Dynamic Risked Equilibria

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The setup: $a = \text{(solar, wind, diesel, consumer)}$
Dynamics and uncertainties

- Scenario tree is data
- $T$ stages (use 6 here)
- Nodes $n \in \mathcal{N}$, $n_+$ successors
- Stagewise probabilities $\mu(m)$ to move to next stage $m \in n_+$
- Uncertainties (wind flow, cloud cover, rainfall, demand) $\omega_a(n)$
- Actions $u_a$ for each agent (dispatch, curtail, generate, shed), with costs $C_a$
- State and shared variables (storage, prices)
- Recursive (nested) definition of expected cost-to-go: $\theta(n) = 
\sum_{m \in n_+} \mu(m) \left( \sum_{a \in \mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$

$t \in 0, 1, 2, 3, 4, 5, 6$
Model

\[
\begin{align*}
\text{SO:} \quad & \min_{(\theta, u, x) \in \mathcal{F}} \sum_{a \in \mathcal{A}} C_a(u_a(0)) + \theta(0) \\
\text{s.t.} \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\
& \theta(n) \geq \sum_{m \in n_+} \mu(m) \left( \sum_{a \in \mathcal{A}} C_a(u_a(m)) + \theta(m) \right) \\
& \sum_{a \in \mathcal{A}} g_a(u_a(n)) \geq 0
\end{align*}
\]

- Actions $u_a$ (dispatch, curtail, generate, shed), with costs $C_a$
- $g_a$ converts actions into energy.
- Storage allows energy to be moved across stages (batteries, pump, compressed air, etc)
- Optimization forces an equality to recover definition of $\theta(n)$
Decomposition (of agents) by prices $\pi$

Split up $\theta$ into agent contributions $\theta_a$ and add weighted constraints into objective:

$$\min_{(\theta, u, x) \in F} \sum_{a \in A} C_a(u_a(0)) + \theta_a(0) - \pi^T (g_a(u_a(n)))$$

s.t. $x_a(n) = x_a(n-) - u_a(n) + \omega_a(n)$

$$\theta_a(n) \geq \sum_{m \in n_+} \mu(m)(C_a(u_a(m)) + \theta_a(m))$$

Problem then decouples into multiple optimizations (over tree)

$$AO(a, \pi): \min_{(\theta, u, x) \in F} Z_a(0) + \theta_a(0)$$

s.t. $x_a(n) = x_a(n-) - u_a(n) + \omega_a(n)$

$$\theta_a(n) \geq \sum_{m \in n_+} \mu(m)(Z_a(m) + \theta_a(m))$$

$$Z_a(n) = C_a(u_a(n)) - \pi(n)g_a(u_a(n))$$
SO equivalent to MOPEC (price takers)

- Perfectly competitive (Walrasian) equilibrium is a MOPEC

\[ \{(u_a(n), \theta_a(n)), n \in \mathcal{N}\} \in \text{arg min } AO(a, \pi) \]

and

\[ 0 \leq \sum_{a \in \mathcal{A}} g_a(u_a(n)) \perp \pi(n) \geq 0 \]

- One optimization per agent, coupled together with solution of complementarity (equilibrium) constraint

- Overall, a (Generalized) Nash Equilibrium problem (or a MOPEC), solvable as a large scale complementarity problem (replacing the optimizations by their KKT conditions) using the PATH solver

- MOPEC(\(\mu\)) equilibrium = SO(\(\mu\)) optimum

- But in practice there is a gap between SO and MOPEC. Why?
One explanation: risk

- Modern approach to modeling risk aversion uses concept of risk measures
  - $\text{CVaR}_\alpha$: mean of upper tail beyond $\alpha$-quantile (e.g. $\alpha = 0.95$)

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}[Z] = \sup_{\mu \in \mathcal{D}} \mu^T Z$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$  
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \leq \lambda_i \leq p_i/(1 - \alpha), \sum_i \lambda_i = 1\}$, then
  $$\rho(Z) = \text{CVaR}_\alpha(Z)$$
Dynamic risk averse equilibrium

Replace each agent’s problem AO by RA:

\[
\text{RA}(a, \pi, D_a): \min_{(\theta, u, x) \in F} Z_a(0) + \theta_a(0)
\]

s.t. \(x_a(n) = x_a(n-) - u_a(n) + \omega_a(n)\)

\[
\theta_a(n) \geq \sum_{m \in n^+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n)
\]

\[
Z_a(n) = C_a(u_a(n)) - \pi(n)g_a(u_a(n))
\]

- \(p_a^k(m)\) are extreme points of the agents’ risk set at \(m\)
- \(\text{RE}(D_A) \equiv \text{RA}(a, \pi, D_a)\) for all \(a \in A\) and market clearing
- \(\text{RE}(D_A)\) equilibrium \(\neq \text{SO}(D_s)\) risk-averse optimum
- Must solve using equilibrium solver
- Attempt to construct agreement on what would be the worst-case outcome by trading risk
Risk trading can recover system optimum

- **Contracts** for trading risk enable agents to enjoy pooled risk
- Perfectly competitive markets can be *inefficient* if such contracts are missing
- Example: Meridian-Genesis swaption contract enables more efficient operation of thermal and hydro plant by decreasing risk for both parties
- Theorem (PFW, 2016; FP, 2018): If markets for risk (using dynamic coherent risk measures) are complete then a perfectly competitive (risk-averse) equilibrium corresponds to a risk-averse social optimum using a social risk measure
- \( \text{RET}(\mathcal{D}_A) \) equilibrium with contracts = \( \text{SO}(\mathcal{D}_s) \) risk-averse optimum

\[
\mathcal{D}_s = \bigcap_{a \in A} \mathcal{D}_a
\]

- In battery problem can recover by trading the system optimal solution (and its properties) since the retailer/generator agent is risk neutral
Cascading hydro-thermal system: XMGD

- Two hydros on same river: '1' is above '2': spill or release with generation
- Thermal generator 'T' and consumer (risk neutral)

Competing firms (collections of consumers, or generators in energy market)
- Each firm minimizes objective independently
- Look at joint ownership issues (firms represented colors: X, M, G)
- Label consumer as 'D' (but can be partitioned into 'D_1', 'D_2', 'D_3')
Equilibria with cascades: water prices

$T_{ab}$ encodes the water network, water prices are multipliers on:

$$x_a(n_-) + \sum_b T_{ab} u_b(n) + \omega_a(n) \geq x_a(n)$$

Allows interaction with other water uses (irrigation, tourism, conservation)

$$\text{AO}(a, \pi, D_a): \min_{(\theta, u, x) \in F} Z_a(0) + \theta_a(0)$$

s.t. $\theta_a(n) \geq \sum_{m \in n^+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n)$

where $Z_a(n)$ is updated to incorporate prices of interactions

$$Z_a(n; u, x) = C_a(u_a(n)) - \pi(n)g_a(u_a(n)) + \alpha_a(n)(x_a(n) - x_a(n_-) - \omega_a(n)) - \sum_{b \in A} \alpha_b(n) T_{ba} u_a(n),$$
Ownership of both hydros is not beneficial with competitive pricing of water
Low inflow 0.1

**XMGD**

TotRA = 62382  
SysRA = 65269  
SysRN = 65375

**MMGD**

TotRA = 62552  
SysRA = 65371  
SysRN = 65375

- **Not true:** risk averse and low inflows shows advantage to co-ownership of hydros
Vertical integration/asset swaps

- SysRN and TotRN in risk neutral case, followed by SysRA and TotRA for three cases depicted on left

- Vertical integration and risk matters!

Base: XMGDEF

Vertical integration: MMGDMG

VI & Asset Swap: GMGDMG
XMGDEF/MMGDMG/GMGDMG (water price differences)

control: gcap = 60, bcap = 125
Other specializations and extensions (see Kim [Fri, # 101])

\[
\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \leq 0, \forall i, f(x, z, \pi) = 0
\]

\[\pi \text{ solves } \text{VI}(h(x, \cdot), C)\]

- NE: Nash equilibrium (no VI coupling constraints, \(g_i(x_i)\) only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: \(z(x_i, x_{-i})\) shared
- Shared constraints: \(f\) is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment
Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what (MOPEC facilitates easy “behavior” description at model level)
- It’s available (in GAMS)
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- Stochastic equilibria - clearing the market in each scenario (risk measures specified via OVF)
- Ability to trade risk using contracts