Optimization, Equilibrium and Computation for Energy Economics

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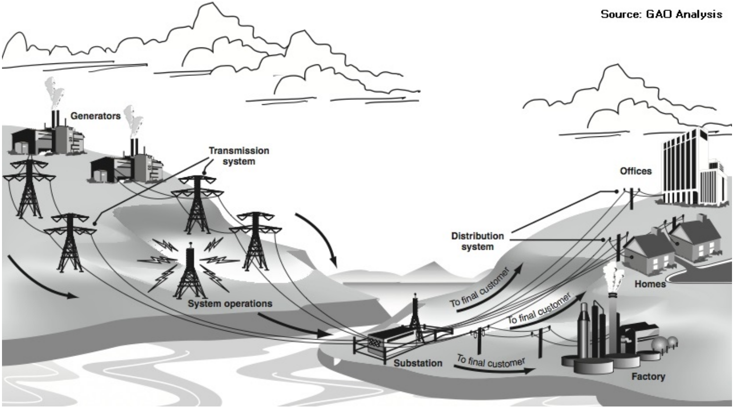
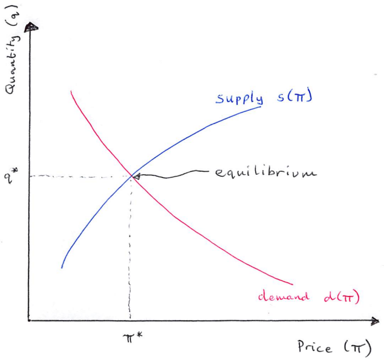
Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $\geq \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

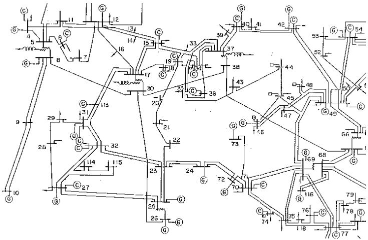
Single market, single good: equilibrium



Walras: $0 \leq s(\pi) - d(\pi) \perp \pi \geq 0$

Market design and rules to foster competitive behavior/efficiency

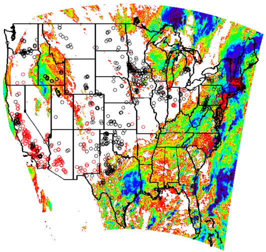
- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network



- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution

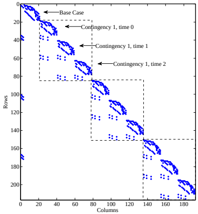
Satellite data, FERC and Reserves

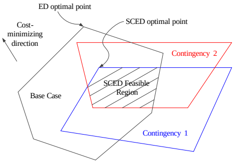
Solar transmittance and power



- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Use 12 hour cloud cover forecasts to reduce reserves

- Federal Energy Regulatory Commission (FERC) contract to build models and data
- Provided on NEOS (Network enabled optimization system)





- Integrate satellite forecast data with power system data and smoke models to provide reliability and savings outcomes

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$
\begin{aligned}
&\min_{x} \; c(x) && \text{cost}\\
&\text{s.t. } Ax \geq q && \text{balance}\\
&\quad\;\; Bx = b, x \geq 0 && \text{technical constr}
\end{aligned}
$$

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$\min_x \; c(x) \qquad \text{cost}$$

$$\text{s.t.} \; Ax \geq d(\pi) \qquad \text{balance}$$

$$Bx = b, x \geq 0 \qquad \text{technical constr}$$

- $q = d(\pi)$: issue is that π is the multiplier on the "balance" constraint
- Such multipliers (LMP's) are critical to operation of market
- Can try to solve the problem iteratively (shooting method):

$$\pi^{new} \in \text{multiplier}(OPF(d(\pi)))$$

Alternative: Form KKT of QP, exposing π to modeler

$$L(x, \mu, \lambda) = c(x) + \mu^T(d(\pi) - Ax) + \lambda^T(b - Bx)$$

$$\begin{aligned}
0 \leq & \; -\nabla_\mu L = Ax - d(\pi) & \perp \quad & \mu \geq 0 \\
0 = & \; -\nabla_\lambda L = Bx - b & \perp \quad & \lambda \\
0 \leq & \; \nabla_x L = \nabla c(x) - A^T\mu - B^T\lambda & \perp \quad & x \geq 0
\end{aligned}$$

- empinfo: dualvar π balance
- Fixed point: replaces $\mu \equiv \pi$

Alternative: Form KKT of QP, exposing π to modeler

$$\begin{aligned} 0 &\leq Ax - d(\pi) & \perp \quad & \pi \geq 0 \\ 0 &= Bx - b & \perp \quad & \lambda \\ 0 &\leq \nabla c(x) - A^T\pi - B^T\lambda & \perp \quad & x \geq 0 \end{aligned}$$

- empinfo: dualvar π balance
- Fixed point: replaces $\mu \equiv \pi$
- LCP/MCP is then solvable using PATH

$$z = \begin{bmatrix} \pi \\ \lambda \\ x \end{bmatrix}, \quad F(z) = \begin{bmatrix} & & A \\ & & B \\ -A^T & -B^T & \end{bmatrix} z + \begin{bmatrix} -d(\pi) \\ -b \\ \nabla c(x) \end{bmatrix}$$

- Existence, uniqueness, stability from variational analysis
- EMP does this automatically from the annotations

Other applications of complementarity

Complementarity can model fixed points and disjunctions

- Economics: Walrasian equilibrium (supply equals demand), taxes and tariffs, computable general equilibria, option pricing (electricity market), airline overbooking
- Transportation: Wardropian equilibrium (shortest paths), selfish routing, dynamic traffic assignment
- Applied mathematics: Free boundary problems
- Engineering: Optimal control (ELQP)
- Mechanics: Structure design, contact problems (with friction)
- Geology: Earthquake propagation

Good solvers exist for large-scale instances of Complementarity Problems

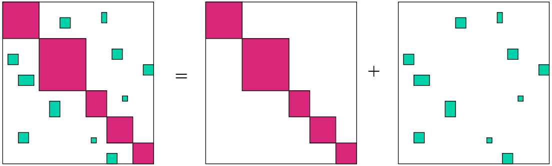
Extension: MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves } h(x, \pi) = 0$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h pi
```

- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



Perfect competition

$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

Cournot: two agents (duopoly)

$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

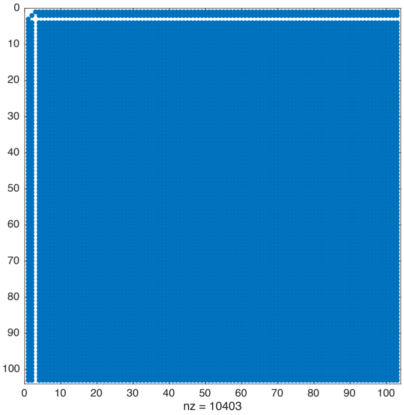
$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect π by choice of x_i
- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot, even Stackleberg)

Computational issue: PATH

- Cournot model: $|\mathcal{A}| = 5$
- Size $n = |\mathcal{A}| * N_a$

Size (n)	Time (secs)
1,000	35.4
2,500	294.8
5,000	1024.6

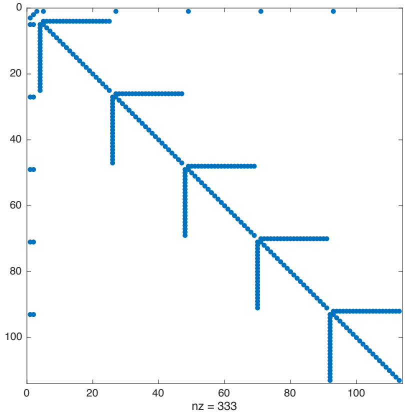


Jacobian nonzero pattern
$n = 100,\ N_a = 20$

Computation: implicit functions and local variables

- Use implicit fn: $z(x) = \sum_j x_j$ (and local aggregation)
- Generalization to $F(z, x) = 0$ (via adjoints)
- empinfo: implicit z F

Size (n)	Time (secs)
1,000	0.5
2,500	0.8
5,000	1.6
10,000	3.9
25,000	17.7
50,000	52.3



Jacobian nonzero pattern
$n = 100,\ N_a = 20$

A Simple Network Model

Load segments s represent electrical load at various instances

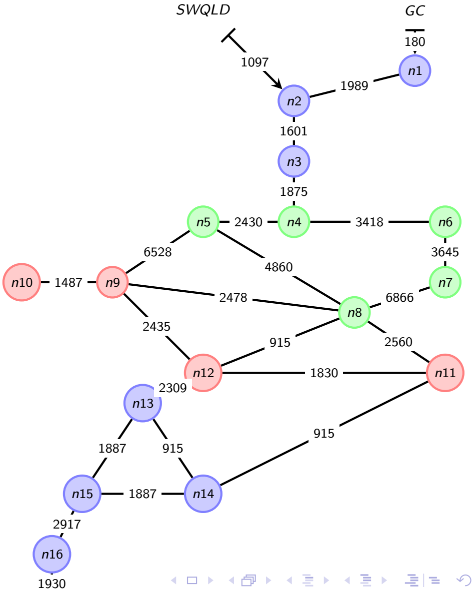
d_n^s Demand at node n in load segment s (MWe)

X_i^s Generation by unit i (MWe)

F_L^s Net electricity transmission on link L (MWe)

Y_n^s Net supply at node n (MWe)

π_n^s Wholesale price ($ per MWhe)



Nodes n, load segments s, generators i, Ψ is node-generator map

$$
\begin{aligned}
\max_{X,F,d,Y} \quad & \sum_s \left(W(d^s(\lambda^s)) - \sum_i c_i(X_i^s) \right) \\
\text{s.t.} \quad & \Psi(X^s) - d^s(\lambda^s) = Y^s \\
& 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_s X_i^s \\
& Y \in \mathcal{X}
\end{aligned}
$$

where the network is described using:

$$
\mathcal{X} = \left\{ Y : \exists F, F^s = \mathcal{H} Y^s, -\overline{F}^s \leq F^s \leq \overline{F}^s, \sum_n Y_n^s \geq 0, \forall s \right\}
$$

- Key issue: decompose. Introduce multiplier π^s on supply demand constraint (and use $\lambda^s := \pi^s$)
- How different approximations of $\mathcal{X}$ affect the overall solution

Case $\mathcal{H}$: Loop flow model

$$\max_{d} \quad \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s)\right)$$

$$+\max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s)\right)$$

$$\text{s.t.} \quad 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_{s} X_i^s$$

$$+\max_{Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad \sum_{i} Y_i^s \ge 0, -\overline{F}^s \le \mathcal{H} Y^s \le \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0$$

Let $\mathcal{A}$ be the node-arc incidence matrix, $\mathcal{H}$ be the shift matrix, $\mathcal{L}$ be the loop constraint matrix. Standard results show:

$$\mathcal{X} = \{Y : \exists F, F = \mathcal{H}Y, F \in \mathcal{F}\}$$
$$\mathcal{X} = \left\{Y : \exists (F, \theta), Y = \mathcal{A}F, B\mathcal{A}^T\theta = F, \theta \in \Theta, F \in \mathcal{F}\right\}$$
$$\mathcal{X} = \{Y : \exists F, Y = \mathcal{A}F, \mathcal{L}F = 0, F \in \mathcal{F}\}$$

Loopflow model (using $\mathcal{A}, \mathcal{L}$)

$$
\begin{aligned}
\max_{d} \quad & \sum_s (W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s)) \\
+ \max_{X} \quad & \sum_s \left(\pi^s \Psi(X^s) - \sum_i c_i(X_i^s) \right) \\
\text{s.t.} \quad & 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\
+ \max_{F,Y} \quad & \sum_s -\pi^s Y^s \\
\text{s.t.} \quad & Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \le F^s \le \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$

Network model

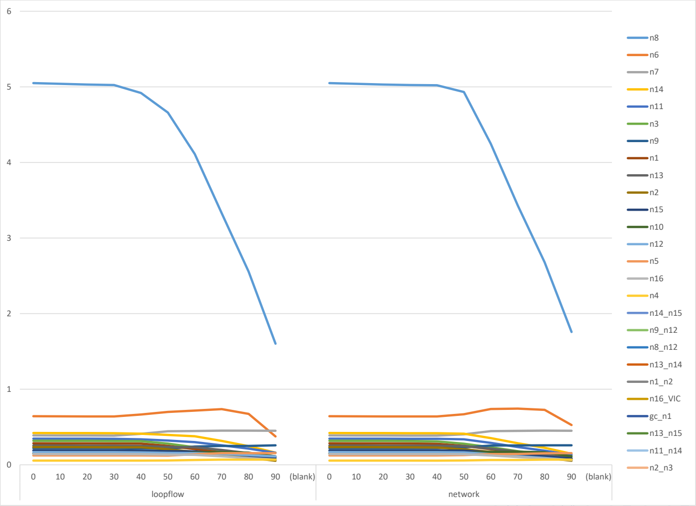
Drop loop constraints:

$$
\begin{aligned}
&\max_{d} && \sum_s \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s)\right) \\
&+\max_{X} && \sum_s \left(\pi^s \Psi(X^s) - \sum_i c_i(X_i^s)\right) \\
&\text{s.t.} && 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\
&+\max_{F,Y} && \sum_s -\pi^s Y^s \\
&\text{s.t.} && Y^s = \mathcal{A}F^s, -\overline{F}^s \le F^s \le \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$

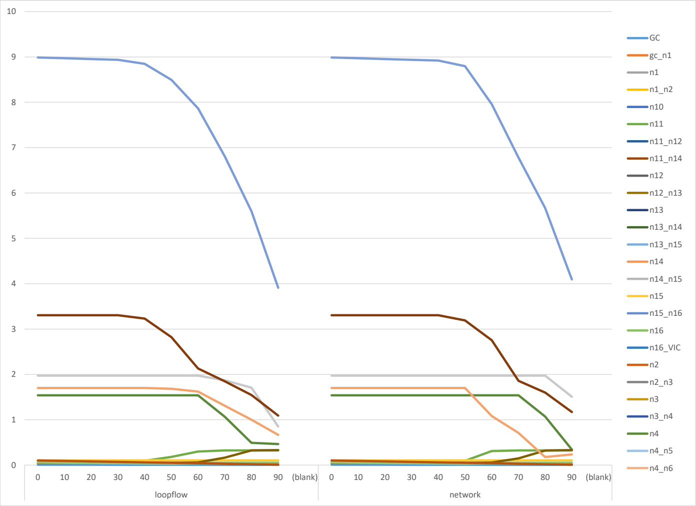
Comparing Network and Loopflow: Demand

Here we look at simulations which impose a proportional reduction in transmission across the network. The *network* and *loopflow* models demonstrate similar responses:



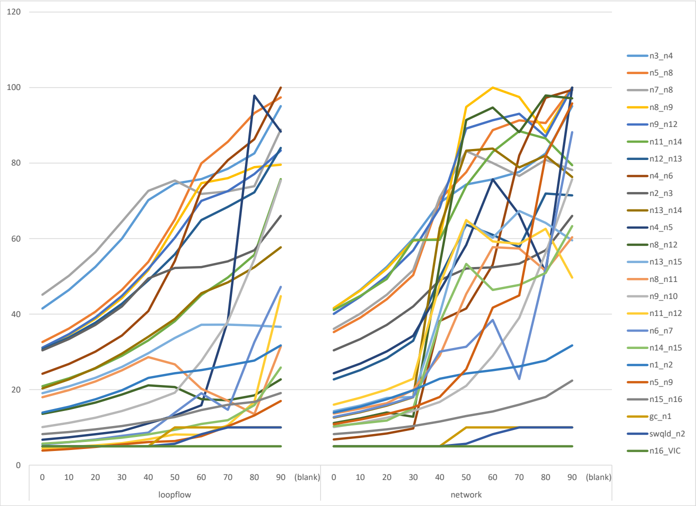
Comparing Network and Loopflow: Generation

Likewise, generation is similar in the two models:



Comparing Network and Loopflow: Transmission

Network transmission levels reveal that the two models are quite different:



The Game: update red, blue and purple components

$$\max_{d} \quad \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s) \right)$$

$$+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right)$$

$$\text{s.t.} \quad 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_{s} X_i^s$$

$$+ \max_{Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad \sum_{i} Y_i^s \ge 0, -\overline{F}^s \le \mathcal{H} Y^s \le \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0$$

Top down/bottom up

- $\lambda^s = \pi^s$ so use complementarity to expose (EMP: dualvar)
- Change interaction via new price mechanisms
- All network constraints encapsulated in (bottom up) NLP (or its approximation by dropping $\mathcal{L}F^s = 0$):

$$\max_{F,Y} \quad \sum_s -\pi^s Y^s$$
$$\text{s.t.} \quad Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s$$

- Could instead use the NLP over Y with $\mathcal{H}$
- Clear how to instrument different behavior or different policies in interactions (e.g. Cournot, etc) within EMP
- Can add additional detail into top level economic model describing consumers and producers
- Can solve iteratively using SELKIE

Pricing

Our implementation of the heterogeneous demand model incorporates three alternative pricing rules. The first is *average cost pricing*, defined by

$$P_{\mathrm{ACP}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s q_{jns}}$$

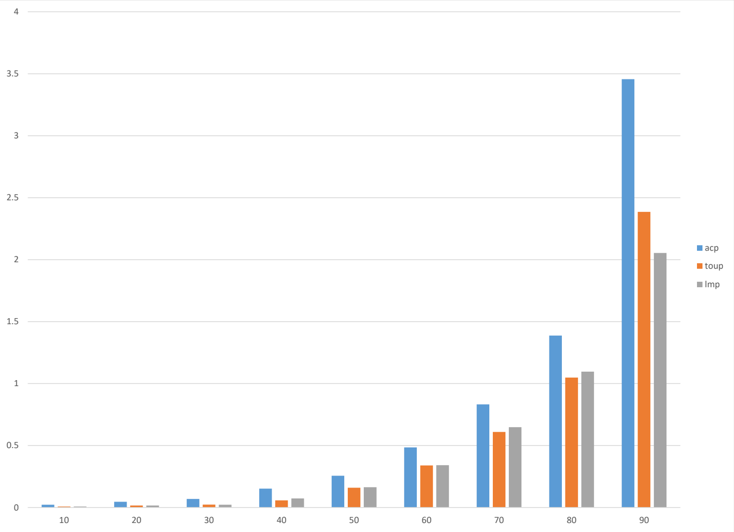
The second is *time of use pricing*, defined by:

$$P_s^{\mathrm{TOU}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} q_{jns}}$$

The third is *location marginal pricing* corresponding to the wholesale prices denoted P_{ns} above. Prices for individual demand segments are then assigned:

$$p_{jns} = \begin{cases} P_{\mathrm{ACP}} & (jn) \in \mathcal{R}_{\mathrm{ACP}} \\ P_s^{\mathrm{TOU}} & (jn) \in \mathcal{R}_{\mathrm{TOU}} \\ P_{ns} & (jn) \in \mathcal{R}_{\mathrm{LMP}} \end{cases}$$

Smart Metering Lowers the Cost of Congestion



Other specializations and extensions

$$\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \leq 0, \forall i, f(x, z, \pi) = 0$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: $z(x_i, x_{-i})$ shared
- Shared constraints: f is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Economic Application

- Model is a partial equilibrium, geographic exchange model.
- Goods are distinguished by region of origin.
- There is one unit of region r goods.
- These goods may be consumed in region r or they may be exported.
- Each region solves:

$$\min_{X, T_r} f_r(X, T) \text{ s.t. } H(X, T) = 0, \; T_j = \bar{T}_j, j \neq r$$

  where $f_r(X, T)$ is a quadratic form and $H(X, T)$ defines X uniquely as a function of T, the taxes and tariffs.
- $H(X, T)$ defines an equilibrium; here it is simply a set of equations, not a complementarity problem
- Applications: Brexit, modified GATT, Russian Sanctions

Model statistics and performance comparison of the EPEC

MCP statistics according to the shared variable formulation		
Replication	Switching	Substitution
12,144 rows/cols 544,019 non-zeros 0.37% dense	6,578 rows/cols 444,243 non-zeros 1.03% dense	129,030 rows/cols 3,561,521 non-zeros 0.02% dense

PATH			Shared variable formulation (major, time)		
crash	spacer	prox	Replication	Switching	Substitution
✓		✓	7 iters 8 secs	20 iters 22 secs	20 iters 406 secs
		✓	24 iters 376 secs	22 iters 19 secs	21 iters 395 secs
	✓		8 iters 28 secs	8 iters 18 secs	8 iters 219 secs

Results

Gauss-Seidel residuals

Iteration	deviation
1	3.14930
2	0.90970
3	0.14224
4	0.02285
5	0.00373
6	0.00061
7	0.00010
8	0.00002
9	0.00000

Tariff revenue

region	SysOpt	MOPEC
1	0.117	0.012
2	0.517	0.407
3	0.496	0.214
4	0.517	0.407
5	0.117	0.012

- Note that competitive solution produces much less revenue than system optimal solution
- Model has non-convex objective, but each subproblem is solved globally (lindoglobal)

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

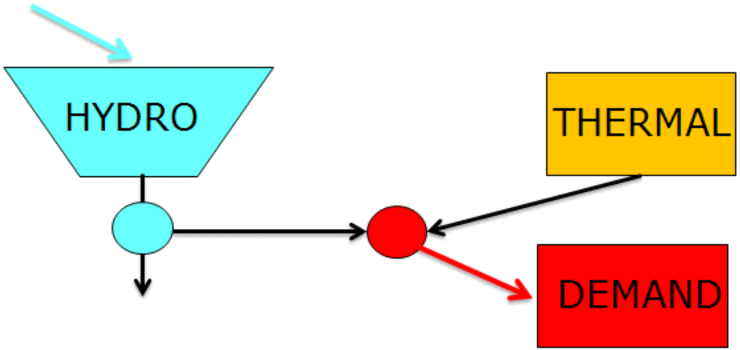
- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- dualvar (use multipliers from one agent as variables for another)
- QS functions (both in objectives and constraints)
- implicit functions and shared constraints

- Currently available within GAMS
- Some solution algorithms implemented in modeling system - limitations on size, decomposition and advanced algorithms
- Can evaluate effects of regulations and their implementation in a competitive environment

Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what
- It's available (in GAMS)
- Allows use and control of dual variables / prices
- MOPEC facilitates easy "behavior" description at model level
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- New decomposition algorithms available to modeler (Gauss Seidel, Randomized Sweeps, Gauss Southwell, Grouping of subproblems)
- Can evaluate effects of regulations and their implementation in a competitive environment
- Stochastic equilibria - clearing the market in each scenario
- Ability to trade risk using contracts

Hydro-Thermal System (Philpott/F./Wets)



- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities

Simple electricity "system optimization" problem

$$
\begin{aligned}
\text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\
& x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}
$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

Decomposition by prices π

$$
\begin{aligned}
\max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
& + \pi^T \left(\sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j - \sum_{k \in \mathcal{K}} d_k \right) \\
\text{s.t.} \quad & x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}
$$

Problem then decouples into multiple optimizations

$$
\begin{aligned}
& \sum_{k \in \mathcal{K}} \max_{d_k \geq 0} \left(W_k(d_k) - \pi^T d_k \right) + \sum_{j \in \mathcal{T}} \max_{v_j \geq 0} \left(\pi^T v_j - C_j(v_j) \right) \\
& \qquad + \sum_{i \in \mathcal{H}} \max_{u_i, x_i \geq 0} \left(\pi^T U_i(u_i) + V_i(x_i) \right) \\
& \qquad \qquad \text{s.t. } x_i = x_i^0 - u_i + h_i^1
\end{aligned}
$$

SO equivalent to CE (price takers)

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\text{CE:} \quad \text{Consumers } k \in \mathcal{K} \text{ solve CP}(k): \max_{d_k \geq 0} \; W_k(d_k) - \pi^T d_k$$

$$\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j): \max_{v_j \geq 0} \; \pi^T v_j - C_j(v_j)$$

$$\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i): \max_{u_i, x_i \geq 0} \; \pi^T U_i(u_i) + V_i(x_i)$$

$$\text{s.t. } x_i = x_i^0 - u_i + h_i^1$$

$$0 \leq \pi \perp \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.$$

But in practice there is a gap between SO and CE. How to explain?

Stochastic: Agents have recourse?

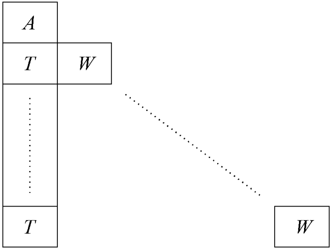
- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP:} \min \quad c(x^1) + \rho[q^T x^2]$$

$$\text{s.t.} \quad Ax^1 = b, \quad x^1 \geq 0,$$

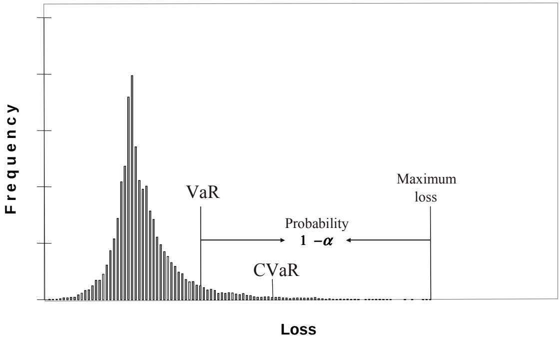
$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

$$\text{CP:} \quad \min_{d^1 \geq 0} \quad p^1 d^1 - W(d^1)$$

$$\text{TP:} \quad \min_{v^1 \geq 0} \quad C(v^1) - p^1 v^1$$

$$\text{HP:} \quad \min_{u^1, x^1 \geq 0} \quad -p^1 U(u^1)$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

Two stage stochastic MOPEC (1,1,1)

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0} \quad p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) \right]$$

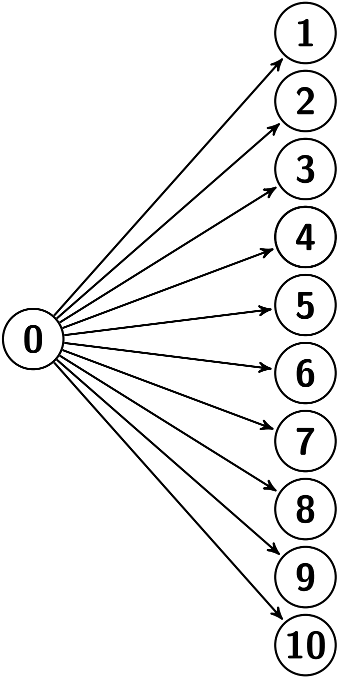
$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0} \quad C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0}} \quad -p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) \right]$$

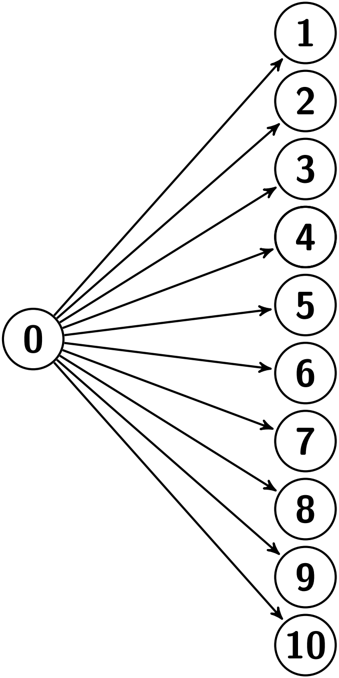
$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1, \quad x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$



- Single hydro, thermal and representative consumer
- Initial storage 10, inflow of 4 to 0, equal prob random inflows of *i* to node *i*
- Risk neutral: SO equivalent to CE (key point is that each risk set is a singleton, and that is the same as the system risk set)



- Single hydro, thermal and representative consumer
- Initial storage 10, inflow of 4 to 0, equal prob random inflows of i to node i
- Risk neutral: SO equivalent to CE (key point is that each risk set is a singleton, and that is the same as the system risk set)
- Each agent has its own risk measure, e.g. $0.8EV + 0.2CVaR$
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

Equilibrium or optimization?

Theorem

If (d, v, u, x) solves (risk averse) SO, then there exists a probability distribution σ_k and prices p so that (d, v, u, x, p) solves (risk neutral) CE(σ)

(Observe that each agent must maximize their own expected profit using probabilities σ_k that are derived from identifying the worst outcomes as measured by SO. These will correspond to the worst outcomes for each agent only under very special circumstances)

- High initial storage level (15 units)
  - Worst case scenario is 1: lowest system cost, smallest profit for hydro
  - SO equivalent to CE
- Low initial storage level (10 units)
  - Different worst case scenarios
  - SO different to CE (for large range of demand elasticities)
- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0} \quad p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0} \quad C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0}} \quad -p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$

Trading risk: pay σ_ω now, deliver 1 later in ω

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0, t^C} \quad \sigma t^C + p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) - t^C_\omega \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0, t^T} \quad \sigma t^T + C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) - t^T_\omega \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0, t^H}} \quad \sigma t^H - p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) - t^H_\omega \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$

$$0 \leq \sigma_\omega \perp t^C_\omega + t^T_\omega + t^H_\omega \geq 0, \forall \omega \quad \sigma = (\sigma_\omega)$$

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

MCP size of equilibrium problems containing shared variables by formulation strategy

Strategy	Size of the MCP
replication	$(n + 2mN)$
switching	$(n + mN + m)$
substitution (explicit)	$(n + m)$
substitution (implicit)	$(n + nm + m)$

$$F_i(z) = \begin{bmatrix} \nabla_{x_i} f_i(x,y) - (\nabla_{x_i} H(y,x))\mu_i \\ \nabla_{y_i} f_i(x,y) - (\nabla_{y_i} H(y,x))\mu_i \\ H(y_i, x) \end{bmatrix}, \quad z_i = \begin{bmatrix} x_i \\ y_i \\ \mu_i \end{bmatrix}.$$

Spacer steps

- Given (x, y, μ) during iterations
- Compute a unique feasible pair $(\tilde{y}, \tilde{\mu})$
- Evaluate the residual at $(x, \tilde{y}, \tilde{\mu})$
- Choose the point if it has less residual than the one of (x, y, μ)

Reserves, interruptible load, demand response

- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Consumers may also be able to reduce consumption for short periods
- Alternative to sharp price increases during peak periods
- Constraints linking energy "bids" and reserve "bids"

$$v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

- Multiple scenarios - linking constraints on bids require "bid curve to be monotone"

Price taking: model is MOPEC

Consumption d_k, demand response r_k, energy v_j, reserves u_j, prices π

$$\text{Consumer} \quad \max_{(d_k, r_k) \in \mathcal{C}} \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{s.t. } v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

$$\text{Transmission} \quad \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

Market clearing

$$0 \leq \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A} f \geq 0$$

$$0 \leq \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \geq 0$$

Large consumer is price making: MPEC

Leader/follower

$$\text{Consumer} \quad \max \text{utility}(d_k) - \pi_d{}^T d_k + \text{profit}(r_k, \pi_r)$$

with the constraints:

$$(d_k, r_k) \in \mathcal{C}$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}'} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{Transmission} \quad \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

$$0 \le \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A}f \ge 0$$

$$0 \le \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \ge 0$$

Solution and observations

- Formulate as MIP, add mononticity constraints and scenarios
- New Zealand (NZEM) data, large consumer at bottom of South Island
- Expected difference percentage between "wait and see" solutions versus model solution (evaluated post optimality with simulation)

Sample Size	1	2	4	6	8
Expected diff	31.34	17.83	9.22	7.35	9.26
Standard dev	22.86	9.62	4.86	7.69	6.59
Bound gap (%)	0	0	0	12.7	24.8

- More samples better(!)
- More research to model/solve more detailed problems

Dual Representation of Risk Measures

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \leq \lambda_i \leq p_i/(1-\alpha), \sum_i \lambda_i = 1\}$, then

$$\rho(Z) = \overline{CVaR}_{\alpha}(Z)$$

- Special case of a Quadratic Support Function

$$\rho(y) = \sup_{u \in U} \langle u, By + b \rangle - \frac{1}{2} \langle u, Mu \rangle$$

- EMP allows any Quadratic Support Function to be defined and facilitates a model transformation to a tractable form for solution

Addition: compose equilibria with QS functions

- Add soft penalties to objectives and/or within constraints:

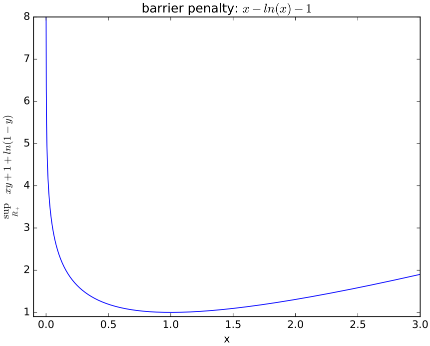
$$\min_{x} \theta(x) + \rho_O(F(x))$$
$$\text{s.t. } \rho_C(g(x)) \leq 0$$

```
QS g rhoC udef B M
...

QSF cvarup F rhoO theta p
```

- $batinclude QSprimal modname using emp min obj
- Allow modeler to compose QS functions automatically

- Can solve using MCP or primal reformulations
- More general conjugate functions also possible:



The link to MOPEC

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\rho(y) = \sup_{u \in U} \langle u, y \rangle - \frac{1}{2} \langle u, Mu \rangle$$

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

$$0 \in \partial\theta(x) + \nabla F(x)^T u + N_X(x)$$

$$0 \in -u + \partial\rho(F(x)) \iff 0 \in -F(x) + Mu + N_U(u)$$

This is a MOPEC, and we have multiple copies of this for each agent