IMPLICIT FUNCTIONS AND SOLUTION MAPPINGS

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Abstract

The classical implicit function theorem revolves around solving an equation $f(p, x) = 0$ for x in terms of a parameter p, and tells us when the solution mapping associated with this equation is a differentiable function. In this talk we move into a much wider territory in replacing equation-solving problems by more complicated problems for "generalized equations." Such problems arise in constrained optimization, models of equilibrium, control theory, and many other areas. An important feature, in contrast to ordinary equations, is that functions obtained implicitly from their solution mappings typically lack differentiability, but often exhibit Lipschitz continuity. It turns out that if we put aside differentiability of the solution and focus on Lipschitz continuity only, we can not only cover more general models but also get estimates of the solution changes resulting from approximations of the model that do not involve derivatives. After a review of Robinson's implicit function theorem and its extensions, we carry further the implicit function theorem paradigm placing it in the framework of Newton's method applied to generalized equations.