Using Resolution for Inference in First-Order Logic

1. Problem Statement

Tony, Mike and Ellen belong to the Alpine Club. Every member of the Alpine Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.

Query: Is there a member of the Alpine Club who is a mountain climber but not a skier?

2. Translation into PC Wffs

Let S(x) mean x is a skier, M(x) mean x is a mountain climber, and L(x,y) mean x likes y, where the domain of the first variable is Alpine Club members, and the domain of the second variable is snow and rain. We can now translate the above sentences into the following wffs:

 $\forall x, S(x) \lor M(x)$ $\neg \exists x, M(x) \land L(x, Rain)$ $\forall x, S(x) \supset L(x, Snow)$ $\forall y, L(Mike, y) \leftrightarrow \neg L(Tony, y)$ L(Tony, Rain)L(Tony, Snow)

Query: $\exists x, M(x) \land \neg S(x)$

3. Conversion to Clause Form

$$\begin{split} S(x_1) \lor M(x_1) \\ \neg M(x_2) \lor \neg L(x_2, Rain) \\ \neg S(x_3) \lor L(x_3, Snow) \\ \neg L(Tony, x_4) \lor \neg L(Mike, x_4), \ L(Tony, x_5) \lor L(Mike, x_5) \\ L(Tony, Rain) \end{split}$$

L(Tony, Snow)

Negation of the Query: $\neg M(x_7) \lor S(x_7)$

4. Construction of Resolution Refutation Tree

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$$\neg M(x_{7}) \lor S(x_{7})$$

$$S(x_{1}) \lor M(x_{1})$$

$$\sigma = \{x_{1} / x_{7}\}$$

$$S(x_{1})$$

$$\neg S(x_{3}) \lor L(x_{3}, Snow)$$

$$\sigma = \{x_{1} / x_{3}\}$$

$$L(x_{1}, Snow)$$

$$\neg L(Tony, x_{4}) \lor \neg L(Mike, x_{4})$$

$$\sigma = \{Snow / x_{4}, Mike / x_{1}\}$$

$$\neg L(Tony, Snow)$$

$$L(Tony, Snow)$$

5. Construction of Answer Extraction Tree

$$\neg M(x_{7}) \lor S(x_{7}) \lor (M(x_{7}) \land \neg S(x_{7}))$$

$$S(x_{1}) \lor M(x_{1})$$

$$\sigma = \{x_{1}/x_{7}\}$$

$$S(x_{1}) \lor (M(x_{1}) \land \neg S(x_{1}))$$

$$\neg S(x_{3}) \lor L(x_{3}, Snow)$$

$$\sigma = \{x_{1}/x_{3}\}$$

$$L(x_{1}, Snow) \lor (M(x_{1}) \land \neg S(x_{1}))$$

$$\neg L(Tony, x_{4}) \lor \neg L(Mike, x_{4})$$

$$\sigma = \{Snow/x_{4}, Mike/x_{1}\}$$

$$\neg L(Tony, Snow)$$

$$(M(Mike) \land \neg S(Mike))$$

$$L(Tony, Snow)$$

$$\sigma = \{\}$$

$$M(Mike) \land \neg S(Mike)$$

Answer to query: Mike!