Optimization and Equilibrium in Energy Economics

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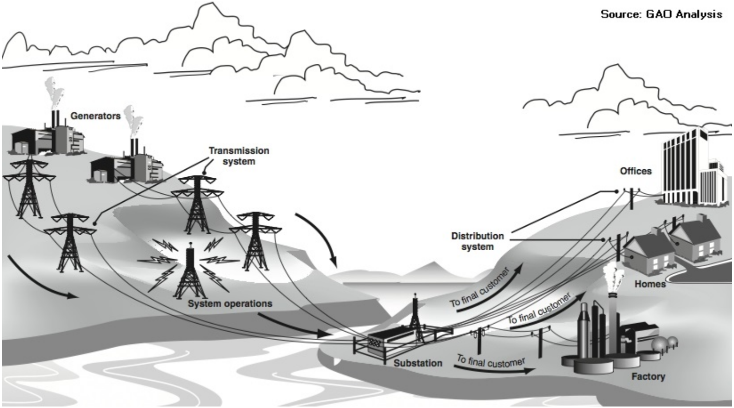
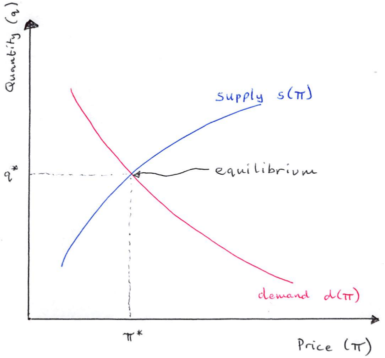
BEAM-ME Meeting, Frankfurt, Germany
September 28, 2017

Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $\geq \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

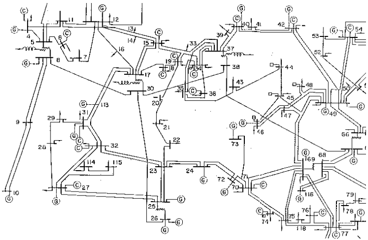
Single market, single good: equilibrium



Walras: $0 \leq s(\pi) - d(\pi) \perp \pi \geq 0$

Market design and rules to foster competitive behavior/efficiency

- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network

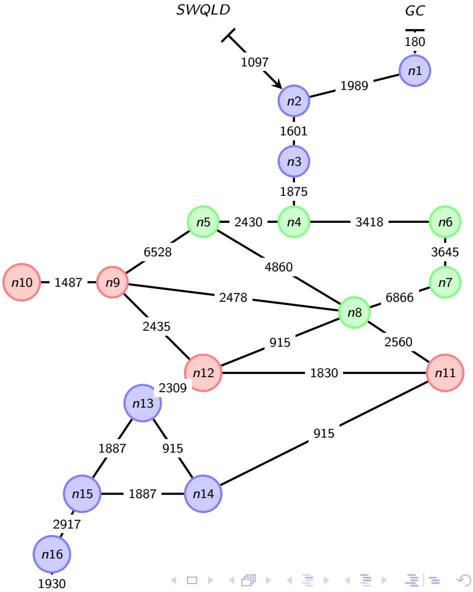


- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution

A Simple Network Model

Load segments s represent electrical load at various instances

- d_n^s Demand at node n in load segment s (MWe)
- X_i^s Generation by unit i (MWe)
- F_L^s Net electricity transmission on link L (MWe)
- Y_n^s Net supply at node n (MWe)
- π_n^s Wholesale price ($ per MWhe)



Nodes n, load segments s, generators i, Ψ is node-generator map

$$\max_{X,F,d,Y} \quad \sum_s \left(W(d^s(\lambda^s)) - \sum_i c_i(X_i^s) \right)$$

$$\text{s.t.} \quad \Psi(X^s) - d^s(\lambda^s) = Y^s$$

$$0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s$$

$$Y \in \mathcal{X}$$

where the network is described using:

$$\mathcal{X} = \left\{ Y : \exists F, F^s = \mathcal{H} Y^s, -\overline{F}^s \le F^s \le \overline{F}^s, \sum_n Y_n^s \ge 0, \forall s \right\}$$

- Key issue: decompose. Introduce multiplier π^s on supply demand constraint (and use $\lambda^s := \pi^s$)
- How different approximations of $\mathcal{X}$ affect the overall solution

Decomposition by prices π

$$\max_{X,d,Y} \quad \sum_s \left(W(d^s(\pi^s)) - \sum_i c_i(X_i^s) + \pi^s(\Psi(X^s) - d^s(\pi^s) - Y^s) \right)$$

$$\text{s.t.} \quad 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_s X_i^s$$

$$\sum_i Y_i^s \geq 0, -\overline{F}^s \leq \mathcal{H} Y^s \leq \overline{F}^s$$

Problem then decouples into multiple optimizations

$$\max_d \sum_s (W(d^s(\pi^s)) - \pi^s d^s(\pi^s)) + \max_X \sum_s \left(\pi^s \Psi(X^s) - \sum_i c_i(X_i^s) \right)$$

$$+ \max_Y \sum_s -\pi^s Y^s$$

$$\text{s.t. } 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_s X_i^s, \sum_i Y_i^s \geq 0, -\overline{F}^s \leq \mathcal{H} Y^s \leq \overline{F}^s$$

Case $\mathcal{H}$: Loop flow model

$$\max_{d} \quad \sum_{s} \left(W(d^s(\pi^s)) - \pi^s d^s(\pi^s) \right)$$

$$+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right)$$

$$\text{s.t.} \quad 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_{s} X_i^s$$

$$+ \max_{Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad \sum_{i} Y_i^s \geq 0, -\overline{F}^s \leq \mathcal{H} Y^s \leq \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\pi^s) - Y^s = 0$$

Example of a MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves } h(x, \pi) = 0$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h pi
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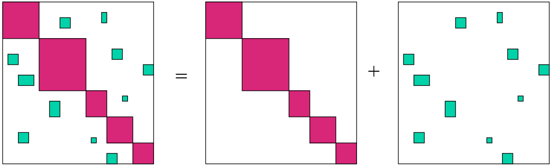
- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



Approximating details of transmission

Let $\mathcal{A}$ be the node-arc incidence matrix, $\mathcal{H}$ be the shift matrix, $\mathcal{L}$ be the loop constraint matrix. Standard results show:

$$\mathcal{X} = \{Y : \exists F, F = \mathcal{H}Y, F \in \mathcal{F}\}$$
$$\mathcal{X} = \left\{Y : \exists (F, \theta), Y = \mathcal{A}F, B\mathcal{A}^T\theta = F, \theta \in \Theta, F \in \mathcal{F}\right\}$$
$$\mathcal{X} = \{Y : \exists F, Y = \mathcal{A}F, \mathcal{L}F = 0, F \in \mathcal{F}\}$$

Loopflow model (using $\mathcal{A}, \mathcal{L}$)

$$\max_{d} \quad \sum_{s} \left(W(d^s(\pi^s)) - \pi^s d^s(\pi^s) \right)$$

$$+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right)$$

$$\text{s.t.} \quad 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_{s} X_i^s$$

$$+ \max_{F,Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad Y^s = \mathcal{A} F^s, \mathcal{L} F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\pi^s) - Y^s = 0$$

Network model

Drop loop constraints:

$$\max_{d} \quad \sum_{s} \left(W(d^s(\pi^s)) - \pi^s d^s(\pi^s) \right)$$

$$+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right)$$

$$\text{s.t.} \quad 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_{s} X_i^s$$

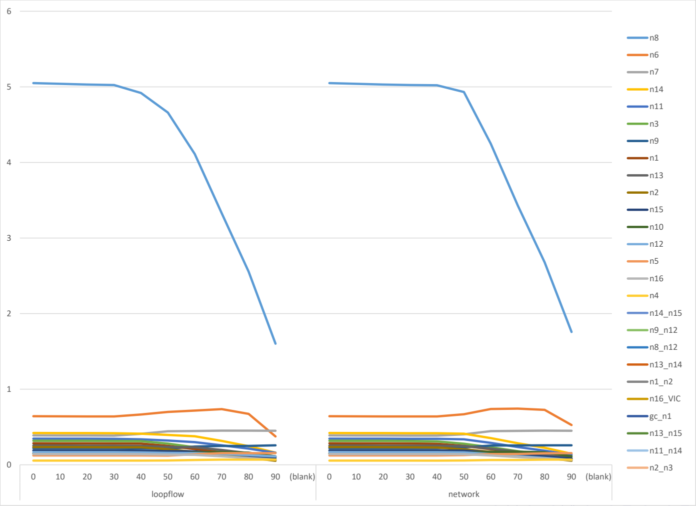
$$+ \max_{F,Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad Y^s = \mathcal{A} F^s, -\overline{F}^s \le F^s \le \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\pi^s) - Y^s = 0$$

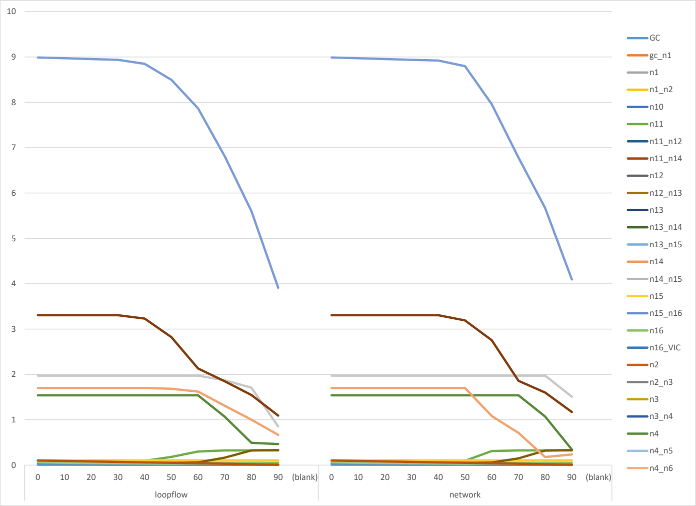
Comparing Network and Loopflow: Demand

Here we look at simulations which impose a proportional reduction in transmission across the network. The *network* and *loopflow* models demonstrate similar responses:



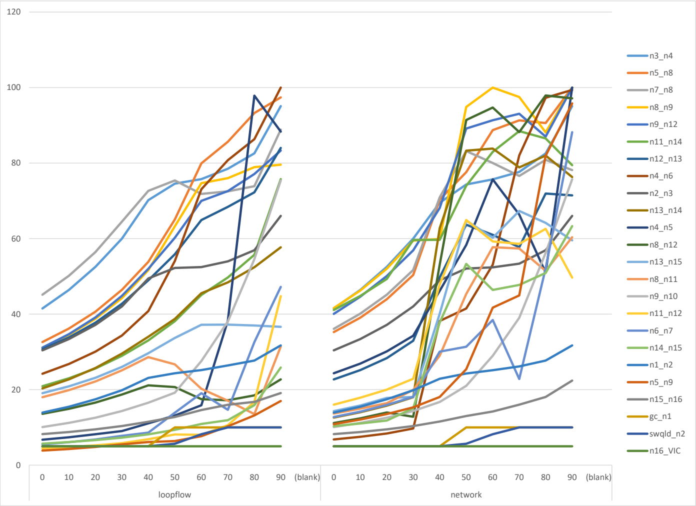
Comparing Network and Loopflow: Generation

Likewise, generation is similar in the two models:



Comparing Network and Loopflow: Transmission

Network transmission levels reveal that the two models are quite different:



The Game: update red, blue and purple components

$$
\begin{aligned}
\max_{d} \quad & \sum_s \left(W(d^s(\pi^s)) - \pi^s d^s(\pi^s)\right) \\
+ \max_{X} \quad & \sum_s \left(\pi^s \Psi(X^s) - \sum_i c_i(X_i^s)\right) \\
\text{s.t.} \quad & 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_s X_i^s \\
+ \max_{Y} \quad & \sum_s -\pi^s Y^s \\
\text{s.t.} \quad & \sum_i Y_i^s \geq 0, -\overline{F}^s \leq \mathcal{H} Y^s \leq \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\pi^s) - Y^s = 0
$$

Top down/bottom up

- $\lambda^s = \pi^s$ so use complementarity to expose multipliers
- Change interaction via new price mechanisms
- All network constraints encapsulated in (bottom up) NLP (or its approximation by dropping $\mathcal{L}F^s = 0$):

$$\max_{F,Y} \quad \sum_s -\pi^s Y^s$$
$$\text{s.t.} \quad Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s$$

- Could instead use the NLP over Y with $\mathcal{H}$
- Clear how to instrument different behavior or different policies in interactions (e.g. Cournot, etc) within EMP
- Can add additional detail into top level economic model describing consumers and producers
- Can solve iteratively using SELKIE

Pricing

Our implementation of the heterogeneous demand model incorporates three alternative pricing rules. The first is *average cost pricing*, defined by

$$P_{\text{ACP}} = \frac{\sum_{jn \in \mathcal{R}_{\text{ACP}}} \sum_s p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\text{ACP}}} \sum_s q_{jns}}$$

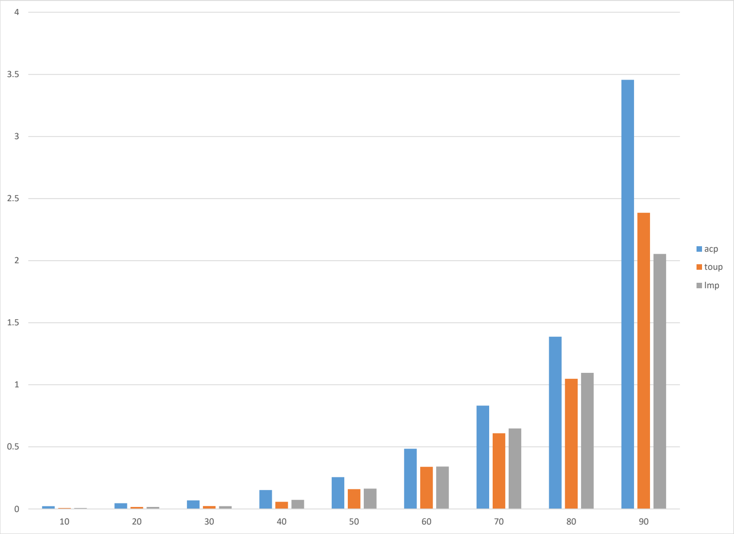
The second is *time of use pricing*, defined by:

$$P_s^{\text{TOU}} = \frac{\sum_{jn \in \mathcal{R}_{\text{TOU}}} p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\text{TOU}}} q_{jns}}$$

The third is *location marginal pricing* corresponding to the wholesale prices denoted P_{ns} above. Prices for individual demand segments are then assigned:

$$p_{jns} = \begin{cases} P_{\text{ACP}} & (jn) \in \mathcal{R}_{\text{ACP}} \\ P_s^{\text{TOU}} & (jn) \in \mathcal{R}_{\text{TOU}} \\ P_{ns} & (jn) \in \mathcal{R}_{\text{LMP}} \end{cases}$$

Smart Metering Lowers the Cost of Congestion



Other specializations and extensions

$$\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \le 0, \forall i, f(x, z, \pi) = 0$$

$$\pi \text{ solves VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: $z(x_i, x_{-i})$ shared
- Shared constraints: f is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Economic Application

- Model is a partial equilibrium, geographic exchange model.
- Goods are distinguished by region of origin.
- There is one unit of region r goods.
- These goods may be consumed in region r or they may be exported.
- Each region solves:

$$\min_{X,T_r} f_r(X,T) \text{ s.t. } H(X,T)=0,\; T_j=\bar{T}_j, j\neq r$$

  where $f_r(X,T)$ is a quadratic form and $H(X,T)$ defines X uniquely as a function of T, the taxes and tariffs.
- $H(X,T)$ defines an equilibrium; here it is simply a set of equations, not a complementarity problem
- Applications: Brexit, modified GATT, Russian Sanctions

Model statistics and performance comparison of the EPEC

MCP statistics according to the shared variable formulation		
Replication	Switching	Substitution
12,144 rows/cols 544,019 non-zeros 0.37% dense	6,578 rows/cols 444,243 non-zeros 1.03% dense	129,030 rows/cols 3,561,521 non-zeros 0.02% dense

PATH			Shared variable formulation (major, time)		
crash	spacer	prox	Replication	Switching	Substitution
✓		✓	7 iters 8 secs	20 iters 22 secs	20 iters 406 secs
		✓	24 iters 376 secs	22 iters 19 secs	21 iters 395 secs
	✓		8 iters 28 secs	8 iters 18 secs	8 iters 219 secs

Results

Gauss-Seidel residuals

Iteration	deviation
1	3.14930
2	0.90970
3	0.14224
4	0.02285
5	0.00373
6	0.00061
7	0.00010
8	0.00002
9	0.00000

Tariff revenue

region	SysOpt	MOPEC
1	0.117	0.012
2	0.517	0.407
3	0.496	0.214
4	0.517	0.407
5	0.117	0.012

- Note that competitive solution produces much less revenue than system optimal solution
- Model has non-convex objective, but each subproblem is solved globally (lindoglobal)

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- dualvar (use multipliers from one agent as variables for another)
- QS functions (both in objectives and constraints)
- implicit functions and shared constraints

- Currently available within GAMS
- Some solution algorithms implemented in modeling system - limitations on size, decomposition and advanced algorithms
- Can evaluate effects of regulations and their implementation in a competitive environment

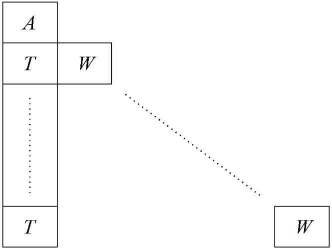
Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what
- It's available (in GAMS)
- Allows use and control of dual variables / prices
- MOPEC facilitates easy "behavior" description at model level
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- New decomposition algorithms available to modeler (Gauss Seidel, Randomized Sweeps, Gauss Southwell, Grouping of subproblems)
- Can evaluate effects of regulations and their implementation in a competitive environment
- Stochastic equilibria - clearing the market in each scenario
- Ability to trade risk using contracts

Stochastic: Agents have recourse?

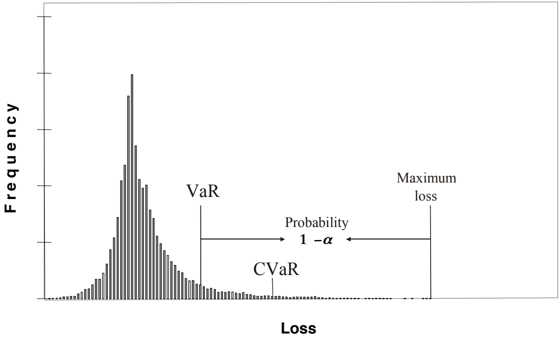
- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$
\begin{aligned}
\text{SP:}\ \min \quad & c(x^1) + \rho[q^T x^2] \\
\text{s.t.} \quad & Ax^1 = b, \quad x^1 \geq 0, \\
& T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega), \\
& x^2(\omega) \geq 0, \forall \omega \in \Omega.
\end{aligned}
$$



Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0} \quad p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0} \quad C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0}} \quad -p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$

Trading risk: pay σ_ω now, deliver 1 later in ω

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0, t^C} \quad \sigma t^C + p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) - t^C_\omega \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0, t^T} \quad \sigma t^T + C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) - t^T_\omega \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0, t^H}} \quad \sigma t^H - p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) - t^H_\omega \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$

$$0 \leq \sigma_\omega \perp t^C_\omega + t^T_\omega + t^H_\omega \geq 0, \forall \omega \quad \sigma = (\sigma_\omega)$$

Reserves, interruptible load, demand response

- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Consumers may also be able to reduce consumption for short periods
- Alternative to sharp price increases during peak periods
- Constraints linking energy "bids" and reserve "bids"

$$v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

- Multiple scenarios - linking constraints on bids require "bid curve to be monotone"

Price taking: model is MOPEC

Consumption d_k, demand response r_k, energy v_j, reserves u_j, prices π

$$\text{Consumer} \quad \max_{(d_k, r_k) \in \mathcal{C}} \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{s.t.} \quad v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

$$\text{Transmission} \quad \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

Market clearing

$$0 \leq \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A} f \geq 0$$

$$0 \leq \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \geq 0$$

Large consumer is price making: MPEC

Leader/follower

$$\text{Consumer} \quad \max \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

with the constraints:

$$(d_k, r_k) \in \mathcal{C}$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}'} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{Transmission} \quad \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

$$0 \le \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A} f \ge 0$$

$$0 \le \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \ge 0$$

Solution and observations

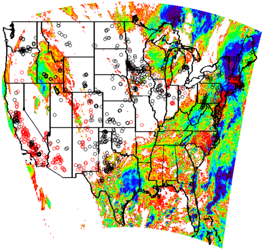
- Formulate as MIP, add mononticity constraints and scenarios
- New Zealand (NZEM) data, large consumer at bottom of South Island
- Expected difference percentage between "wait and see" solutions versus model solution (evaluated post optimality with simulation)

Sample Size	1	2	4	6	8
Expected diff	31.34	17.83	9.22	7.35	9.26
Standard dev	22.86	9.62	4.86	7.69	6.59
Bound gap (%)	0	0	0	12.7	24.8

- More samples better(!)
- More research to model/solve more detailed problems

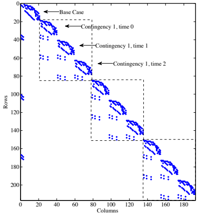
Satellite data, FERC and Reserves

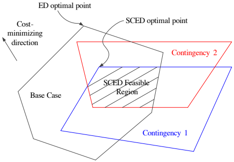
Solar transmittance and power



- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Use 12 hour cloud cover forecasts to reduce reserves

- Federal Energy Regulatory Commission (FERC) contract to build models and data
- Provided on NEOS (Network enabled optimization system)





- Integrate satellite forecast data with power system data and smoke models to provide reliability and savings outcomes