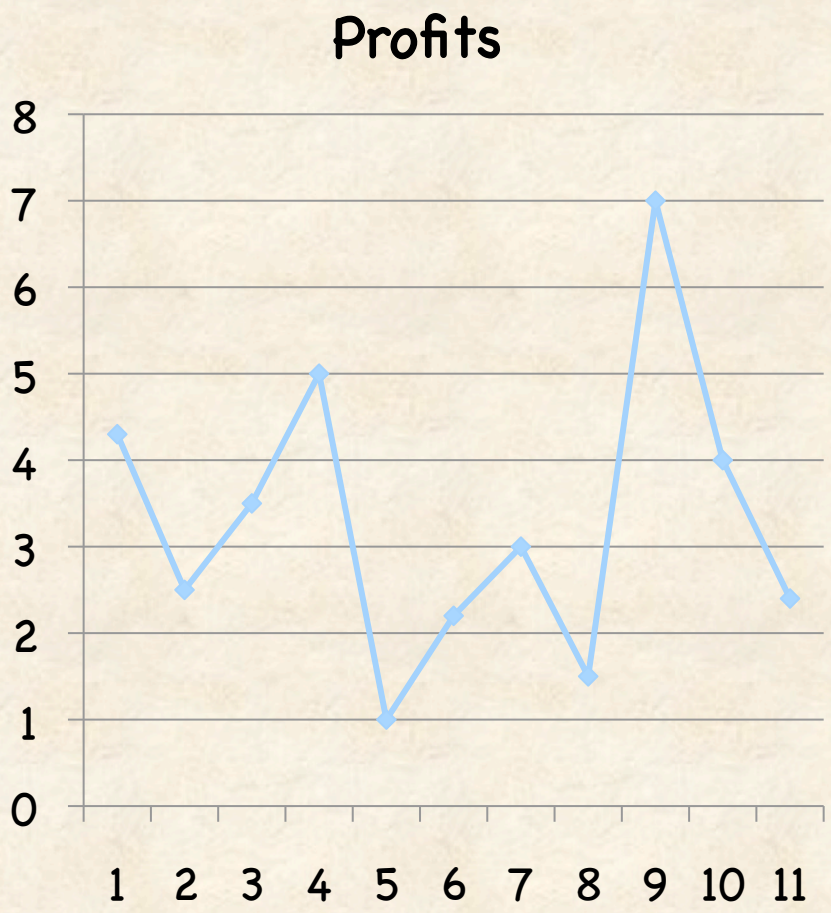
- Someone magically knows f()
- f() might be very complex…

How would you set the price to maximize profit?

How can *you* use computation to find the optimal price?



Naive Optimization Approach



Evaluate set of options and pick best

- Pick some prices (x values) (every $1 thru $11)
- Compute profit = f(price) and store in profit List
- Naïve approach: Search through entire list for maximum
  - Linear or Binary search?

Why isn't this approach always a good idea?

Why doesn't Naïve approach always work?

What if too many data points to test all?

- MB (10^6), GB (10^9), TB (10^{12}) of data on disk

What if very costly to compute f(x)?

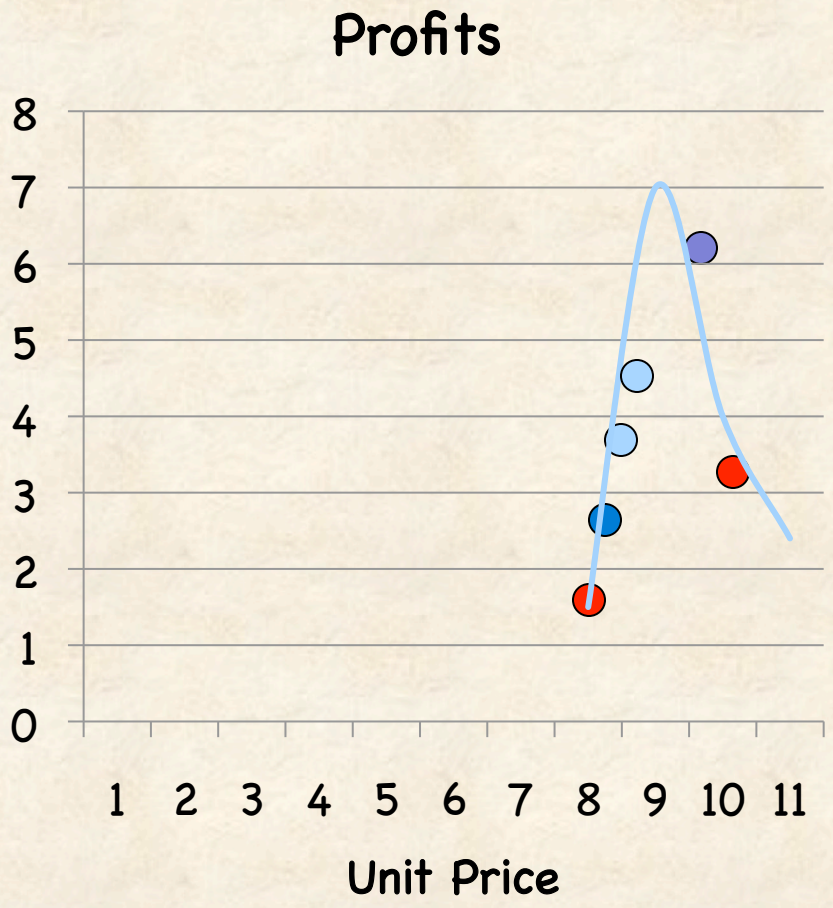
- Computing f(x) could be running a day-long simulation on 1000 processors

Can't explore entire data set

- Must explore only small number of points

Approach: Assume function has particular shape...

Gradient Ascent: Find Max



Pick 1st random point

Pick 2nd point in direction A

Worse! What should we do?

Switch direction

Pick 3rd point in direction B

Better! What should we do?

Keep going

Pick 4th point in dir B

Better! What should we do?

Keep going

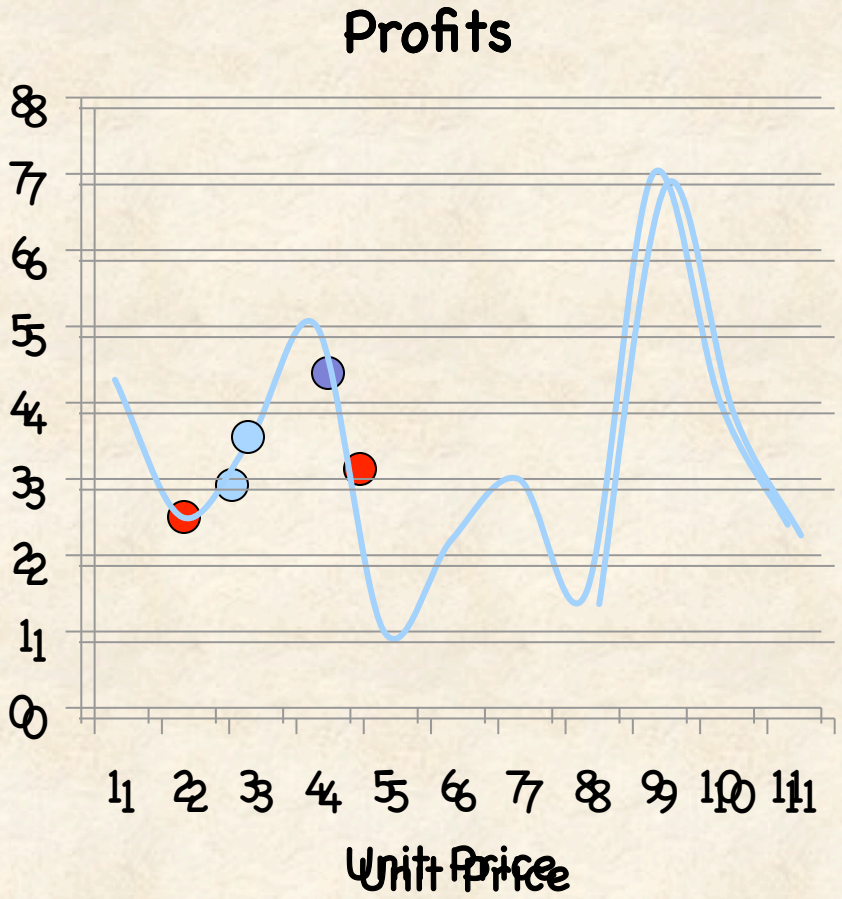
Pick 5th point in dir B

- Worse! What should we do?
- Switch directions

Pick 6th point

Better! Stop, good enough...

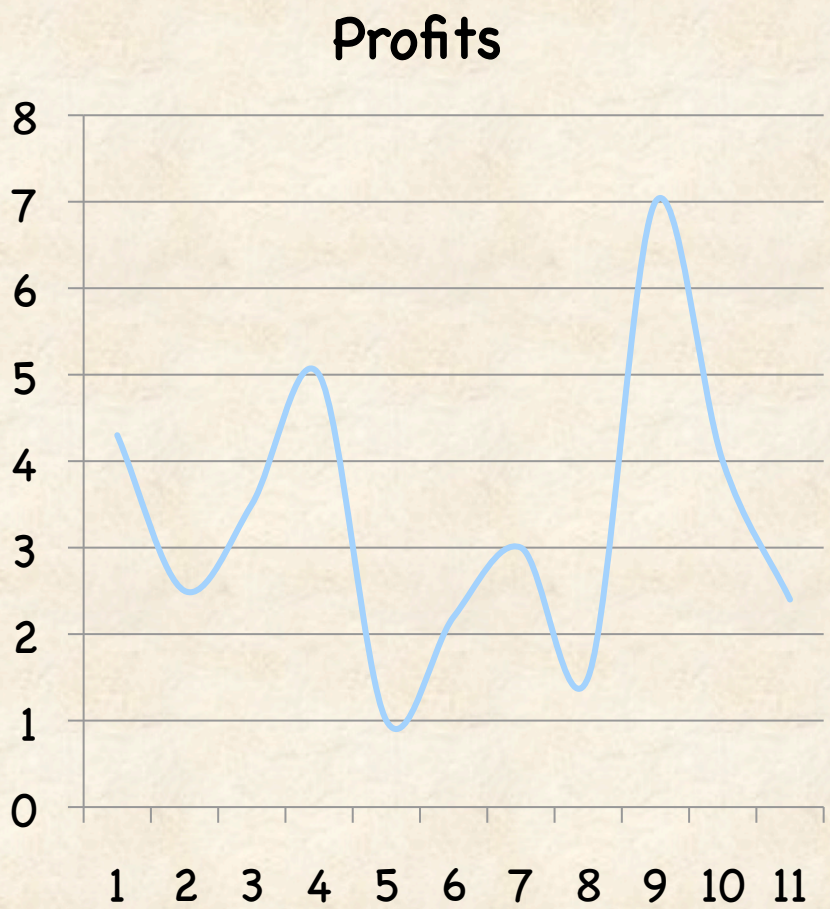
What Can Go Wrong?



Can find local maxima instead of global!

How could you improve algorithm?

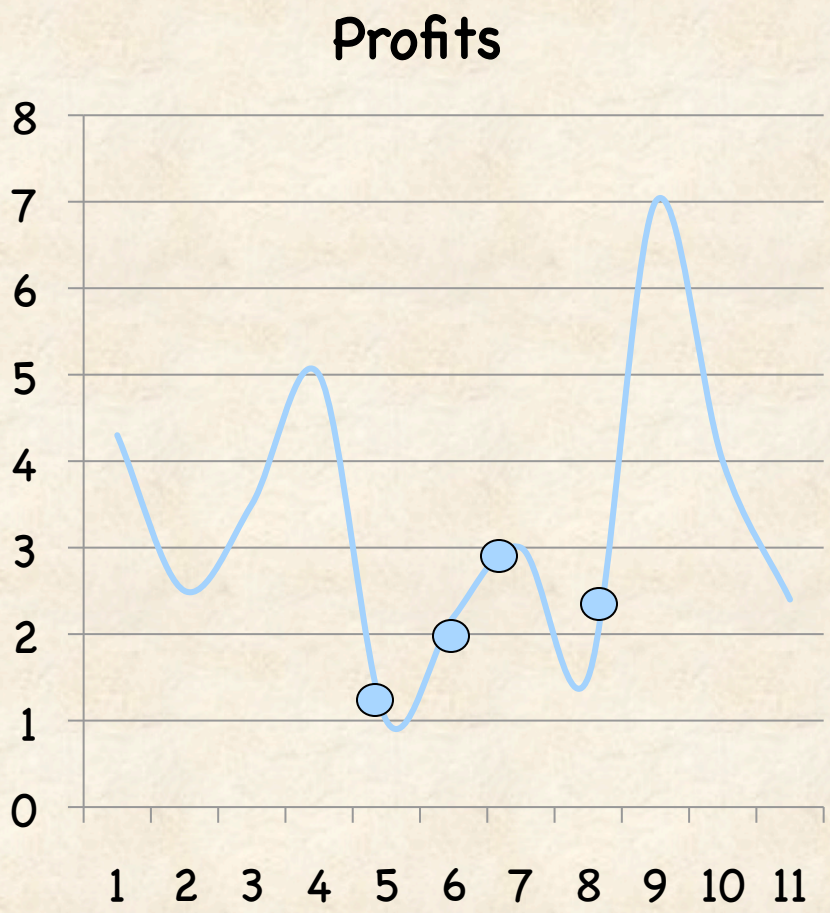
What Improvements Could *You* Make?



Approach #1: Try multiple times

- Different random starts
- Keep best result

What Improvements Could *You* Make?



Approach #2:

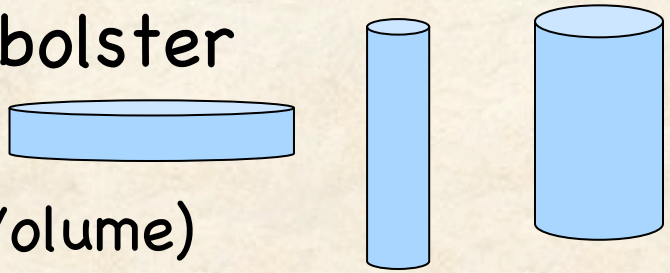
"Simulated annealing"

- Gradient ascent, but sometimes continue in same direction when see worse results
- Pick worse point with some probability p
- Gradually decrease p over time

Implementing Optimization in Scratch

Scenario: You own a business upholstering cushions

Customer wants you to cover a bolster (cylindrical cushion) for $30



- Must contain $10m^3$ of stuffing (Volume)
- Don't care about the dimensions (radius or height)

Your costs include material

- Less material you use, the more profit you'll make…
- Goal: Minimize Surface Area

Useful formulas:

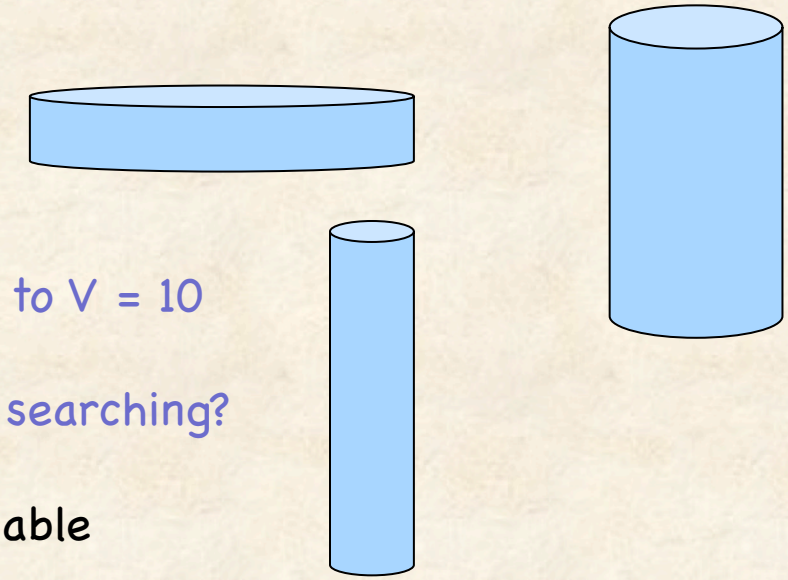
$$V = \pi \cdot r^2 \cdot h$$

$$SA = 2\pi r^2 + 2\pi rh$$

How to Solve Bolster Problem?

$$V = \pi \cdot r^2 \cdot h$$

$$SA = 2\pi r^2 + 2\pi rh$$



Need to find values of r and h that lead to V = 10 and minimize SA

How can you solve with computation? By searching?

- Vary one variable
- Compute SA as function of that variable
- Pick value that gives minimal SA

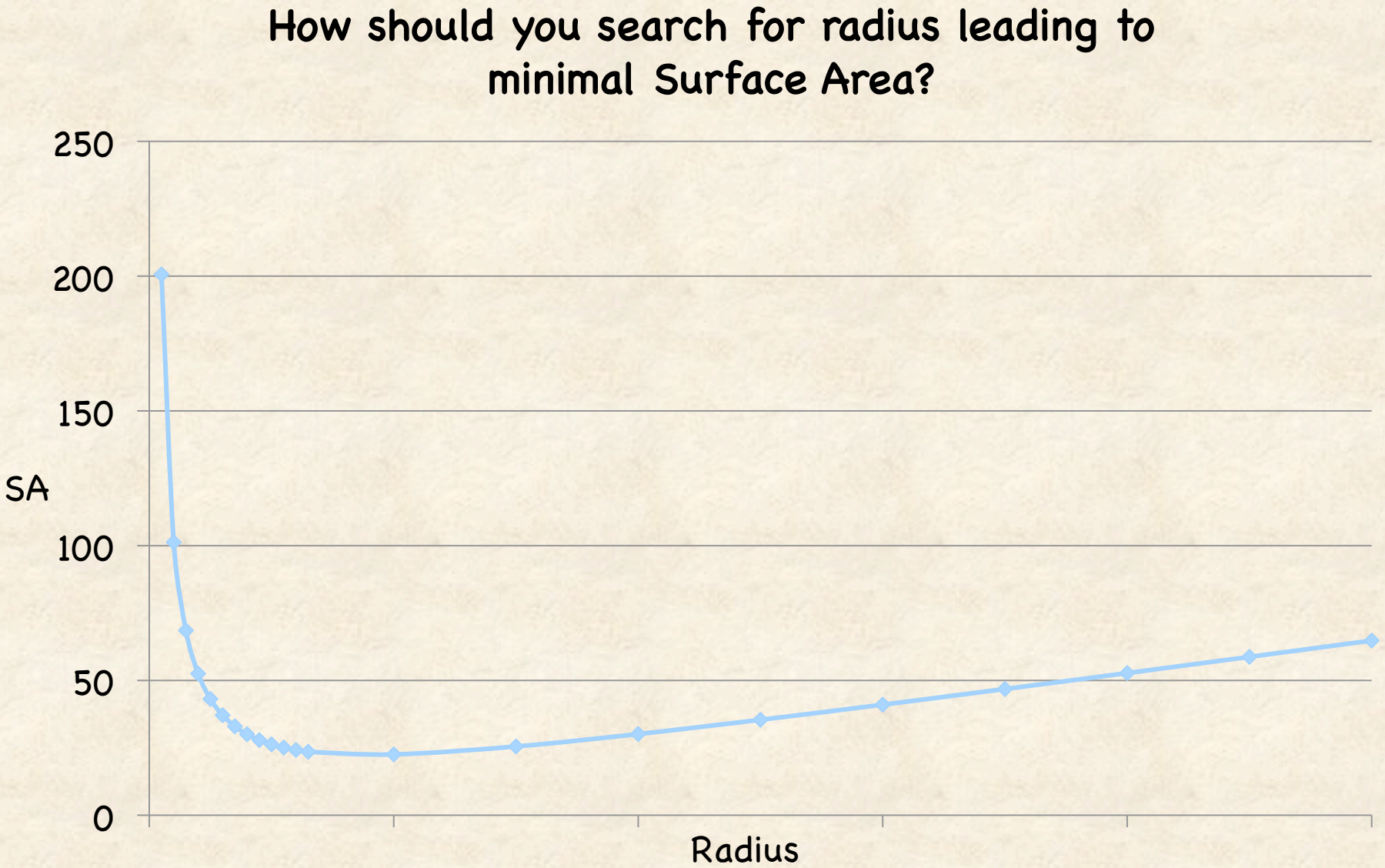
What is our one variable?

- Specify h in terms of r (given fixed V)
- Substitute for h in SA equation

$$h = \frac{V}{\pi r^2}$$

$$SA = 2\pi r^2 + \frac{2V}{r}$$

What does function look like?



Questions to Ask and Answer

What variable are we varying?

- Radius

What variable are we trying to minimize?

- Surface area

How do we know the value of the surface area?

$$SA = 2\pi r^2 + \frac{2V}{r}$$

How should we initialize radius?

- Don't know
- Ask the user!

How much should we increase it on each iteration?

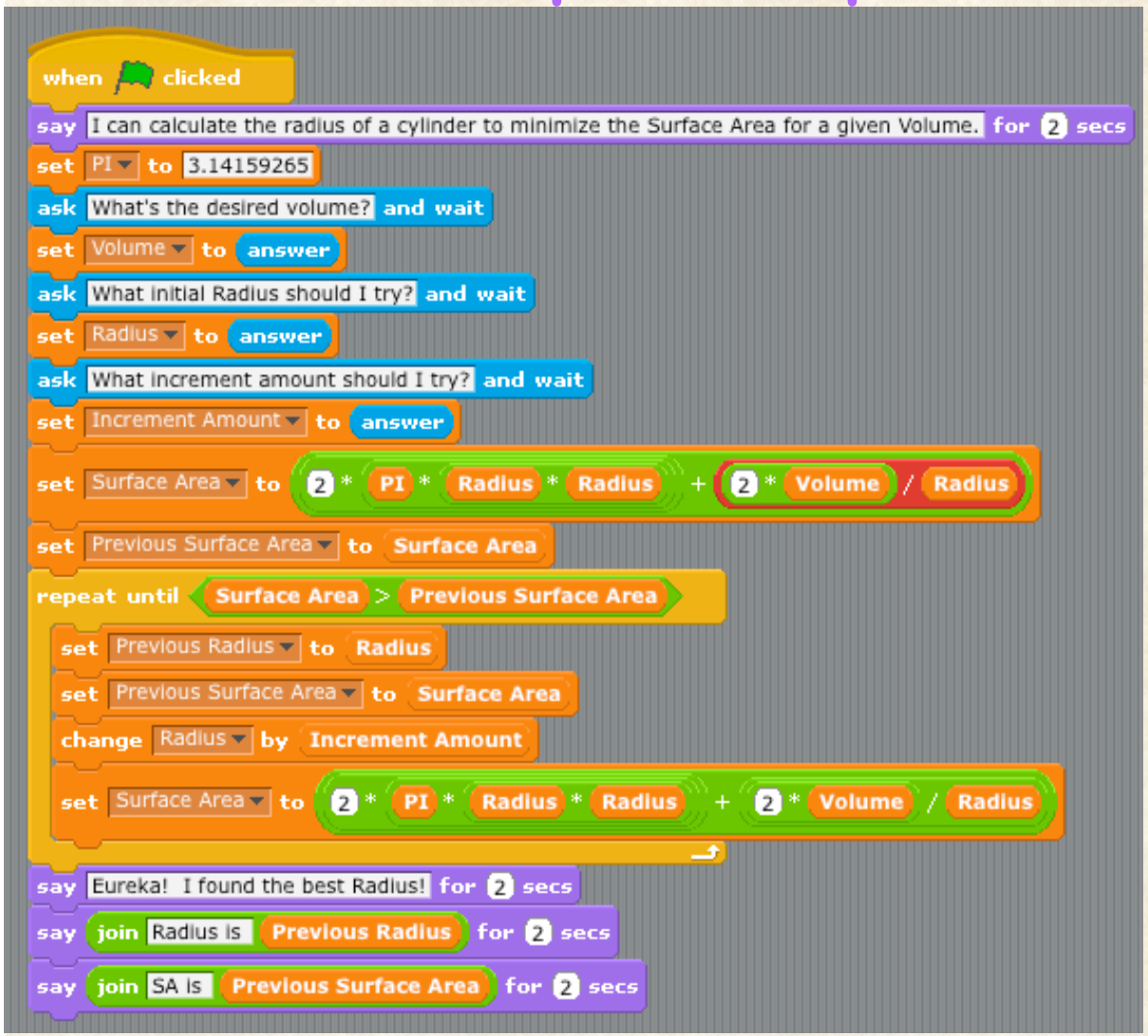
- Don't know, so ask the user!

How do we know we've found the minimum?

- SA for new radius > SA for prev radius
- Previous radius is the minimum

Simple Implementation

$$SA = 2\pi r^2 + \frac{2V}{r}$$



Keep trying larger values of Radius

until SA > prev SA

For V= 10m^3

Best radius = 1.2m

Leads to SA of 25.71m^2

Height = 2.21m

Code structure similar for any optimization problem

- Replace f(radius)

Today's Summary

Optimization

- Many engineers and scientists use computation for optimization
- How to use computation find parameters leading to best (max, or min) result
- Simplest: Search through parameter space linearly, stop at max/min; more sophisticated techniques to find global optimum

Announcements

- Programming Project 1: Draft due today at 5pm in Scratch Gallery