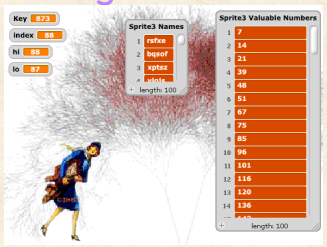
UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

CS 202
Introduction to Computation

Professor Andrea Arpaci-Dusseau
Fall 2010

Lecture 20: How can computation… find what you're looking for - faster?



Motivating Exercise

Play 20 questions in pairs

- Repeat few times
  - Person A thinks of a number between 1 and 100
  - Person B guesses number with YES/NO questions
  - Record how many guesses needed
- Switch roles and play few times more

Repeat with numbers between 1 and 1000

- Handout sheet of number grid may be useful
- Cross off guesses or numbers that secret can't be

How many guesses on average did it take you?

- What algorithm works well?

Use Number Grid to Track Guesses

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420
421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440
441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460
461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480
481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500
501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560
561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580
581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600
601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620
621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640
641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660
661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680
681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700
701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720
721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740
741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760
761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780
781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800
801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820
821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840
841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860
861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880
881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900
901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920
921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940
941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960
961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980
981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000

Best algorithm for searching?

BINARY SEARCH

Guess number midway between "lo" and "hi"

(lo starts out at 1, hi at 1000, midway = 500)

Ask "Lower than this midway number?"

If Yes then

- Set hi = midway - 1
- Guess number ½ btwn lo and hi (< 250?)

ELSE

- Set lo = midway
- Guess number ½ btwn lo and hi (< 750?)

Repeat

Play guessing game again with 1000 numbers – should need 10 or fewer guesses!

How would you implement Binary Search for Key?

Exercise Guessing Game:

What was the secret to be guessed?

Integer between 1 and 1000 partner was thinking of

Binary Search for Specified Key:

What is the secret to be guessed?

Secret is index in List holding key we are looking for

Different Assumptions for Linear vs. Binary Search?

Binary search assumes list is sorted!

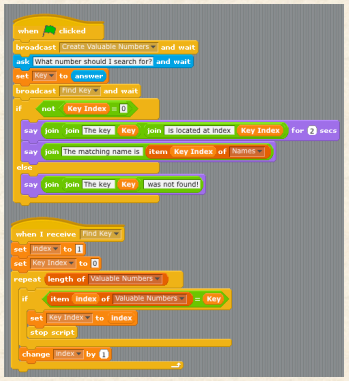
Data organization very important

- Can you think of other organization techniques for helping people look up data?

Trade-off: Should application pay cost to sort data or not?

- Will look at sorting later…

Review: Linear Search



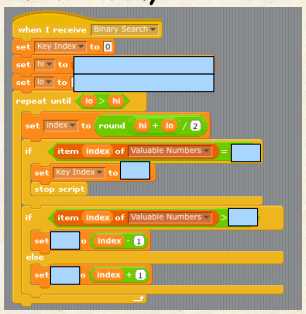
Algorithm checks every element in list (in order) to see if it is the one…

Variables

- Key: Input – What we are searching for
- Key Index: Output – Index where we found Key
- index: local variable

Binary Search in Scratch

Use index to skip around List efficiently



Invariant (condition always holds true)

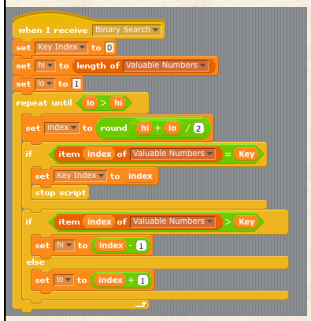
lo <= Index of Secret key <= hi

True before loop begins

True every time after

Running Binary Search: Ex 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	15	28	31	32	33	49	60	62	68	69	74	81	85	96	97

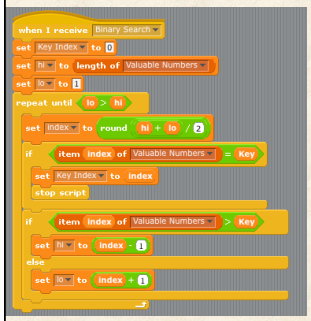


Looking for key 85

Loop	index	Item	Item > Key?	Lo	Hi
Initial					
1					
2					
3					
4					

Running Binary Search: Ex 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	15	28	31	32	33	49	60	62	68	69	74	81	85	96	97

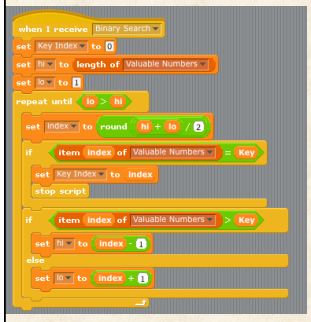


Looking for key 33

Loop	index	Item	Item > Key?	Lo	Hi
Initial					
1					
2					
3					
4					

Running Binary Search: Ex 3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	15	28	31	32	33	49	60	62	68	69	74	81	85	96	97

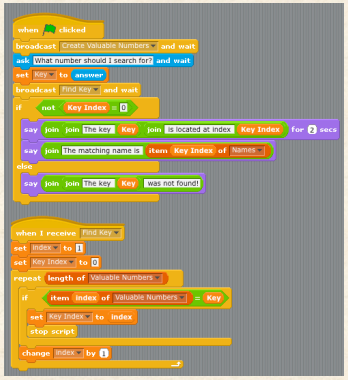


Looking for key 34

Loop	index	Item	Item > Key?	Lo	Hi
Initial					
1					
2					
3					
4					

Not found! Key Index still 0

How many guesses to find Key with Linear Search?



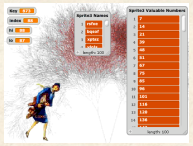
How many loops?

N = Elements in List

- Best case (minimum)?
  - 1 loop!
- Worst case (maximum)?
  - N loops
- Average case?
  - N/2 loops
  - O(N)
  - Just like Find Max

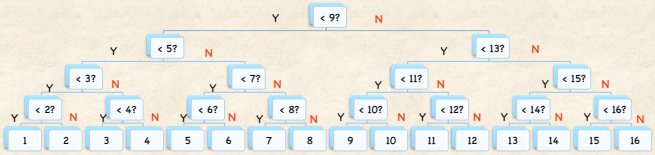
How many Guesses Needed?

N	16	32	64	128	256	512	1024	2048	4096
Linear									
Binary									



Decision Tree for 1..16

How many questions needed to find answer between 1 and 16 (16 numbers)?

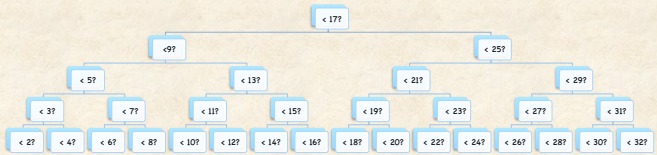


Only 4 questions for 16 numbers

Decision Tree for 1..32

How many questions needed to find answer between 1 and 32 (32 numbers)?

Double range of numbers covered



Just one more question --> 5 questions

Decision Tree for 1..64

How many questions needed to find answer between 1 and 64 (64 numbers)?

Double again range of numbers covered

6 questions

Decision Tree for N items

How many questions needed for N integers?

Each level of tree corresponds to one question
How deep (or high) is tree of N integers?

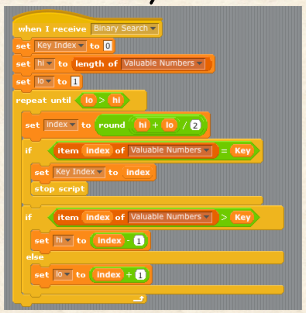
1 question -> 2 numbers
2 questions -> 4 numbers
3 questions -> 8 numbers
4 questions → 16 numbers
5 questions -> 32
6 questions -> 64
Q questions -> ??
2^Q numbers

How to go from N to Q?

$Q = \log_2 N$

Binary Search in Scratch

Use index to skip around List efficiently



How many iterations?

Worst case:
Operations: O ($\log_2$ N)

Modification to decision tree
(2 if's per loop)
Stops early if item at index = key

20 Questions

How many objects can you choose between with 20 questions?

2^20
This is approximately 1 million objects! **(1,048,576)**

How to be a good player of 20 questions?

Today's Summary

Today's Topics

- How to efficiently search for element in a List
- O(N) guesses to find using Linear Search
- O(Log_2 N) guesses to find using Binary Search (depth of tree)
  - Assumes data is sorted!

Reading:

- Invitation pp 55-66 and 80-88 (Searching and complexity)

Announcements

- Exam 1 Returned Friday
- Project 1 Draft uploaded to Website Gallery by Friday 5pm
  - Comment on others by Monday at 5pm