Dynamic Risked Equilibria

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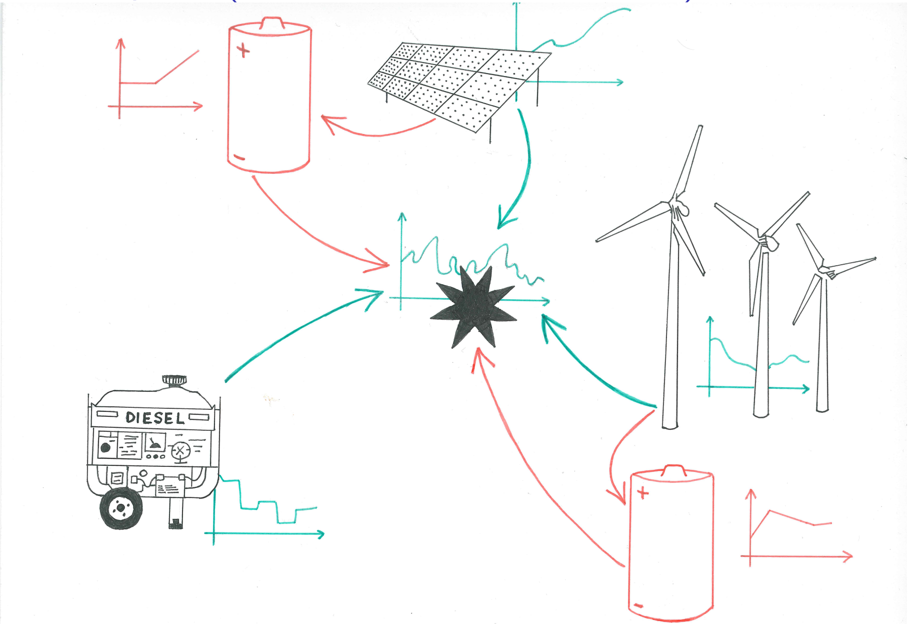
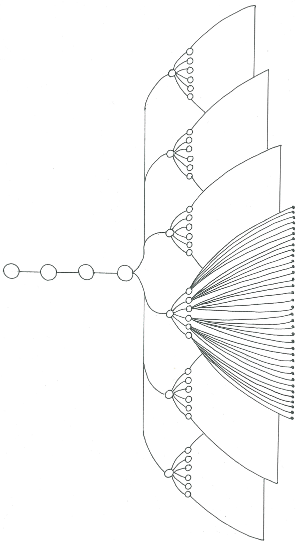
The setup: a=(solar, wind, diesel, consumer)



Dynamics and uncertainties

- Scenario tree is data
- T stages (use 6 here)
- Nodes $n \in \mathcal{N}$, n_+ successors
- Stagewise probabilities $\mu(m)$ to move to next stage $m \in n_+$
- Uncertainties (wind flow, cloud cover, rainfall, demand) $\omega_a(n)$
- Actions u_a for each agent (dispatch, curtail, generate, shed), with costs C_a
- State and shared variables (storage, prices)
- Recursive (nested) definition of expected cost-to-go: $\theta(n) = \sum_{m \in n_+} \mu(m) \left(\sum_{a \in \mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$

$t \in 0, 1, 2, 3, 4, 5, 6$



Model

$$
\begin{aligned}
\text{SO:} \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad & \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0) \\
\text{s.t.} \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\
& \theta(n) \geq \sum_{m\in n_+} \mu(m) \left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m) \right) \\
& \sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0
\end{aligned}
$$

- Actions u_a (dispatch, curtail, generate, shed), with costs C_a
- g_a converts actions into energy.
- Storage allows energy to be moved across stages (batteries, pump, compressed air, etc)
- Optimization forces an equality to recover definition of $\theta(n)$

Decomposition (of agents) by prices π

Split up θ into agent contributions θ_a and add weighted constraints into objective:

$$
\begin{aligned}
\min_{(\theta,u,x)\in\mathcal{F}} \quad & \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta_a(0) - \pi^T\left(g_a(u_a(n))\right) \\
\text{s.t. } \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\
& \theta_a(n) \geq \sum_{m\in n_+} \mu(m)\left(C_a(u_a(m)) + \theta_a(m)\right)
\end{aligned}
$$

Problem then decouples into multiple optimizations (over tree)

$$
\begin{aligned}
\mathrm{AO}(a,\pi): \min_{(\theta,u,x)\in\mathcal{F}} \quad & Z_a(0) + \theta_a(0) \\
\text{s.t. } \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\
& \theta_a(n) \geq \sum_{m\in n_+} \mu(m)(Z_a(m) + \theta_a(m)) \\
& Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n))
\end{aligned}
$$

SO equivalent to MOPEC (price takers)

- Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\{(u_a(n), \theta_a(n)), n \in \mathcal{N}\} \in \arg\min \mathrm{AO}(a, \pi)$$

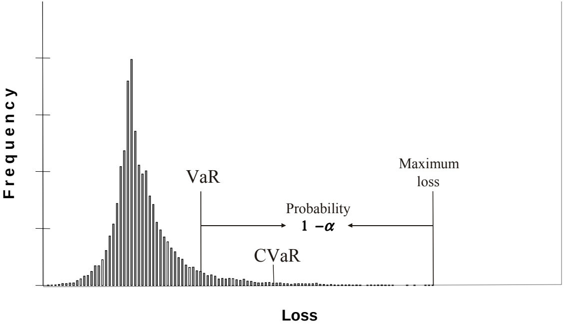
and

$$0 \leq \sum_{a \in \mathcal{A}} g_a(u_a(n)) \perp \pi(n) \geq 0$$

- One optimization per agent, coupled together with solution of complementarity (equilibrium) constraint
- Overall, a (Generalized) Nash Equilibrium problem (or a MOPEC), solvable as a large scale complementarity problem (replacing the optimizations by their KKT conditions) using the PATH solver
- MOPEC(μ) equilibrium = SO(μ) optimum
- But in practice there is a gap between SO and MOPEC. Why?

One explanation: risk

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z] = \sup_{\mu \in \mathcal{D}} \mu^T Z$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \leq \lambda_i \leq p_i/(1-\alpha), \sum_i \lambda_i = 1\}$, then

$$\rho(Z) = \overline{CVaR}_\alpha(Z)$$

Dynamic risk averse equilibrium

Replace each agents problem AO by RA:

$$
\begin{aligned}
\mathrm{RA}(a, \pi, \mathcal{D}_a): \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad & Z_a(0) + \theta_a(0) \\
\text{s.t.} \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\
& \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n) \\
& Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n))
\end{aligned}
$$

- $p_a^k(m)$ are extreme points of the agents risk set at m
- $\mathrm{RE}(\mathcal{D}_{\mathcal{A}}) \equiv \mathrm{RA}(a, \pi, \mathcal{D}_a)$ for all $a \in \mathcal{A}$ and market clearing
- $\mathrm{RE}(\mathcal{D}_{\mathcal{A}})$ equilibrium $\neq$ $\mathrm{SO}(\mathcal{D}_s)$ risk-averse optimum
- Must solve using equilibrium solver
- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Risk trading can recover system optimum

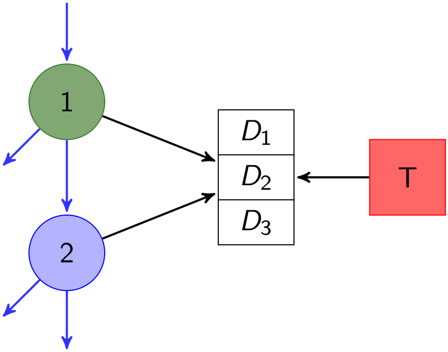
- Contracts for trading risk enable agents to enjoy pooled risk
- Perfectly competitive markets can be inefficient if such contracts are missing
- Example: Meridian-Genesis swaption contract enables more efficient operation of thermal and hydro plant by decreasing risk for both parties
- Theorem (PFW, 2016; FP, 2018): If markets for risk (using dynamic coherent risk measures) are complete then a perfectly competitive (risk-averse) equilibrium corresponds to a risk-averse social optimum using a social risk measure
- RET($\mathcal{D}_{\mathcal{A}}$) equilibrium with contracts = SO($\mathcal{D}_s$) risk-averse optimum

$$\mathcal{D}_s = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a$$

- In battery problem can recover by trading the system optimal solution (and its properties) since the retailer/generator agent is risk neutral

Cascading hydro-thermal system: XMGD

- Two hydros on same river: '1' is above '2': spill or release with generation
- Thermal generator 'T' and consumer (risk neutral)



- Competing firms (collections of consumers, or generators in energy market)
- Each firm minimizes objective independently
- Look at joint ownership issues (firms represented colors: X, M, G)
- Label consumer as 'D' (but can be partitioned into 'D_1','D_2','D_3')

Equilibria with cascades: water prices

T_{ab} encodes the water network, water prices are multipliers on:

$$x_a(n_-) + \sum_b T_{ab} u_b(n) + \omega_a(n) \geq x_a(n)$$

Allows interaction with other water uses (irrigation, tourism, conservation)

$$\text{AO}(a, \pi, \mathcal{D}_a): \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad Z_a(0) + \theta_a(0)$$
$$\text{s.t. } \theta_a(n) \geq \sum_{m\in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n)$$

where $Z_a(n)$ is updated to incorporate prices of interactions

$$Z_a(n; u, x) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) + \alpha_a(n)(x_a(n) - x_a(n_-) - \omega_a(n)) - \sum_{b\in\mathcal{A}} \alpha_b(n) T_{ba} u_a(n),$$

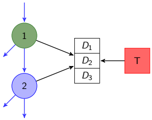
Average inflow 0.6

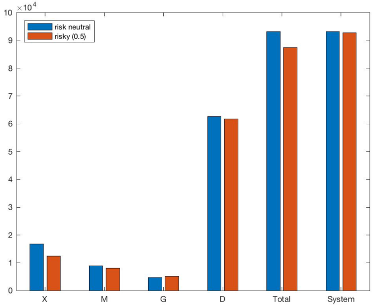
XMGD

TotRA $= 87351$

SysRA $= 92763$

SysRN $= 93109$





MMGD

TotRA $= 87351$

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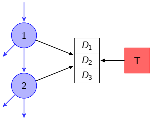
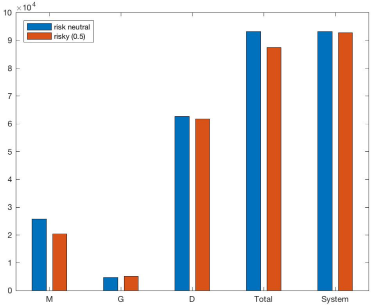
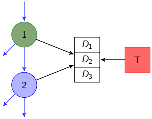




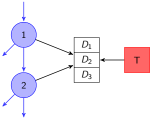
- Ownership of both hydros is not beneficial with competitive pricing of water

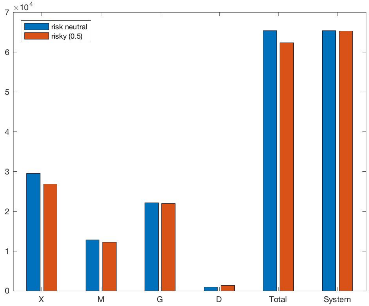
Low inflow 0.1

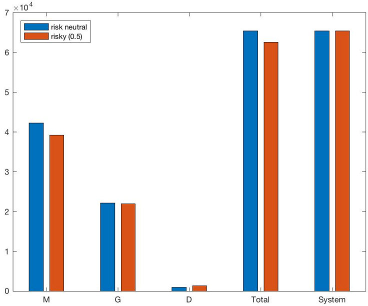
XMGD
TotRA = 62382
SysRA = 65269
SysRN = 65375



MMGD
TotRA = 62552
SysRA = 65371
SysRN = 65375



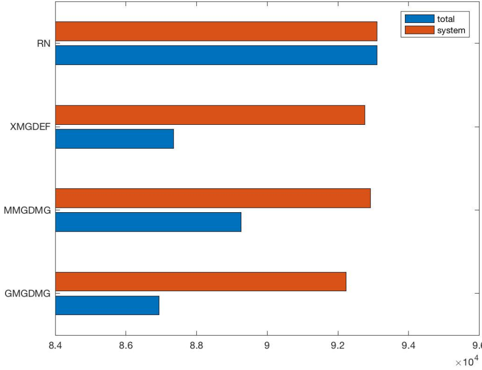




- Not true: risk averse and low inflows shows advantage to co-ownership of hydros

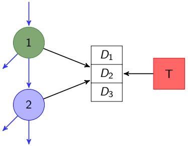
Vertical integration/asset swaps

- SysRN and TotRN in risk neutral case, followed by SysRA and TotRA for three cases depicted on left

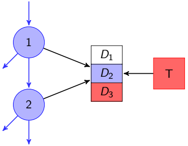


- Vertical integration and risk matters!

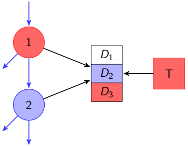
Base: XMGDEF



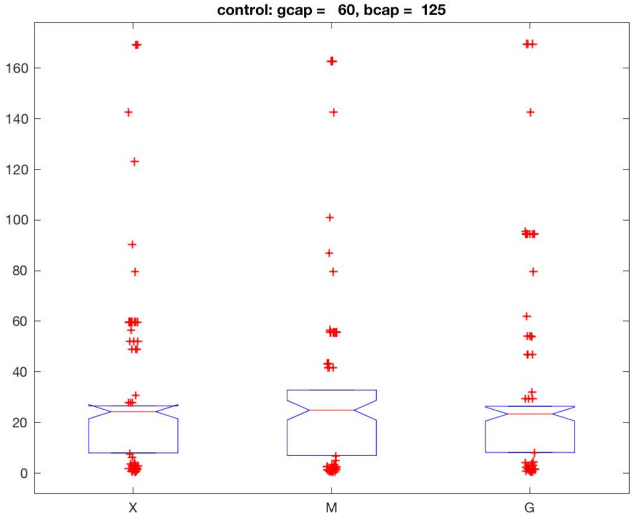
Vertical integration: MMGDMG



VI & Asset Swap: GMGDMG



XMGDEF/MMGDMG/GMGDMG (water price differences)



Other specializations and extensions (see Kim [Fri, # 101])

$$\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \leq 0, \forall i, f(x, z, \pi) = 0$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: $z(x_i, x_{-i})$ shared
- Shared constraints: f is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what (MOPEC facilitates easy "behavior" description at model level)
- It's available (in GAMS)
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- Stochastic equilibria - clearing the market in each scenario (risk measures specified via OVF)
- Ability to trade risk using contracts