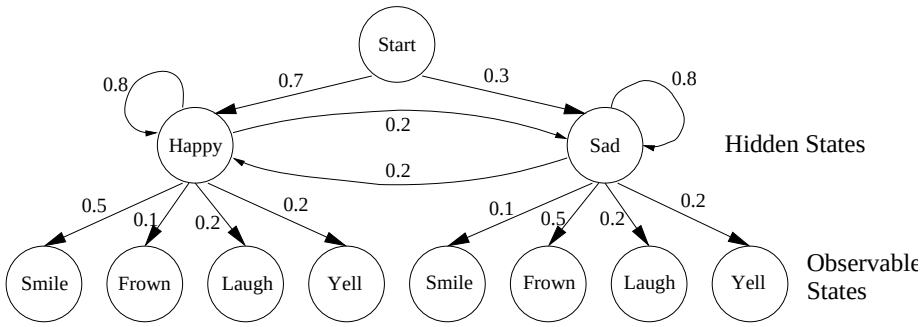
An HMM



$P(q_2$=Happy) = ?

$$P(q_2 = H) = P(q_2 = H \mid q_1 = H)P(q_1 = H) + P(q_2 = H \mid q_1 = S)P(q_1 = S)$$

(by Conditioning rule)

$$= (.8)(.7) + (.2)(.3) = 0.62$$

$P(q_3$=Happy) = ?

$$P(q_3 = H) = P(q_3 = H \mid q_1 = H, q_2 = H)P(q_1 = H, q_2 = H)$$
$$+ P(q_3 = H \mid q_1 = H, q_2 = S)P(q_1 = H, q_2 = S)$$
$$+ \text{ 2 more cases} \quad \text{(by Conditioning rule)}$$
$$= P(q_3 = H \mid q_2 = H)P(q_1 = H, q_2 = H) + \ldots$$
$$= P(q_3 = H \mid q_2 = H)P(q_2 = H \mid q_1 = H)P(q_1 = H) + \ldots$$
$$\text{(by Chain rule)}$$

$$= (.7)(.8)(.8) + (.7)(.2)(.2) + (.3)(.2)(.8) + (.3)(.8)(.2)$$

$P(o_1$=Laugh, o_2=Frown) = ?

$$P(o_1 = L, o_2 = F) = P(o_1 = L, o_2 = F \mid q_1 = H, q_2 = H)P(q_1 = H, q_2 = H)$$
$$+ P(o_1 = L, o_2 = F \mid q_1 = H, q_2 = S)P(q_1 = H, q_2 = S)$$
$$+ P(o_1 = L, o_2 = F \mid q_1 = S, q_2 = H)P(q_1 = S, q_2 = H)$$
$$+ P(o_1 = L, o_2 = F \mid q_1 = S, q_2 = S)P(q_1 = S, q_2 = S)$$
$$\text{by Conditioning rule}$$
$$= P(o_1 = L \mid o_2 = F, q_1 = H, q_2 = H)P(q_1 = H, q_2 = H) + \ldots$$
$$\text{by Conditionalized Chain rule}$$
$$= P(o_1 = L \mid q_1 = H)P(o_2 = F \mid q_2 = H)P(q_2 = H \mid q_1 = H)P(q_1 = H) + \ldots$$
$$\text{by independence of } o_1 \text{and } o_2, \text{and } o_1 \text{and } q_2$$
$$= P(q_1 = H)P(o_1 = L \mid q_1 = H)P(q_2 = H \mid q_1 = H)P(o_2 = F \mid q_2 = H) + \ldots$$
$$= (.7)(.2)(.8)(.1) + \ldots$$

$P(q_1=S \mid o_1=F) = ?$

$$P(q_1 = S \mid o_1 = F) = \frac{P(o_1 = F \mid q_1 = S)P(q_1 = S)}{P(o_1 = F)} \quad \text{by Bayes's rule}$$

$$= \frac{(.5)(.3)}{P(o_1 = F \mid q_1 = S)P(q_1 = S) + P(o_1 = F \mid q_1 = H)P(q_1 = H)}$$

$$= \frac{(.5)(.3)}{(.5)(.3) + (.1)(.7)}$$

$P(q_3=H \mid o_1=F, o_2=L, o_3=Y)=?$

$$\begin{aligned} P(q_3 = H \mid o_1 = F, o_2 = L, o_3 = Y) = {} & P(q_3 = H, q_2 = H, q_1 = H \mid ...) \\ & + P(q_3 = H, q_2 = H, q_1 = S \mid ...) \\ & + P(q_3 = H, q_2 = S, q_1 = H \mid ...) \\ & + P(q_3 = H, q_2 = S, q_1 = S \mid ...) \end{aligned}$$

where

$$P(q_3 = H, q_2 = H, q_1 = H \mid o_1 = F, o_2 = L, o_3 = Y)$$

$$= \frac{P(o_1 = F, o_2 = L, o_3 = Y \mid q_1 = H, q_2 = H, q_3 = H)P(q_1 = H, q_2 = H, q_3 = H)}{P(o_1 = F, o_2 = L, o_3 = Y)}$$

by Bayes's rule

$$= \frac{P(o_1 = F \mid q_1 = H)P(o_2 = L \mid q_2 = H)P(o_3 = Y \mid q_3 = H)P(q_3 = H \mid q_2 = H)P(q_2 = H \mid q_1 = H)P(q_1 = H)}{P(o_1 = F, o_2 = L, o_3 = Y)}$$