UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

CS 202
Introduction to Computation

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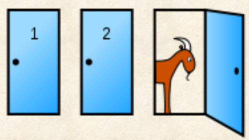
Fall 2010

Lecture 28:
How can computation…
guess what will usually happen?



Motivating Exercise: Monty Hall Problem

Suppose you're on a game show

You're given choice of prize behind 1 of 3 closed doors:

- Behind one door is a car
- Behind the other two doors are goats.

You pick a door, say Number 1. The host, who knows what's behind the doors, opens another door, say Number 3, which has a goat.

He asks, "Do you want to switch to door Number 2?"

Should you switch your choice???

Official (Non-ambiguous) Phrasing

Suppose you're on a game show and you're given the choice of three doors. Behind one door is a car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show.

The rules of the game show are as follows:

After you've chosen a door, the door remains closed for the time being.

The game show host, Monty Hall, who knows what is behind the doors, now must open one of the two remaining doors, and the door he opens must have a goat behind it.

If both remaining doors have goats behind them, he chooses one randomly.

After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door.

Imagine that you chose Door 1 and the host opens Door 3, which has a goat.

He then asks you "Do you want to switch to Door Number 2?"

Is it to your advantage to change your choice?

What is your probability of winning if you don't switch? If you do switch?

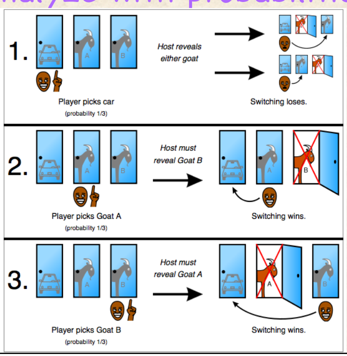
Three Approaches to Solving Monty Hall Problem

1. Analyze with probabilities
2. Play game many times with people
3. Simulate with computation

Option 1:
Analyze with probabilities

Chance of winning if keep choice?

1/3 chance of winning



Chance of winning if switch choice?

2/3 chance of winning

Option 2:
Play Game with People

Two people participate, alternate roles

- Contestant
- Game show host

Simulate behavior of game multiple times

Each contestant tries both strategies 10 times

- Keep vs. switch
- Record number of times win vs lose w/ each strategy

	Keep	Switch
Win Tally		
Lose Tally		

Think about algorithm both contestant and host are using

Record Success Rate

	Don't Switch	Switch
Win Tally	3	6
Lose Tally	7	4

	Don't Switch	Switch
Win Tally	4	8
Game Tally	6	2

Why not best to use people?

Slow to do many trials

People bad at picking random numbers

People give inadvertent clues

Cards might look different -> can guess car

Option 3: Computer Simulation

As perform more trials, by law of large numbers, percentage win approximates probability of winning

Repeat Many Trials (Strategy == Switch or Keep)

- Car placed behind random door
- Contestant picks random door
- If (Contestant door == Car door)
  - Monty opens 1 of 2 other doors at random
- Else (Contestant did not pick car door)
  - Monty opens goat door
- If (Strategy == Switch)
  - Contestant switches choice to closed door
- If (Contestant door == Car door)
  - Increment Win Tally

Monty Hall: Scratch Program

6 trials: expect to win how many times?

- If Strategy == Keep?
  - 2 times
- If Strategy == Switch?
  - 4 times
- What actually happens?

100 trials: expect to win how many times?

- If Strategy == Keep?
  - 33 times
- If Strategy == Switch?
  - 67 times
- What actually happens?

Monty Hall: Better Intuition?

Easier to understand when scale number of doors

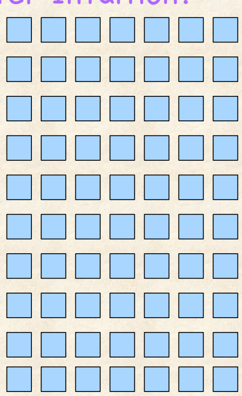
Imagine 70 doors

You pick 1 (purple)

Monte Hall opens 68 doors revealing goats (black)

Do you switch or not?

Do you think car is behind 1 you originally picked or 1 he is not showing you??

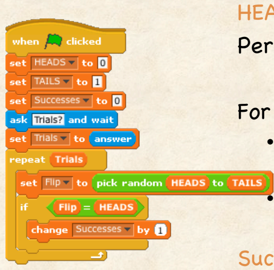


Probability Simulations in other Domains

Any game of chance: cards, dice, coin flips, luck-based board games

How to measure probability of getting heads or tails?

Coin Flips: Version 1



HEADS and TAILS: Constants

Perform multiple Trials

For each trial:

- Generate Data
- Evaluate Success

Success/Trials approximates probability with many trials

Coin Flips: Version 2

What is probability of getting all heads?

- As a function of the number of flips?

Probability of N=4 heads in a row?

- HHHH = ½ * ½ * ½* ½ = 1/16
- Probability = $1/2^N$

Coin Flips: Version 2

HEADS and TAILS: Constants

Perform multiple Trials

For each trial:

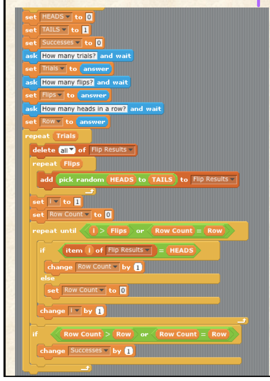
- Generate Data
  - List of coin flips!
- Evaluate Success
  - Count number of HEADS in list
  - Success if all HEADS

Coin Flips: Version 3

What is the probability of obtaining a continuous string of N heads in M coin flips?

- N = 3, M = 4
- Success? HHTH
- Success? HHHT

Coin Flips: Version 3



HEADS and TAILS: Constants

Perform multiple Trials

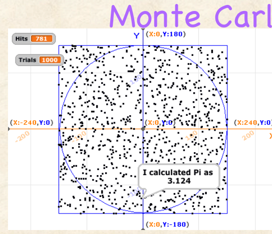
Perform M random coin Flips

Measure often get N (Row) heads in a row

- Row Count shows how many
- Reset to 0 on TAILS

Success percentage approximates probability with many trials

Calculation of Pi: Monte Carlo Simulation



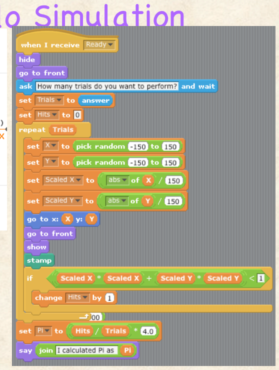
Calculate pi given ratio of samples falling in unit circle vs square

Circle area: πr^2

Square area: $2r*2r = 4r^2$

Hits / Trials = πr^2 / $4r^2$

Π = Hits / Trials * 4



Sport Simulations

Vary win rate (k%) for games

Repeat large number of trials

Pick random game winner with given probability

See who wins set

Today's Summary

Today's topic

- Computation can be use to simulate behavior of simple systems with random component
- Measured successes after many trials approximates probability

Announcements

- Homework 6 late turn in: 5pm today (lab 12-2)
- Homework 7 will be available today (pencil and paper)