Agent Decision Rules, Batteries, and Carbon Pricing

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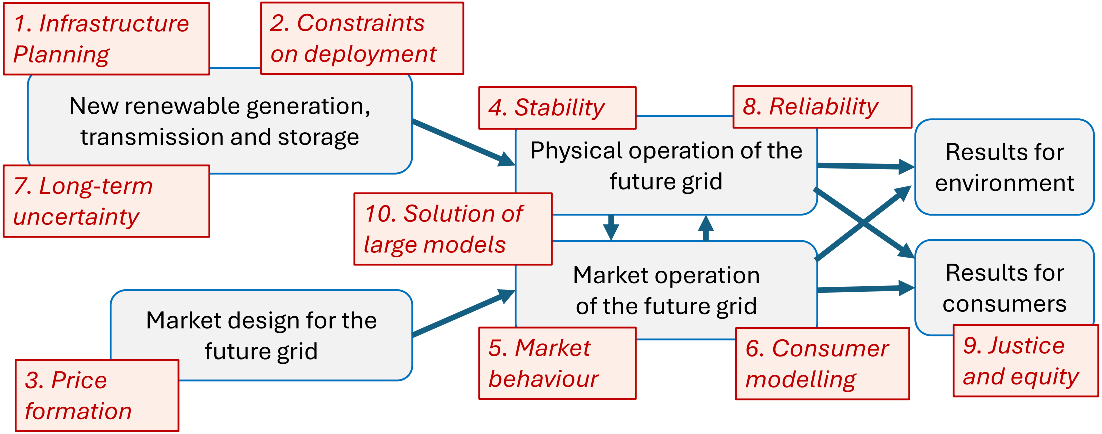
Research performed while the authors were participating in the Architecture of Green Energy Systems Program hosted by the Institute for Mathematical and Statistical Innovation (IMSI), and supported by the National Science Foundation (Grant No. DMS-1929348).

Paper downloadable at `https://www.epoc.org.nz/papers/ADRPaperForEPOC.pdf`

10 challenges for mathematical modeling of the green-energy transition



http://dx.doi.org/10.1007/s40518-025-00274-9

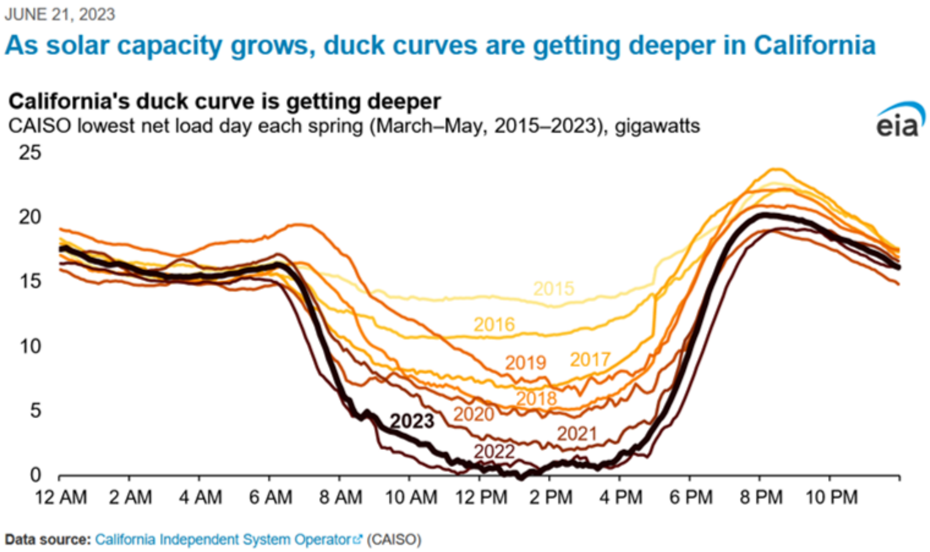
Challenge	Core Issue	Potential Research Directions
1. Policy choices for infrastructure planning	Coordinating investment decisions under uncertainty and multiple objectives	- Dynamic multi-agent models for investment under policy uncertainty - Mechanism design for co-optimizing public and private objectives - Game theory treatment of government, transmission planner and generator interaction
2. Constraints on deployment	Time-dependent constraints on build-out rates and system integration	- Spatiotemporal deployment optimization models - Dynamic constraints in capacity planning
7. Long-term uncertainty	Structural uncertainty in policy, technology, and demand	- Distributionally robust and adaptive optimization in energy systems - Stochastic modeling with scenario discovery - Learning models in multi-decade planning problems

Challenge	Core Issue	Potential Research Directions
4. Stability	Maintaining voltage, frequency, and system control with low inertia	- Control-theoretic models for low-inertia grids - Dynamic models for inverter-based systems under large disturbances - Probabilistic stability analysis with high renewable penetration
8. Reliability	System resilience to extreme events, correlated failures, and weather-driven variability	- Probabilistic reliability modeling with climate-informed risk - Cascading failure simulation and mitigation strategies - Planning with resilience constraints
9. Justice and equity	Distributional impacts of energy policies and transition pathways	- Multi-objective modeling with equity constraints - Quantitative metrics for fairness in cost/benefit allocation - Spatially resolved impact assessments across demographic groups

Challenge	Core Issue	Potential Research Directions
3. Price formation	Ensuring efficient and stable price signals in evolving market designs	- Coupled physical-economic dispatch models - Integration of flexibility markets with energy and capacity pricing - Evaluation of forward and real-time market interactions under volatility
5. Market behavior	Strategic bidding, market power, and competition in mixed-asset environments	- Equilibrium modeling with renewables and storage - Game-theoretic models with learning and bounded rationality - Empirical validation of strategic behavior
6. Consumer modeling	Electrification, demand-side flexibility, and response to new tariffs	- Behavioral models of flexible demand and automation - Aggregator optimization with household level constraints and preferences - Models linking consumer heterogeneity to pricing and adoption

Our setting

- Renewable energy (wind and solar) growing in scale.
- Grid-connected storage increasing.
- Stochastic multiperiod dispatch and pricing being proposed.
  - Pricing rules for minimizing uplift payments.
  - Pricing the option value of storage.
  - Consistency of prices from multiperiod solutions.
  - Consensus on system operator's scenarios?
- Proposal: return to single-period dispatch but use decision rules defined by a dynamic programming policy.



California Independent System Operator (CAISO) Duck curves [https://www.caiso.com]

Energy & Environment   Energy Transitions   Renewables & Advanced Energy   United States and Canada

New Atlanticist | May 13, 2024

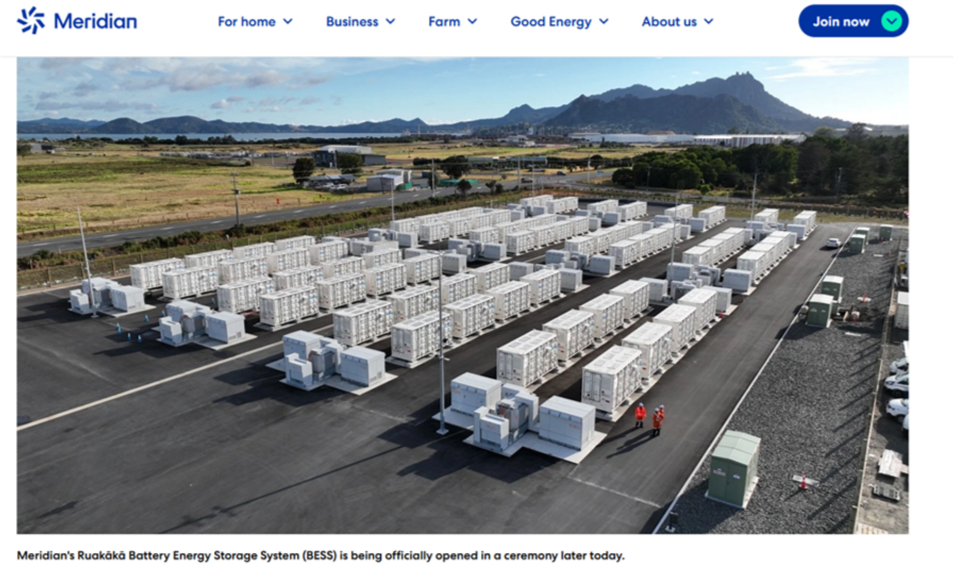
California's battery boom is a case study for the energy transition

By Joseph Webster

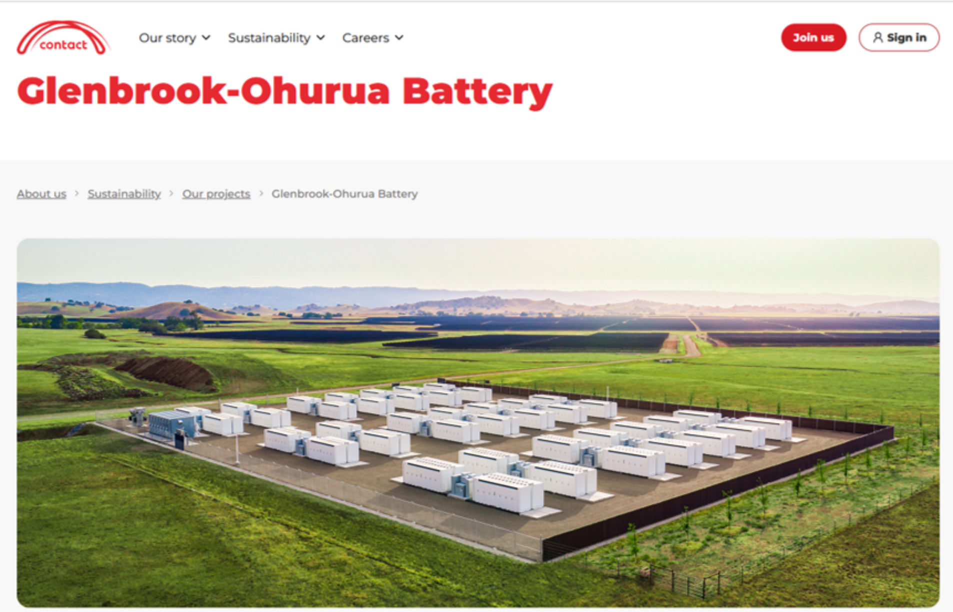
California is the country's largest and most mature solar market, but it's also changing in important ways. On April 25, California marked a major milestone, as it became the first state to deploy 10 gigawatts (GW) of battery storage capacity. This large-scale deployment of lithium-ion storage batteries is leading to lower solar "curtailment," or when electricity generation is suppressed due to price signals or physical oversupply. Curtailment is a problem because it means solar power stations, for example, are producing less electricity than they could, contributing less to the overall energy mix than they otherwise might.

CAISO battery boom [New Atlanticist, May 2024]

Rotohiko Battery Energy Storage System (BESS) (35 MWh, December, 2023)
[https://infratec.co.nz]



Ruakaka BESS (100 MW, May, 2025) [https://www.meridianenergy.co.nz]



Contact Energy Glenbrook BESS (100 MW, March 2026) [https://contact.co.nz]

Economic dispatch example

$x_i(t)$ = dispatch of generator $i \in \mathcal{G}$ in period t;

$\bar{x}_i$ = dispatch of generator i in period $t-1$;

$y_j(t)$ = storage in battery $j \in \mathcal{B}$ at end of period t;

$\bar{y}_j$ = storage in battery j at end of period $t-1$;

u_j = discharge from battery j in period t;

v_j = charge input to battery j in period t;

$$\mathcal{X}_i(\bar{x}) = \{x \mid 0 \leq x \leq q_i, x - \bar{x}_i \leq \rho_i, \bar{x}_i - x \leq \sigma_i\},$$

$$\begin{aligned}\mathcal{Y}_j(\bar{y}) = \{(y, u, v) | 0 \leq y \leq E_j, 0 \leq u \leq r_j, 0 \leq v \leq s_j,\\ y = \bar{y}_j - u + \eta_j v\}.\end{aligned}$$

Economic dispatch and pricing: period t

$$
\begin{aligned}
\text{EP}(t): \quad & \min \sum_{i \in \mathcal{G}} c_i(x_i(t)) + Lz(t) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) \geq d^t, \ [\pi(t)] \\
& x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G}, \\
& (y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B}, \\
& z(t) \in [0, d^t].
\end{aligned}
$$

[Here $c_i(x)$ is a convex increasing function of x; L is VOLL.]

Self dispatch versus central dispatch

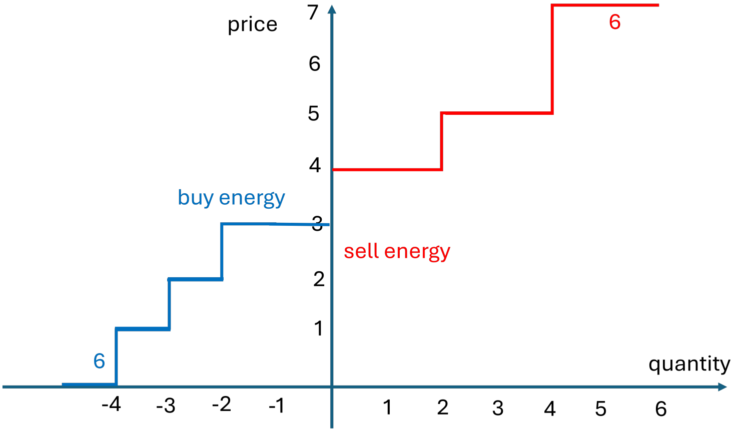
Self dispatch

- Battery forecasts/models prices and solves an optimization problem to maximize revenue from storage.
- System operator forecasts exogenous battery operation as part of net demand.

Central dispatch

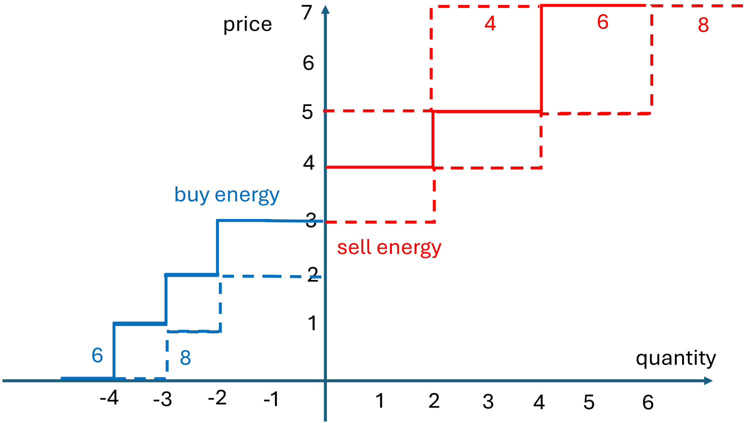
- Battery provides supply/demand curve defining what battery will sell/buy as price increases.
- System operator co-optimizes single-period dispatch and endogenous battery operation.

Supply/demand offer curve for battery



Supply-demand curve for battery with capacity 10 and charge 6.

Supply/demand offer curves for battery



Curves depend on battery charge.

Multiperiod economic dispatch and pricing

$$\text{ED:}\quad \min \sum_{t=1}^{T} \sum_{i \in \mathcal{G}} c_i(x_i(t)) + Lz(t)$$

$$\text{s.t.}\quad \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) \geq d^t, \ \ [\pi(t)]$$

$$x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G},$$

$$(y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B},$$

$$z(t) \in [0, d^t], \quad t = 1, \ldots T$$

System stage problem and expected future cost

$$
\begin{aligned}
\mathrm{EP}(t)\colon \min \quad & \sum_{i \in \mathcal{G}} c_i(x_i(t)) + Lz(t) + C^t(x, y) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) \geq d^t, \\
& x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G}, \\
& (y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B}, \\
& z(t) \in [0, d^t].
\end{aligned}
$$

- This is basis of dynamic programming algorithm

Agent decision rules (ADRs)

- System operator collects future cost functions (ABFs) $G_a^t(x_a)$ and $B_a^t(y_a)$ from agents and uses them in place of $C^t(x, y)$.
- An agent decision rule (ADR) is a mapping from any known parameter of the stage t problem, and a's state (storage) at end of t, to an energy offer in period t.
- We can define an ADR for battery a using observed price $\pi(t)$ and its initial storage y_a and B_a^t.

$$\begin{array}{rl} \min_{u,v\geq 0} & \pi(t)(v-u)+B_a^t(y_a-u+\eta v) \\ \text{s.t.} & 0\leq y_a-u+\eta v\leq E_a \end{array}$$

Single-node electricity dispatch with ABFs

Given ABFs for (generators and) battery agents $\mathcal{B}$, and state of charge $y(t-1)$:

$$
\begin{aligned}
\text{ADR}(t)\text{: min} \quad & \sum_{a \in \mathcal{G}} c_a x_a(t) + L z(t) + \sum_{a \in \mathcal{G}} G_a^t(x_a) + \sum_{a \in \mathcal{B}} B_a^t(y_a) \\
\text{s.t.} \quad & \sum_{a \in \mathcal{G}} x_a(t) + \sum_{a \in \mathcal{B}} u_a(t) - \sum_{a \in \mathcal{B}} v_a(t) + z(t) \geq d^t, \quad [\pi(t)] \\
& x_a(t) \in \mathcal{X}_a(x_a(t-1)), \quad a \in \mathcal{G}, \\
& (y_a(t), u_a(t), v_a(t)) \in \mathcal{Y}_a(y_a(t-1)), \quad a \in \mathcal{B}, \\
& z(t) \in [0, d^t].
\end{aligned}
$$

New dispatch process with ADRs

- Generator agents provide system operator with marginal costs and ABFs G_a^t.
- Battery agent a provides system operator with ADR defined by convex ABF B_a^t.
- System operator solves single-stage problem ADR(t) and computes dispatch and system marginal price $\pi(t)$.
- Generator i is paid $\pi(t)x_i(t)$.
- Battery j is paid $\pi(t)(u_j(t) - v_j(t))$.

Example: one battery, one ramping generator

$x(t)$ = dispatch of generator in period t;

$\bar{x}$ = dispatch of generator in period $t-1$;

$y(t)$ = storage in battery at end of period t;

$\bar{y}$ = storage in battery at end of period $t-1$;

u = discharge from battery in period t;

v = charge input to battery in period t;

$$\begin{aligned}
\mathcal{X}(\bar{x}) &= \{x \mid 0 \leq x \leq q, x - \bar{x} \leq \rho, \bar{x} - x \leq \sigma\}, \\
\mathcal{Y}(\bar{y}) &= \{(y, u, v) | 0 \leq y \leq E, 0 \leq u \leq r, 0 \leq v \leq s, \\
&\quad y = \bar{y} - u + \eta v\}.
\end{aligned}$$

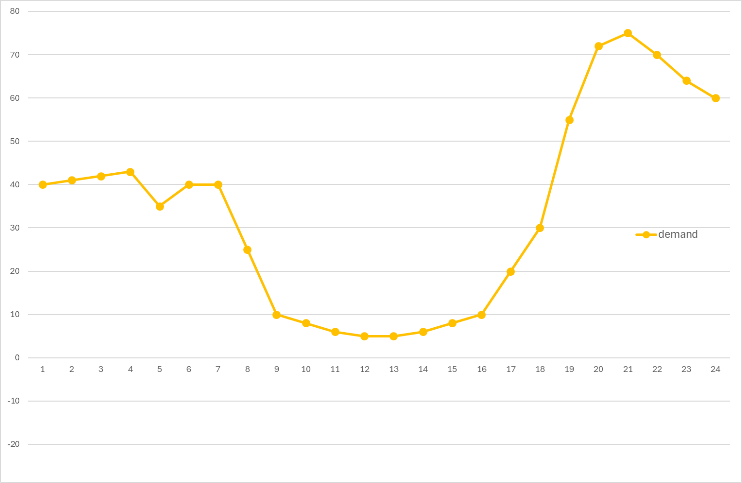
An example: one battery, one ramping generator

Assume $T = 24$, $c(x) = 70.0x$, $\sigma = \infty$. Other parameters are as follows.

$q = 70.0$	$E = 30.0$	$\eta = 0.8$
$r = 10.0$	$s = 10.0$	$\rho = 10.0$
$L = 500.0$	$x^0 = 35.0$	$y^0 = 4.0$

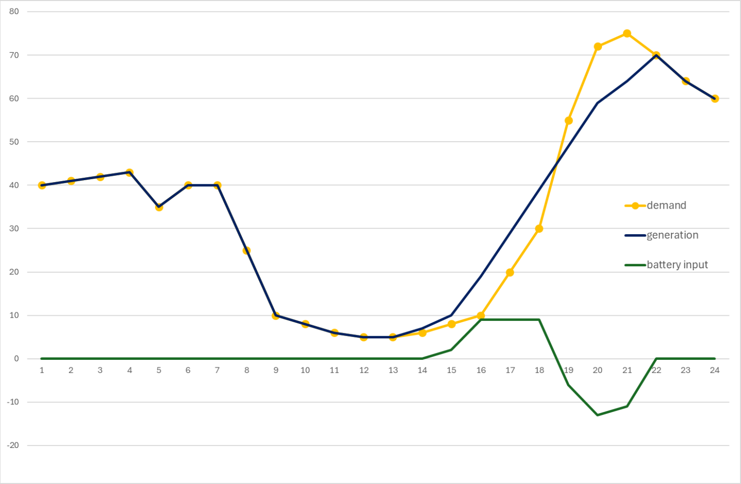
Parameter values for example

Example demand



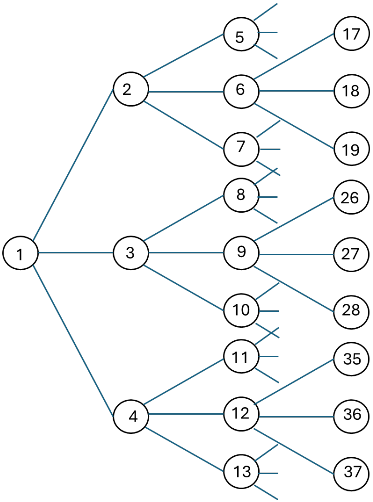
Example values of d^t for $t = 1, 2, \ldots, 24$.

Deterministic solution



SDP solution showing generation x superimposed on demand, and battery net input $v - u$ for $t = 1, 2, \ldots, 24$. Cost is \$48,470.

Uncertain net demand modeled by a scenario tree.



A scenario tree. We write n_- for the parent of node n, for example, $8_- = 3$.

Economic dispatch and pricing in a scenario tree

$$\text{SP:} \quad \min \sum_{n\in\mathcal{N}} P(n) \left(\sum_{i\in\mathcal{G}} c_i(x_i(n)) + Lz(n) \right)$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} x_i(n) + \sum_{j\in\mathcal{B}} u_j(n) - \sum_{j\in\mathcal{B}} v_j(n) + z(n) \geq d^n, \qquad [P(n)\pi(n)], \quad n \in \mathcal{N},$$

$$x_i(1) = x_0, \quad x_i(n) \in \mathcal{X}_i(x(n_-)), \quad \forall i, n \in \mathcal{N} \setminus \{1\},$$

$$y_j(1) = y_0, \quad (y_j(n), u_j(n), v_j(n)) \in \mathcal{Y}_j(y(n_-)), \forall j, n \in \mathcal{N} \setminus \{1\},$$

$$z(n) \in [0, d^n], \quad n \in \mathcal{N}.$$

SDP versus ADR with noise

- We add stagewise independent random noise chosen from -4.0, -2.0, 0.0, 2.0, 4.0 with equal probability
- Solve SDP with independent noise. Expected cost =\$52,377, and optimal value function $V^t(x, y), t = 1, \ldots, 24$.
- Let $(\tilde{x}^t, \tilde{y}^t)$ denote average values of generation and average values of battery storage at each stage.
- ADR(t) uses separable approximation of the system Bellman function:

$$V^t(x, \tilde{y}^t) + V^t(\tilde{x}^t, y).$$

- Simulated ADR policy gives \$54,255 $\pm$ 57. Some social optimality is lost since $(\tilde{x}^t, \tilde{y}^t) \neq (x^*(t), y^*(t))$ (varying with each sample path).

Optimal dispatch gives energy prices π

- Dual variables on demand constraints are $P(n)\pi(n)$ that decouple SP into agent problems. [Ferris & P., 2022]

$$
\begin{aligned}
\text{GP}(i): \quad & \max \quad \sum_{n \in \mathcal{N}} P(n)(\pi(n)x_i(n) - c_i(x_i(n))) \\
& \text{s.t.} \quad x_i(1) = x_0, \quad x_i(n) \in \mathcal{X}_i(x(n_-)), \forall i, n
\end{aligned}
$$

$$
\begin{aligned}
\text{CO}: \quad & \max \quad \textstyle\sum_{n \in \mathcal{N}} P(n)(\pi(n) - L)z(n) \\
& \text{s.t.} \quad 0 \le z(n) \le d^n, \forall n
\end{aligned}
$$

$$
\begin{aligned}
\text{BP}(j): \quad & \max \quad \sum_{n \in \mathcal{N}} P(n)\pi(n)(u_j(n) - v_j(n)) \\
& \text{s.t.} \quad y_j(1) = y_0, \quad (y_j(n), u_j(n), v_j(n)) \in \mathcal{Y}_j(y(n_-)), \forall j, n
\end{aligned}
$$

- Can solve as a stochastic equilibrium problem
- Can extend to coherent risk measures with risk trading

Multi-stage equilibrium theory

- Assume agent i maximizes risk-adjusted expected profit using price process $\pi(n)$, $n \in \mathcal{N}$
  - uses a nested coherent risk measure with one-step polyhedral risk sets $\mathcal{D}_i(n)$ in the interior of the positive orthant;
  - can trade in each node n using a complete market of Arrow-Debreu securities;
  - at each node $n \in \mathcal{N}$, $\bigcap_i \mathcal{D}_i(n) \neq \emptyset$.

Theorem

A. *If $\mathcal{D}(n) = \bigcap_i \mathcal{D}_i(n)$ then the prices $\pi(n)$, $n \in \mathcal{N}$, and actions from (RA)SP define a Walrasian equilibrium where each agent maximizes risk-adjusted expected profit*

B. *Suppose the prices $\pi(n)$, $n \in \mathcal{N}$,give optimal risk adjusted actions for each agent that forms an equilibrium. Then these actions solve (RA)SP where $\mathcal{D}(n) = \bigcap_i \mathcal{D}_i(n)$*

- Idealized; solving equilibrium problem with limited securities shows gap

Remarks

- ADR(t) is a deterministic convex optimization problem (assuming no unit commitment).
- This means prices $\pi(t)$ gives budget balance for system operator (i.e. revenue adequacy).
- Prices $\pi(t)$ defines a perfectly competitive equilibrium for stage t, so agents recover costs.
- Does dispatch problem ADR(t) yield social optimum?
- If all agents and system operator agree on probability distribution of future demand then ADRs can recover social optimum.

ADRs can be system optimal

Theorem

Suppose given $(x(t-1), y(t-1))$, that each agent a makes a perfect forecast $(\tilde{x}^t, \tilde{y}^t)$ of $(x^*(t), y^*(t))$ (for example they might all solve SDDP model with the same shared data). Then

1. the solution for ADR(t) using $\sum_{i \in \mathcal{G}} \tilde{G}_i^t(x_i) + \sum_{j \in \mathcal{B}} \tilde{B}_j^t(y_j)$ is optimal for EP(t) with $C^t(x, y)$;
2. prices from EP(t) and the solution to ADR(t) defines a perfectly competitive equilibrium where all agents optimize profit in period t at system prices accounting for their ADR.

Extensions

- Supply functions offers are simple ADRs.
- Transmission system can be included in dispatch.
- Pumped storage is same as a battery.
- Dispatchable demand is a demand function bid ADR.
- ADRs for flexible demand, hydroelectric generators, reserve and frequency regulation
- Open: unit commitment and minimum uptime?

Conclusions

- In a perfectly competitive, convex, complete market where all agents share the same probability distribution, there is a set of ADRs for each agent that maximizes expected welfare.
- Agents with different probability distributions put their money where their mouths are.
- System operator does not influence future prices through their imperfect forecasts of demand - this falls on aggregating agents' views.
- Changes required to single-period dispatch are relatively simple, but form of offer data is more complicated.

But agents have different probability distributions?

An ADR partial equilibrium occurs when the stochastic process of energy prices assumed by each agent in building their ADR yields the same prices in market clearing. In a scenario tree $\mathcal{T}$ with the same outcomes but different $\mathbb{P}_a$ for each agent $a \in \mathcal{G} \cup \mathcal{B}$:

1. for each $n \in \mathcal{T}$, $x^*(n)$ solves the stage dispatch problem with prices $\pi^*(n)$;
2. *for each* $n \in \mathcal{T}$ *and* $a \in \mathcal{G} \cup \mathcal{B}$, $x_a^*(n)$ *solves*

$$\max_{x_a \in \mathcal{X}_a(x^*(n-))} \{\pi^*(n)^\top x_a - c_a^n(x_a) + W_a^n(x_a)\};$$

3. *for each* $n \in \mathcal{T}$ *that is not a leaf node and* $a \in \mathcal{G} \cup \mathcal{B}$,

$$W_a^n(x_a^*(n)) = \sum_{m \in n+} \mathbb{P}_a(m \mid n)\{\pi^*(m)^\top x_a^*(m) - c_a^n(x_a^*(m)) + W_a^m(x_a^*(m))\}.$$

Note that W_a^n is the welfare (or negative of cost ABF)

A mathematical modelling approach to planning

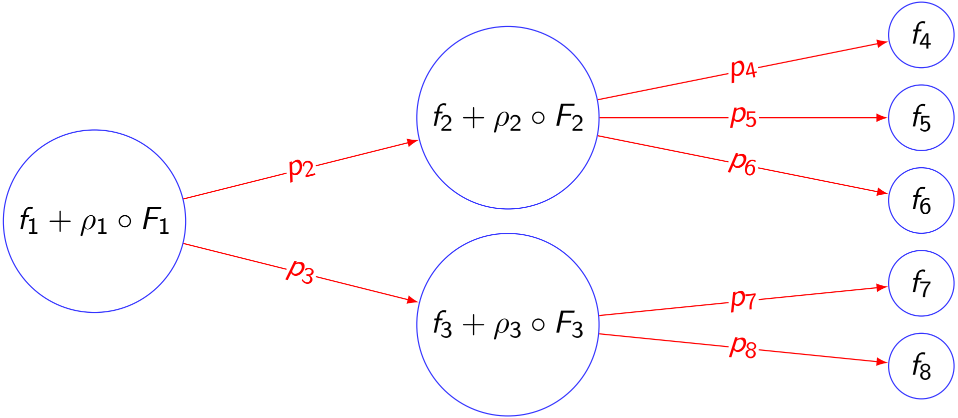
- Build and solve a social planning model that optimizes electricity design and operation with environmental constraints
- Social planning solution should be stochastic: i.e. account for future uncertainty
- Social planning solution should be risk-averse: because the industry is
- Approximate the outcomes of the social plan by a competitive equilibrium with risk-averse players
- Compensate for market failures from imperfect competition or incomplete markets

Need to be able to solve both stochastic optimization and equilibrium formulations

Leading to...

Challenge	Core Issue	Potential Research Directions
10. Solution of large models	Scalability and tractability of complex, multi-sector, multi-scale models	- Decomposition and parallel computing techniques for scalable multi-stage optimization - Machine-learning surrogates for simulation - Validation, benchmarking, and stakeholder trust

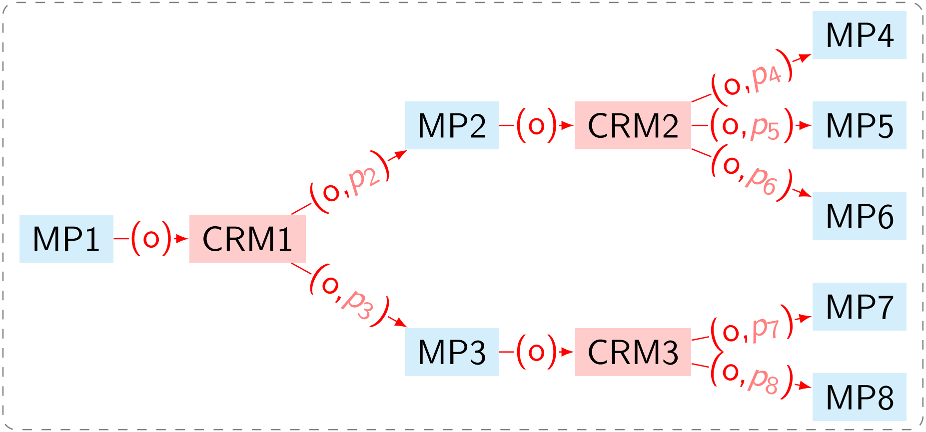
Shared data: (irregular) scenario tree



Recursively:

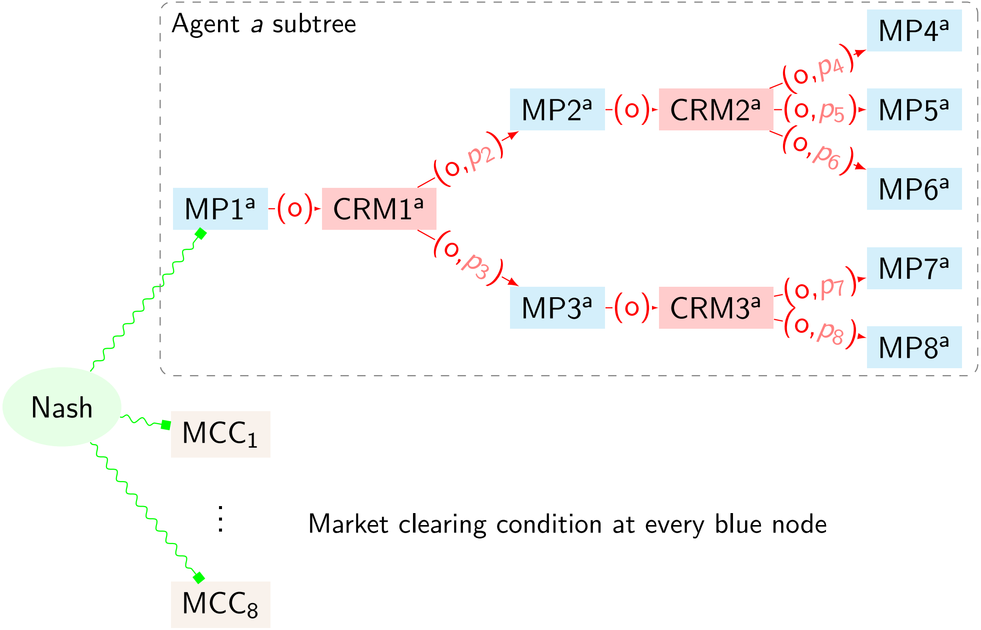
$$\min_{x_\cdot \in \mathcal{X}_0} f_1(x_1)$$
$$+ \rho_1([f_j(x_j) + \rho_j([f_\ell(x_\ell)]_{\ell \in j_+})]_{j \in 1_+})$$

Structured problems



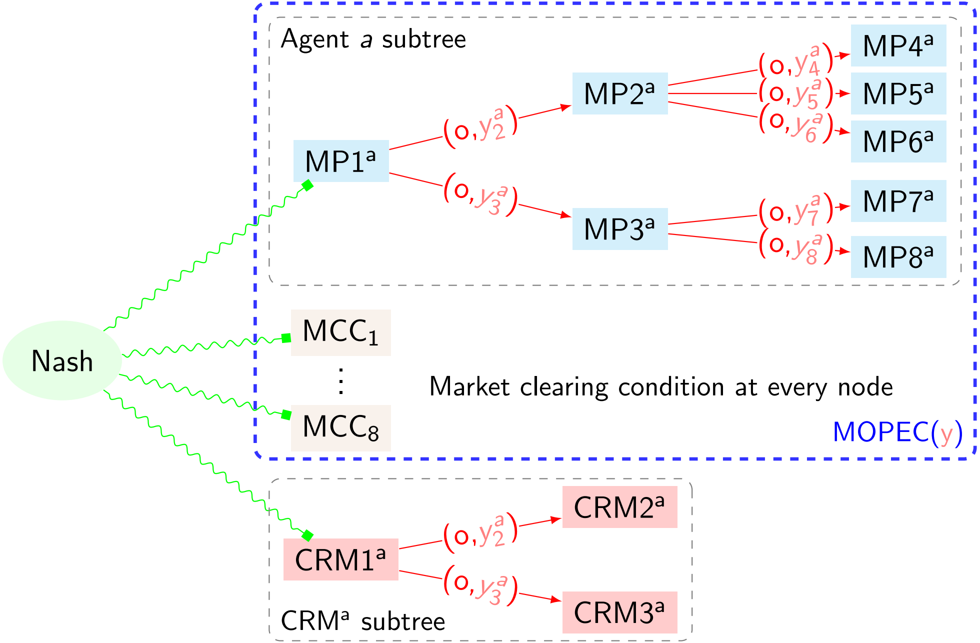
- Red nodes implement the coherent risk measure: in this case a max problem
- ReSHOP enables structured description of problem

Stochastic equilibrium



Market clearing condition at every blue node

SMOPEC (telescope + equilibrium)



Computational results (economic dispatch)

ϵ	λ	PATH	PATH-RN	PD	PD-PATH	PD-CC-PATH
	0.1	11	13	0	97	100
	0.3	0	0	0	63	86
0	0.5	0	0	0	43	74
	0.7	0	0	0	11	36
	0.9	0	0	0	0	10
	0.1	24	93	25	100	100
	0.3	0	0	2	99	100
0.01	0.5	0	0	0	86	97
	0.7	0	0	0	59	90
	0.9	0	0	0	16	51
	0.1	9	99	54	100	100
	0.3	3	54	13	100	100
0.1	0.5	0	1	3	100	100
	0.7	0	0	1	100	100
	0.9	0	0	0	97	100

The End

Any questions?

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For the paper go to https://www.epoc.org.nz/papers/ADRPaperForEPOC.pdf

Carbon pricing: see Jiang, Huber et al, ACM e-Energy 2026, to appear.