UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

CS 202: Introduction to Computation

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How can computation… guess what will usually happen?

Motivating Exercise: Monty Hall Problem

Suppose you're on a game show

You're given choice of prize behind 1 of 3 closed doors:

- Behind one door is a car
- Behind the other two doors are goats.

You pick a door, say Number 1. The host, who knows what's behind the doors, opens another door, say Number 3, which has a goat.

He asks, "Do you want to switch to door Number 2?"

Should you switch your choice???

Three Approaches to Solving Monty Hall Problem

1. Analyze with probabilities
2. Play game many times with people
3. Simulate with computation

Official (Non-ambiguous) Phrasing

Suppose you're on a game show and you're given the choice of three doors. Behind one door is a car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show.

The rules of the game show are as follows:

After you've chosen a door, the door remains closed for the time being.

The game show host, Monty Hall, who knows what is behind the doors, now must open one of the two remaining doors, and the door he opens must have a goat behind it.

If both remaining doors have goats behind them, he chooses one randomly.

After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door.

Imagine that you chose Door 1 and the host opens Door 3, which has a goat.

He then asks you "Do you want to switch to Door Number 2?"

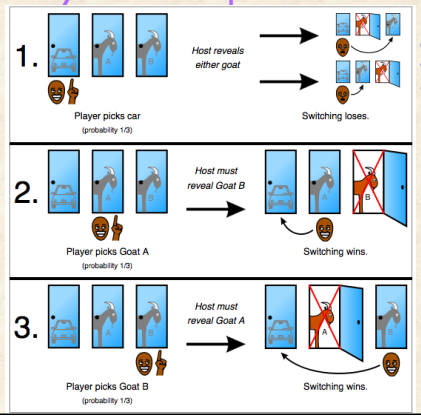
Is it to your advantage to change your choice?

What is your probability of winning if you don't switch? If you do switch?

Option 1: Analyze with probabilities

Chance of winning if keep choice?

1/3 chance of winning



Chance of winning if switch choice?

2/3 chance of winning

Monty Hall: Better Intuition?

Easier to understand when scale number of doors

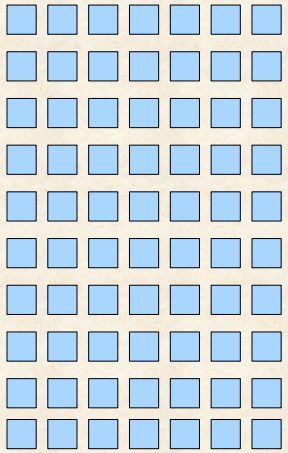
Imagine 70 closed doors

You pick 1 door (purple)

Monte Hall opens 68 doors revealing goats (black)

Do you switch or not?

Do you think car is behind 1 you originally picked or 1 he is not showing you??

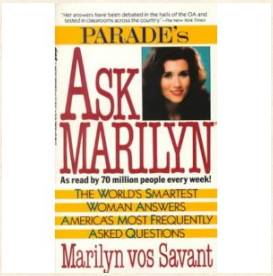


Intuition can lead you astray…

Ask Marilyn column:

10000 people tell her this correct solution was wrong

1000 people with PhDs!



Option 2: Play Game with People

Two people participate, alternate roles

- Contestant
- Game show host – Draw 2 goats, car on cards

Simulate behavior of game multiple times

Each contestant tries both strategies 10 times

- Keep vs. switch
- Record number of times win vs lose w/ each strategy

	Keep	Switch
Win Tally		
Lose Tally		

Think about algorithm both contestant and host are using

Record Success Rate

	Don't Switch	Switch
Win Tally	3	6
Lose Tally	7	4

	Don't Switch	Switch
Win Tally	4	8
Game Tally	6	2

Why not best to use people?

Slow to do many trials

People bad at picking random numbers

People give inadvertent clues

Cards might look different -> can guess car

Option 3: Computer Simulation

Strategy = Switch or Keep

Repeat Many Trials

- Car placed behind random door; goats behind others
- Contestant picks random door
- If (Contestant door == Car door)
  - Monty opens 1 of 2 other doors at random
- Else (Contestant did not pick car door)
  - Monty opens goat door
- If (Strategy == Switch)
  - Contestant switches choice to closed door
- If (Contestant door == Car door)
  - Increment Win Tally

Monty Hall: Scratch Program

6 trials: expect to win how many times?

- If Strategy == Keep?
  - 2 times
- If Strategy == Switch?
  - 4 times
- What actually happens?

100 trials: expect to win how many times?

- If Strategy == Keep?
  - 33 times
- If Strategy == Switch?
  - 67 times
- What actually happens?

Probability Simulations in other Domains

Any game of chance: cards, dice, coin flips, luck-based board games

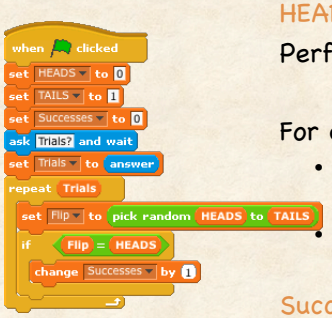
- With more trials, by law of large numbers, win percentage approximates probability of winning

All probability simulations have similar structure:

- Hold some number of trials
- Generate data (using random numbers in some way)
- Evaluate success

Example: How to measure probability of getting heads or tails?

Coin Flips: Version 1



HEADS and TAILS: Constants

Perform multiple Trials

For each trial:

- Generate Data
- Evaluate Success

Success/Trials approximates probability with many trials

Coin Flips: Version 2

What is probability of getting all heads?

- As a function of the number of flips?

Probability of N=4 heads in a row?

- HHHH = ½ * ½ * ½* ½ = 1/16
- Probability = $1/2^N$

How to extend previous probability simulation?

Coin Flips: Version 2

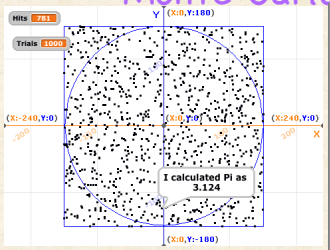
HEADS and TAILS: Constants

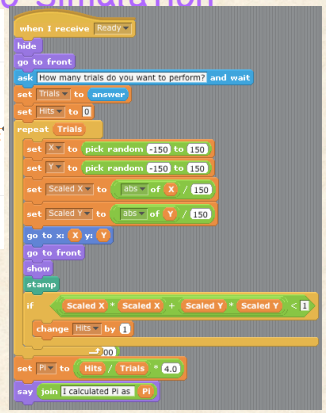
Perform multiple Trials

For each trial:

- Generate Data
  - Flip
  - Count number of heads
- Evaluate Success
  - Success if all HEADS

Calculation of Pi: Monte Carlo Simulation





Calculate pi given ratio of samples falling in unit circle vs square
Circle area: πr^2
Square area: $2r*2r = 4r^2$
Hits / Trials = $\pi r^2 / 4r^2$
Π = Hits / Trials * 4

Check-Up

In probability simulation, what 3 things must one do?

- Hold trials
- Generate data
- Evaluate success

What is missing in script?

- set Hits to 0
- repeat Trials times
- change Hits by 1

Announcements

Exam 1 – One week from today (in lecture)

- Closed notes
- Quantitative, objective answers
- Do you understand? (Not: have you memorized?)
  - Ex: Walk thru scripts w/ variables (or, scripts equivalent?)
- Questions similar to Check-Up's in Lecture Notes
- Practice exam on Friday

No Homework until after Exam

Extra credit for Homework 4

- 1 point for voting in all rounds
- 1 point for "winning"