Optimization, Wisconsin Institutes of Discovery, and Conservation: Is there a link?

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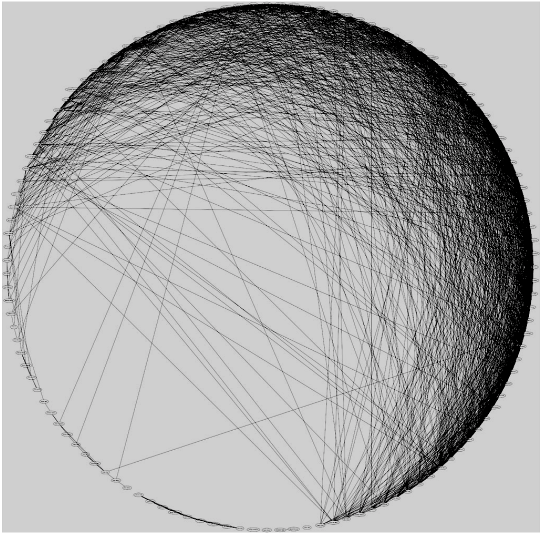
Collaboration with Jeff Linderoth, Jim Luedtke, Ben Recht, Steve Wright and others.

Facebook: a friend wheel



- visual representation of relationships between the friends of any one person
- constructed by placing friends equidistant from each other on circumference of circle
- line segments are drawn between each point if those people are friends with each other
- Order to reduce amount of ink used

QAP (Koopmans and Beckman)

Given n facilities $\{f_1, \ldots, f_n\}$, n locations $\{l_1, \ldots, l_n\}$:
Determine to which location each facility must be assigned
$p : \{1, \ldots, n\} \mapsto \{1, \ldots, n\}$ is an assignment whose cost is

$$c(p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i,j} d_{p(i),p(j)}$$

$$\text{QAP} : \min c(p) \text{ subject to } p \in \Pi_n$$

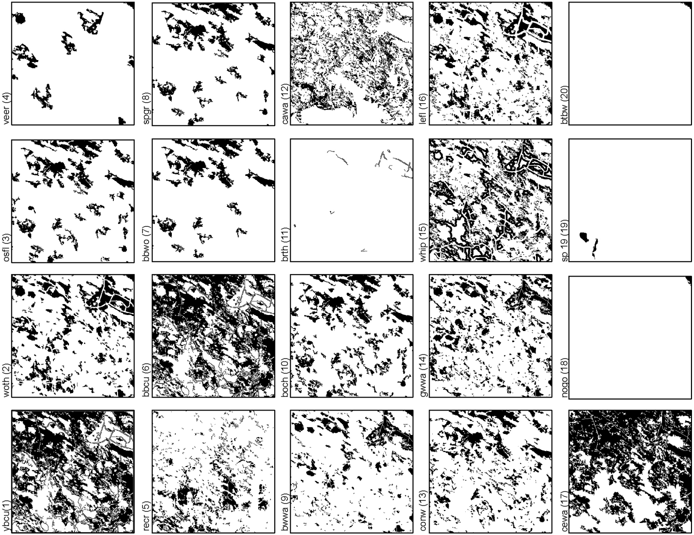
QAP is known to be strongly NP-hard

- n is the number of friends of a given individual
- $w_{i,j} = 1$ if i is a friend of j, and 0 otherwise
- $d_{r,s}$ is the distance from location r on the circle circumference to location s

Northern Wisconsin: Conservation

Golden-winged Warbler. Species maps are 14,309 columns by 11437 rows.

Northern Wisconsin: There's More



Some species require complementary habitats

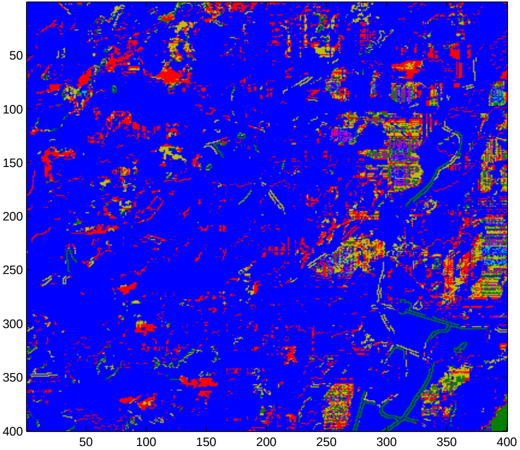
The specs

- GIS data (77 million pixels with indicator that land type in 30 by 30 meter square can support species)
- Incompatibility matrix (cannot have certain species co-habiting)
- Threshold values (how much land required)
- Compact regions, limit total land conserved!

$$x_{s,i,j} = \begin{cases} 1 & \text{if } (i,j) \text{ conserved for species } s \\ 0 & \text{else} \end{cases}$$

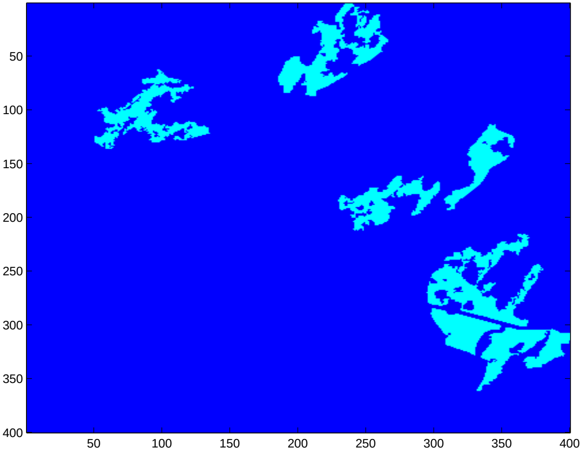
- Example of an assignment model (e.g. Sudoku, etc)

A poor solution



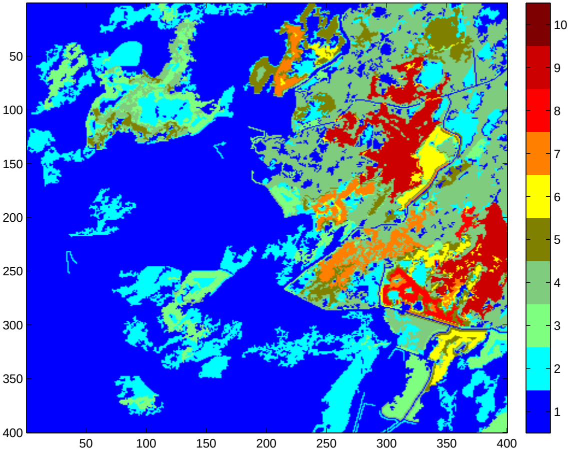
Mip solution needs enormous time, does not get compact boxes or multiple use [Use CPLEX with Matlab tool to visualize solution]

Alternative approach



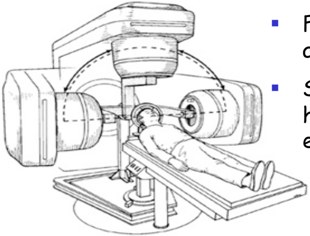
Data reduction (via largest connected components). Solve for these in parallel using network simplex.

Reassembling solution

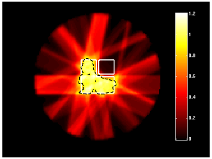


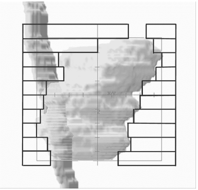
Choose clusters for each species; ensure complementary habitat is satisfied; optimize multiple species overlap

Conformal Radiotherapy

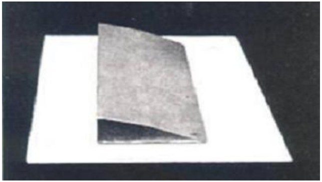


- Fire from multiple angles
- Superposition allows high dose in target, low elsewhere





- Beam shaping via collimator
- Gradient across beam via wedges



Extended Mathematical Programs

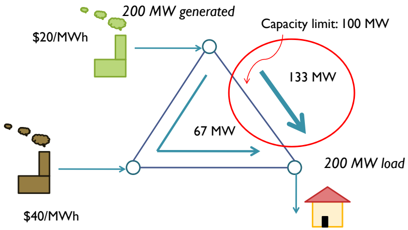
- Optimization models improve understanding of underlying systems and facilitate operational/strategic improvements under resource constraints
- Problem format is old/traditional

$$\min_x f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0$$

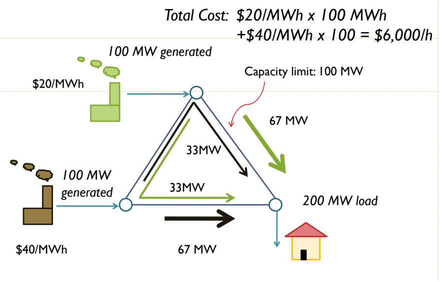
- Extended Mathematical Programs allow annotations of constraint functions to augment this format.
- Give three examples of this: disjunctive programming, bilevel programming and multi-agent competitive models

Transmission switching

Opening lines in a transmission network can reduce cost



(a) Infeasible due to line capacity



(b) Feasible dispatch

Need to use expensive generator due to power flow characteristics and capacity limit on transmission line

Determine which subset of lines to open at any given hour

The basic model

$$\begin{array}{lll} \min_{g,f,\theta} & c^T g & \text{generation cost} \\ \text{s.t.} & g - d = Af,\, f = BA^T\theta & A \text{ is node-arc incidence} \\ & \bar{\theta}_L \leq \theta \leq \bar{\theta}_U & \text{bus angle constraints} \\ & \bar{g}_L \leq g \leq \bar{g}_U & \text{generator capacities} \\ & \bar{f}_L \leq f \leq \bar{f}_U & \text{transmission capacities} \end{array}$$

with transmission switching (within a smart grid technology) we modify as:

$$\begin{array}{lll} \min_{g,f,\theta} & c^T g & \\ \text{s.t.} & g - d = Af & \\ & \bar{\theta}_L \leq \theta \leq \bar{\theta}_U & \\ & \bar{g}_L \leq g \leq \bar{g}_U & \\ \text{either} & f_i = (BA^T\theta)_i,\, \bar{f}_{L,i} \leq f_i \leq \bar{f}_{U,i} & \text{if } i \text{ closed} \\ \text{or} & f_i = 0 & \text{if } i \text{ open} \end{array}$$

Use EMP to facilitate the disjunctive constraints (several equivalent formulations, including LPEC)

Issues

- Models are critical to making hard business decisions
- Model needs enough detail so solutions are realistic
- Computation is hard - many possibilities!
- Need large scale solvers
- How to obtain data, get data into model, verify data integrity - more tools and models
- Interplay between model, data and decision maker is critical
- Visualization helps in motivating the answers

Nash equilibria: modeling competition

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, q), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.
- Quantities q given exogenously, or via complementarity:

$$0 \leq H(x, q) \ \perp \ q \geq 0$$

- Can solve large instances of these problems
- Model competing agents, etc

EMP(iii): Embedded models

- Model has the format:

$$\begin{array}{lll} \text{Agent o:} & \min\limits_{x} & f(x,y) \\ & \text{s.t.} & g(x,y) \leq 0 \quad (\perp \lambda \geq 0) \\ & & \\ \text{Agent v:} & H(x,y,\lambda) = 0 & \quad (\perp y \text{ free}) \end{array}$$

- Difficult to implement correctly (multiple optimization models)
- Can do automatically - simply annotate equations

  empinfo: equilibrium

  min f x defg

  vifunc H y dualvar λ defg
- EMP tool automatically creates an MCP

$$\nabla_x f(x,y) + \lambda^T \nabla g(x,y) = 0$$
$$0 \leq -g(x,y) \perp \lambda \geq 0$$
$$H(x,y,\lambda) = 0$$

Competing agent models

- Competing agents (consumers)
- Each agent maximizes objective independently (utility)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention)

The model details: Brown, Demarzo, Eaves

Each agent maximizes:

$$u_h = -\sum_s \pi_s \left(\kappa - \prod_l c_{h,s,l}^{\alpha_{h,l}} \right)$$

Time 0:

$$d_{h,0,l} = c_{h,0,l} - e_{h,0,l}, \;\; \sum_l p_{0,l} d_{h,0,l} + \sum_k q_k z_{h,k} \leq 0$$

Time 1:

$$d_{h,s,l} = c_{h,s,l} - e_{h,s,l} - \sum_k D_{s,l,k} * z_{h,k}, \;\; \sum_l p_{s,,l} d_{h,s,l} \leq 0$$

Additional constraints (complementarity) outside of control of agents:

$$0 \leq -\sum_h z_{h,k} \perp q_k \geq 0, \;\; 0 \leq -\sum_h d_{h,s,l} \perp p_{s,l} \geq 0$$

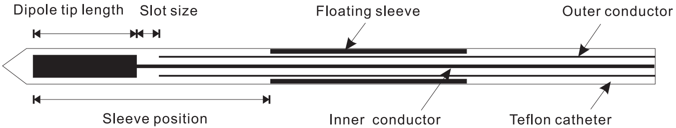
Issues

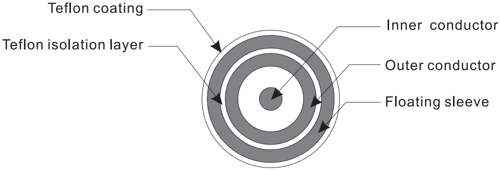
- New model paradigms available
- Models with continuous, discrete, categorical variables necessary
- Size matters
- Can solve realistic scale instances
- Data collection remains hard - new tools help
- Models are critical to making hard decisions

IV: Simulation-based optimization problems

- Computer simulations are used as substitutes to evaluate complex real systems.
- Simulations are widely applied in epidemiology, engineering design, manufacturing, supply chain management, medical treatment and many other fields.
- The goal: Optimization finds the best values of the decision variables (design parameters or controls) that minimize some performance measure of the simulation.
- Other applications: calibration, design optimization, inverse optimization

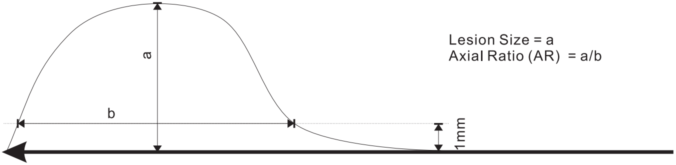
Design a coaxial antenna for hepatic tumor ablation





Simulation of the electromagnetic radiation profile

Finite element models (COMSOL MultiPhysics v3.2) are used to generate the electromagnetic (EM) radiation fields in liver given a particular design



Metric	Measure of	Goal
Lesion radius	Size of lesion in radial direction	Maximize
Axial ratio	Proximity of lesion shape to a sphere	Fit to 0.5
S_{11}	Tail reflection of antenna	Minimize

Issues

- Complex interactions of different types of models
- Large scale solution, in “real time”
- Models to aid in data collection/verification
- Uncertainties in data and model
- Moving effective models into practice - getting the checks done!

Conclusions

- Optimization models effective for large scale planning/operations
- Design optimization possible in conjunction with "expert" simulations
- Must treat uncertainties both in data and model
- New model paradigms (e.g. complementarity, conic programming, stochastic programming) effective for treating uncertainties and competition
- Engaged teams (including embedded optimizers) are most effective for timely, relevant solutions