Coupled Optimization Models for Planning and Operation of Power Systems on Multiple Scales

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Computational Needs for the Next Generation Electric Grid,
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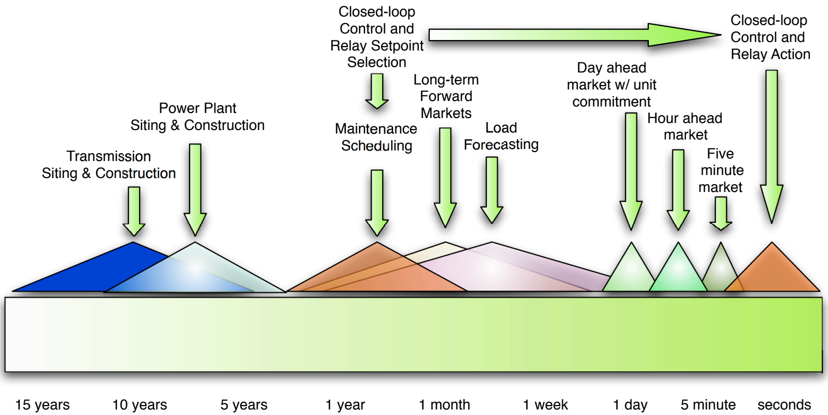
The premise

- The next generation electric grid will be more dynamic, flexible, constrained, and more complicated.
- Decision processes (in this environment) are predominantly hierarchical.
- Models to support such decision processes must also be layered or hierachical.
- Optimization and computation facilitate adaptivity, control, treatment of uncertainties and understanding of interaction effects.
- Coupling of smaller models with well defined interfaces allows validation, understanding, and enhanced solution techniques.

Representative decision-making timescales in electric power systems

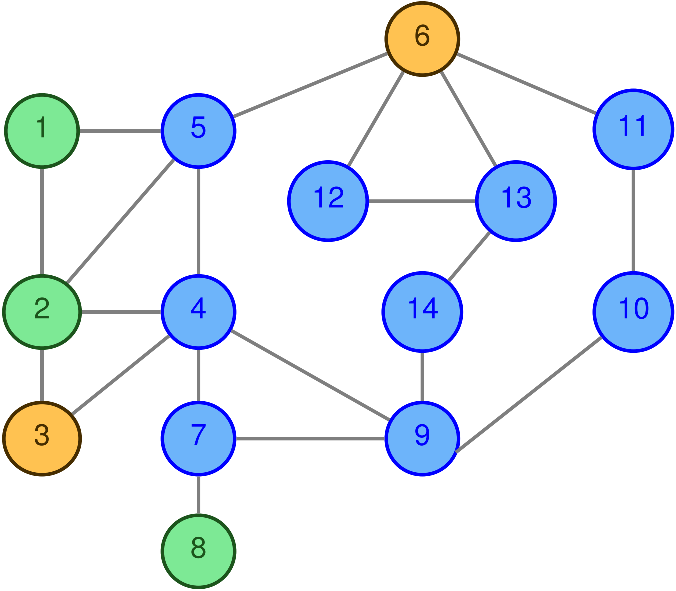


A monster model is difficult to validate, inflexible, prone to errors.

Transmission Line Expansion Model (1)

$$\begin{array}{lll} \min\limits_{x \in X} & \sum\limits_{\omega} \pi_\omega \sum\limits_{i \in N} d_i^\omega p_i^\omega(x) & \\ s.t. & Ax \leq b & \text{(RTO budget (and other) constraints)} \end{array}$$

N:	The set of all nodes
X:	The set of all line expansions being considered
x:	Amount of investment in line $x \in X$
ω:	Demand scenarios
π_ω:	Probability of scenario ω occuring
d_i^ω:	Demand of load node i in in scenario ω
$p_i^\omega(x)$:	Price (LMP) at load node i in scenario ω as a function of x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Generator Expansion (2)

$$\forall f \in F : \quad \min_{y_f} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j)$$

$$\text{s.t.} \quad \sum_{j \in G_f} y_j \leq h_f \qquad \text{(budget cons)}$$

$$y_f \geq 0$$

F: The set of firms
G_f: The set of all generators belonging to firm f
ω: Demand scenarios
y_j: Amount of investment in generator j
q_j: Real power generated at bus j
C_j: Cost function of generator j as a function of y_j and q_j
r: Interest Rate

Market Clearing Model (3)

$$
\begin{aligned}
\forall \omega: \quad & \min_{z,\theta,q} \sum_f \sum_{j \in G_f} C_j(y_j, q_j^\omega) & & \\
s.t. \quad & q_j^\omega - d_j^\omega = \sum_{i \in I(j)} z_{ij} & \forall j \in N (\perp p_j^\omega) \quad & \text{(flow balance)} \\
& z_{ij} = \Omega_{ij}(\theta_i - \theta_j) & \forall (i,j) \in A \quad & \text{(line data)} \\
& -b_{ij}(x) \leq z_{ij} \leq b_{ij}(x) & \forall (i,j) \in A \quad & \text{(line capacity)} \\
& -\underline{u}_j(y_j) \leq q_j \leq \bar{u}_j(y_j) & & \text{(gen capacity)}
\end{aligned}
$$

z_{ij}:	Real power flowing along the i-j arc
q_j:	Real power generated at bus j
θ_i:	Voltage phase angle at bus i
Ω_{ij}:	Susceptance of line i-j
$b_{ij}(x)$:	Line capacity as a function of x
$\underline{u}_j(x)$, $\bar{u}_j(x)$:	Generator j limits as a function of y

Flow of information

$$\begin{array}{lll} \min\limits_{x} & \text{Transmission line expansion (1)} & \mathbf{E}(f(p^{\omega}(x))) \\ s.t. & \forall f \in F \text{ Generator expansion (2)} & \Longrightarrow KKT(2) \\ & & x, q^{\omega} \mapsto y \\ & \forall \omega \text{ Market clearing (3)} & \Longrightarrow KKT(3) \\ & & x, y \mapsto q^{\omega}, p^{\omega}(x) \end{array}$$

Why isn't this just a monster model?

Solution methods

- Use deriviative free method for the upper level problem (1)
- Models (2) and (3) form an MCP (via EMP)
- Solve (3) as NLP using global solver, per scenario (SNLP)
- Solve (KKT(2) + KKT(3)) using EMP and PATH, then repeat
- Alternative: Can show (due to specific problem structure that there is a (convex) NLP whose KKT conditions are that MCP
- Useful for theoretical analysis
- Resulting problem is too large for NLP solvers
- Can show that "Gauss-Seidel/Jacobi" method on problems in (2) and (3) converges in this case - decoupling makes problem tractable for large scale instances

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

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SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.34	4.62	5.01	3.99
ω_2		4.71	4.07	4.25	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	0.00	622.02			377.98
f_2			283.22	216.79	

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
Probability	0.2	0.2	0.25	0.1	0.25
Demand Multiplier	4	6.5	9.5	8	8.9

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1		3.40	3.31	2.77	
ω_2		4.35	3.83	3.88	3.35
ω_3	3.53	5.30	4.66	5.04	3.99
ω_4	2.89	4.55	4.00	4.12	3.41
ω_5	3.27	5.00	4.41	4.68	3.73

Firm	y_1	y_2	y_3	y_6	y_8
f_1	194.39	469.99			335.61
f_2			292.89	207.11	

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
Probability	0.2	0.2	0.25	0.1	0.25
Demand Multiplier	4	6.5	9.5	8	8.9

EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1		5.04	4.45	0.00	
ω_2		4.37	3.86	3.83	3.35
ω_3	3.46	5.33	4.71	5.00	4.01
ω_4	0.00	5.31	4.67	4.97	3.99
ω_5	3.22	5.04	4.45	4.64	3.75

Firm	y_1	y_2	y_3	y_6	y_8
f_1	145.45	507.45			347.05
f_2			320.54	179.46	

- Coupling collections of (sub)-models with well defined (information sharing) interfaces facilitates:
  - appropriate detail and consistency of sub-model formulation (each of which may be very large scale, of different types (mixed integer, semidefinite, nonlinear, variational, etc) with different properties (linear, convex, discrete, smooth, etc))
  - ability for individual subproblem solution verification and engagement of decision makers
  - ability to treat uncertainty by stochastic and robust optimization at submodel level and with evolving resolution
  - ability to solve submodels to global optimality (by exploiting size, structure and model format specificity)

(A monster model that mixes several modeling formats loses its ability to exploit the underlying structure and provide guarantees on solution quality)

Extended Mathematical Programs

- Optimization models improve understanding of underlying systems and facilitate operational/strategic improvements under resource constraints
- Problem format is old/traditional

$$\min_{x} f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0$$

- Extended Mathematical Programs allow annotations of constraint functions to augment this format.
- Developing interfaces and exploiting hierarchical structure using computationally tractable algorithms will provide overall solution speed, understanding of localized effects, and value for the coupling of the system.

Nash Equilibria

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, q), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.
- Quantities q given exogenously, or via complementarity:

$$0 \leq H(x, q) \perp q \geq 0$$

- empinfo: equilibrium
  min loss(i) x(i) cons(i)
  vifunc H q
- Key difference: optimization assumes you control the complete system
- Equilibrium determines what activities run, and who produces what

Supply function equilibria

$$\begin{array}{lll} \text{OPF}(\alpha): & \min_y & \text{energy dispatch cost } (y, \alpha) \\ & \text{s.t.} & \text{conservation of power flow at nodes} \\ & & \text{Kirchoff's voltage law, and simple bound constraints} \end{array}$$

α are (given) price bids, parametric optimization

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$$\begin{array}{lll} \text{Leader}(\bar{\alpha}_{-i}): & \max_{\alpha_i, y, \lambda} & \text{firm } i\text{'s profit } (\alpha_i, y, \lambda) \\ & \text{s.t.} & 0 \le \alpha_i \le \hat{\alpha}_i \\ & & y \text{ solves OPF}(\alpha_i, \bar{\alpha}_{-i}) \end{array}$$

Note that objective involves multiplier from OPF problem

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This is an example of an MPCC since KKT form complementarity constraints

Multi-player EPEC and security constraints

- $(\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_m)$ is an equilibrium if

$$\bar{\alpha}_i \text{ solves Leader}(\bar{\alpha}_{-i}), \quad \forall i$$

- (Nonlinear) Nash Equilibrium where each player solves an MPCC

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- $(\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_m)$ is an equilibrium if

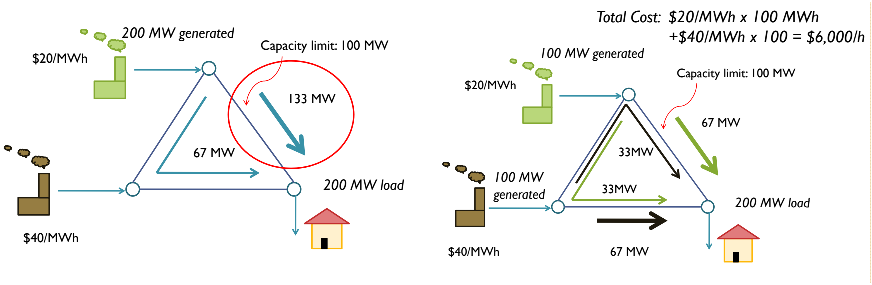
$$\bar{\alpha}_i \text{ solves Leader}(\bar{\alpha}_{-i}), \quad \forall i$$

- (Nonlinear) Nash Equilibrium where each player solves an MPCC
- MPCC is hard (lacks a constraint qualification)
- Nash Equilibrium is PPAD-complete (Chen et al, Papadimitriou et al)
- In practice, also require contingency (scenario) constraints imposed in the OPF problem

- Leader/follower game: Stackleberg
- Supply chains with "market leader"

Transmission switching

Opening lines in a transmission network can reduce cost



(a) Infeasible due to line capacity (b) Feasible dispatch

Need to use expensive generator due to power flow characteristics and capacity limit on transmission line

The basic model

$$
\begin{array}{lll}
\min_{g,f,\theta} & c^T g & \text{generation cost} \\
\text{s.t.} & g - d = Af, f = BA^T\theta & A \text{ is node-arc incidence} \\
& \bar{\theta}_L \le \theta \le \bar{\theta}_U & \text{bus angle constraints} \\
& \bar{g}_L \le g \le \bar{g}_U & \text{generator capacities} \\
& \bar{f}_L \le f \le \bar{f}_U & \text{transmission capacities}
\end{array}
$$

with transmission switching (within a smart grid technology) we modify as:

$$
\begin{array}{lll}
\min_{g,f,\theta} & c^T g & \\
\text{s.t.} & g - d = Af & \\
& \bar{\theta}_L \le \theta \le \bar{\theta}_U & \\
& \bar{g}_L \le g \le \bar{g}_U & \\
\text{either} & f_i = (BA^T\theta)_i, \bar{f}_{L,i} \le f_i \le \bar{f}_{U,i} & \text{if } i \text{ closed} \\
\text{or} & f_i = 0 & \text{if } i \text{ open}
\end{array}
$$

Use EMP to facilitate the disjunctive constraints (several equivalent formulations, including LPEC)

But who cares?

- Why aren't you using my *********** algorithm?
  (Michael Ferris, Boulder, CO, 1994)

But who cares?

- Why aren't you using my *********** algorithm?
  (Michael Ferris, Boulder, CO, 1994)
- Show me on a problem like mine
- Must run on defaults
- Must deal graciously with poorly specified cases
- Must be usable from my environment (Matlab, R, GAMS, ...)
- Must be able to model my problem easily

EMP provides annotations to an existing optimization model that convey new model structures to a solver

NEOS is soliciting case studies that show how to do the above, and will provide some tools to help

Stochastic competing agent models (with Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent maximizes objective independently (utility)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancillary services in energy market)
- Can investigate new instruments to move to system optimal solutions from equilibrium (or market) solutions

The model details: c.f. Brown, Demarzo, Eaves

Each agent maximizes:

$$u_h = -\sum_s \pi_s \left(\kappa - \prod_l c_{h,s,l}^{\alpha_{h,l}} \right)$$

Time 0:

$$\sum_l p_{0,l} c_{h,0,l} + \sum_k q_k z_{h,k} \leq \sum_l p_{0,l} e_{h,0,l}$$

Time 1:

$$\sum_l p_{s,l} c_{h,s,l} \leq \sum_l p_{s,l} \sum_k D_{s,l,k} * z_{h,k} + \sum_l p_{s,l} e_{h,s,l}$$

Additional constraints (complementarity) outside of control of agents:

$$0 \leq -\sum_h z_{h,k} \perp q_k \geq 0$$

$$0 \leq -\sum_h d_{h,s,l} \perp p_{s,l} \geq 0$$

Stochastic programming and risk measures

$$
\begin{aligned}
\text{SP:}\ \min\quad & c^\top x + \mathbb{R}[d^\top y] \\
\text{s.t.}\quad & Ax = b \\
& T(\omega)x + W(\omega)y(\omega) \geq h(\omega), \qquad \text{for all } \omega \in \Omega, \\
& x \geq 0, \quad y(\omega) \geq 0, \qquad \text{for all } \omega \in \Omega.
\end{aligned}
$$

Annotations are slightly more involved but straightforward:

- Need to describe probability distribution
- Define (multi-stage) structure (what variables and constraints belong to each stage)
- Define random parameters and process to generate scenarios
- Can also define risk measures on variables

Automatic reformulation (deterministic equivalent), solvers such as DECIS, etc.

Additional techniques requiring extensive computation

- Chance constraints: $Prob(T_i x + W_i y_i \geq h_i) \geq 1 - \alpha$ - can reformulate as MIP and adapt cuts
- Use of discrete variables (in submodels) to capture logical or discrete choices
- Optimization of simulation or noisy functions
- Robust or stochastic programming
- Decomposition approaches to exploit underlying structure identified by EMP
- Nonsmooth penalties and reformulation approaches to recast problems for existing or new solution methods
- Conic or semidefinite programs - alternative reformulations that capture features in a manner amenable to global computation

Conclusions

- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Extended Mathematical Programming available within the GAMS modeling system
- Able to pass additional (structure) information to solvers
- Embedded optimization models automatically reformulated for appropriate solution engine
- Exploit structure in solvers
- Extend application usage further