Integrated Modeling for Optimization of Energy Systems

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A Simple Network Model

Load segments s represent the electrical load at various instances

- d_n^s Demand at node n in load segment s (MWe)
- X_i^s Generation by unit i (MWe)
- F_L^s Net electricity transmission on link L (MWe)
- Y_n^s Net supply at node n (MWe)
- π_n^s Wholesale price (\$ per MWhe)

Nodes n, load segments s, generators i, Ψ is node-generator map

$$\begin{aligned}
\max_{X,F,d,Y} \quad & \sum_s \left(W(d^s(\lambda^s)) - \sum_i c_i(X_i^s) \right) \\
\text{s.t.} \quad & \Psi(X^s) - d^s(\lambda^s) = Y^s \\
& 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\
& Y \in \mathcal{X}
\end{aligned}$$

where

$$\mathcal{X} = \left\{ Y : \exists F, F^s = \mathcal{H} Y^s, -\overline{F}^s \le F^s \le \overline{F}^s, \sum_n Y_n^s \ge 0, \forall s \right\}$$

- Key issue: decompose. Introduce multiplier π^s on supply demand constraint (and use $\lambda^s := \pi^s$)
- How different approximations of $\mathcal{X}$ affect the overall solution

Case $\mathcal{H}$: Loop flow model

$$\max_{d} \quad \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s) \right)$$

$$+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right)$$

$$\text{s.t.} \quad 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_{s} X_i^s$$

$$+ \max_{Y} \quad \sum_{s} -\pi^s Y^s$$

$$\text{s.t.} \quad \sum_{i} Y_i^s \geq 0, -\overline{F}^s \leq \mathcal{H} Y^s \leq \overline{F}^s$$

$$\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0$$

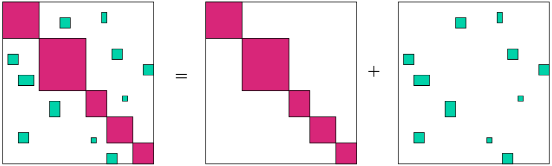
MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h pi cons
```

- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



Other specializations and extensions

$$\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \le 0, \forall i, f(x, z, \pi) = 0$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: $z(x_i, x_{-i})$ shared
- Shared constraints: f is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Let $\mathcal{A}$ be the node-arc incidence matrix, $\mathcal{H}$ be the shift matrix, $\mathcal{L}$ be the loop constraint matrix. Standard results show:

$$\mathcal{X} = \{Y : \exists F, F = \mathcal{H}Y, F \in \mathcal{F}\}$$

$$\mathcal{X} = \left\{Y : \exists (F, \theta), Y = \mathcal{A}F, B\mathcal{A}^T\theta = F, \theta \in \Theta, F \in \mathcal{F}\right\}$$

$$\mathcal{X} = \{Y : \exists F, Y = \mathcal{A}F, \mathcal{L}F = 0, F \in \mathcal{F}\}$$

Using different data ($\mathcal{A}$ is node-arc incidence, $\mathcal{L}$ is loop constraint matrix):

$$\begin{aligned} \max_{X,F,d,Y} \quad & \sum_s \left(W(d^s(\lambda^s)) - \sum_i c_i(X_i^s) \right) \\ \text{s.t.} \quad & \Psi(X^s) - d^s(\lambda^s) = Y^s \\ & 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\ & Y \in \mathcal{X} \end{aligned}$$

where

$$\mathcal{X} = \left\{ Y : \exists F, Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \le F^s \le \overline{F}^s, \forall s \right\}$$

$$\mathcal{X} = \left\{ Y : \exists F, F^s = \mathcal{H}Y^s, -\overline{F}^s \le F^s \le \overline{F}^s, \sum_n Y_n^s \ge 0, \forall s \right\}$$

- $\sum_n Y_n^s = \sum_n (\mathcal{A}F^s)_n = 0$ by properties of $\mathcal{A}$, so drop $\sum_n Y_n^s \ge 0$

Case $\mathcal{A}, \mathcal{L}$

$$
\begin{aligned}
\max_{d} \quad & \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s)\right) \\
+ \max_{X} \quad & \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s)\right) \\
\text{s.t.} \quad & 0 \leq X_i^s \leq \overline{X}_i, \quad \overline{G}_i \geq \sum_{s} X_i^s \\
+ \max_{F,Y} \quad & \sum_{s} -\pi^s Y^s \\
\text{s.t.} \quad & Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s
\end{aligned}
$$

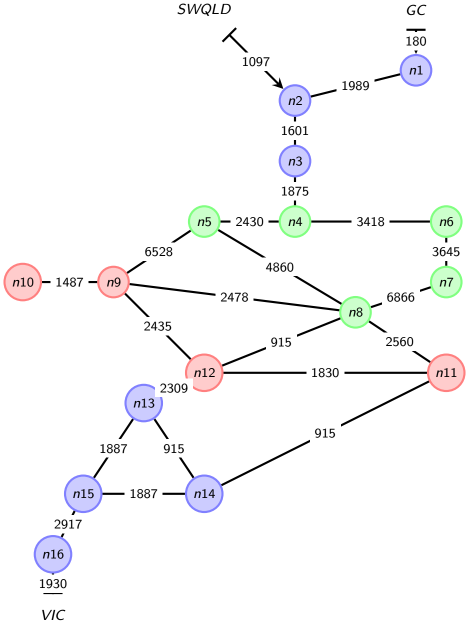
$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$

Network model

Drop loop constraints:

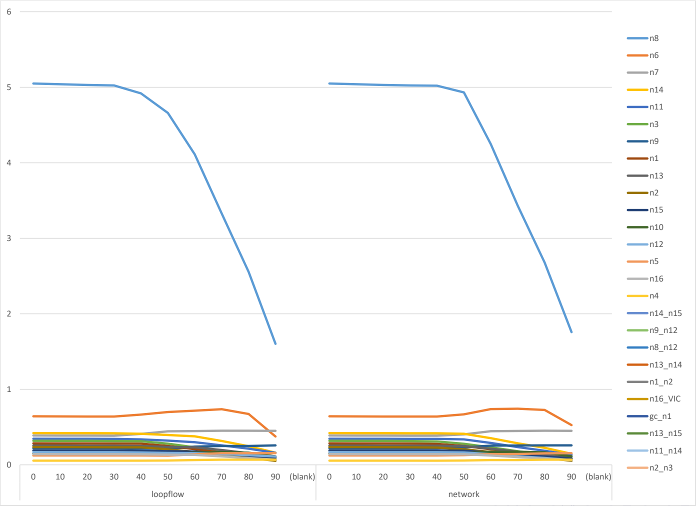
$$
\begin{aligned}
&\max_{d} && \sum_s \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s) \right) \\
+&\max_{X} && \sum_s \left(\pi^s \Psi(X^s) - \sum_i c_i(X_i^s) \right) \\
&\text{s.t.} && 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\
+&\max_{F,Y} && \sum_s -\pi^s Y^s \\
&\text{s.t.} && Y^s = \mathcal{A} F^s, -\overline{F}^s \le F^s \le \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$


SWQLD
GC
180
1097
n1
1989
n2
1601
n3
1875
n5
2430
n4
3418
n6
6528
4860
3645
n10
1487
n9
2478
n7
6866
n8
2435
915
2560
n12
1830
n11
2309
n13
915
1887
915
n15
1887
n14
2917
n16
1930
VIC

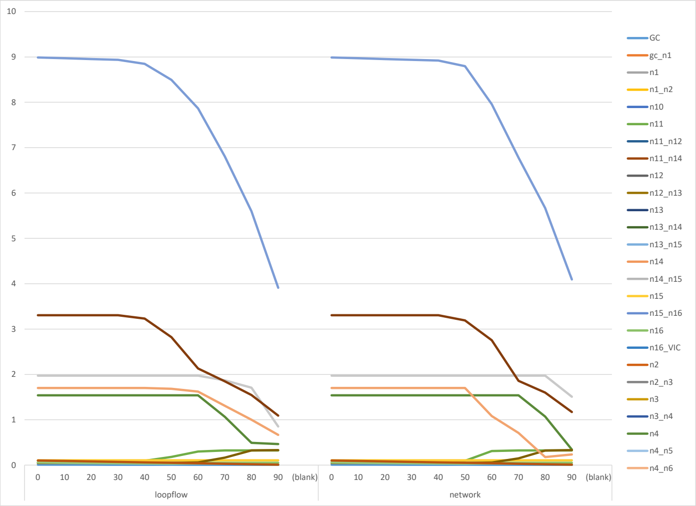
Comparing Network and Loopflow: Demand

Here we look at simulations which impose a proportional reduction in transmission across the network. The *network* and *loopflow* models demonstrate similar responses:



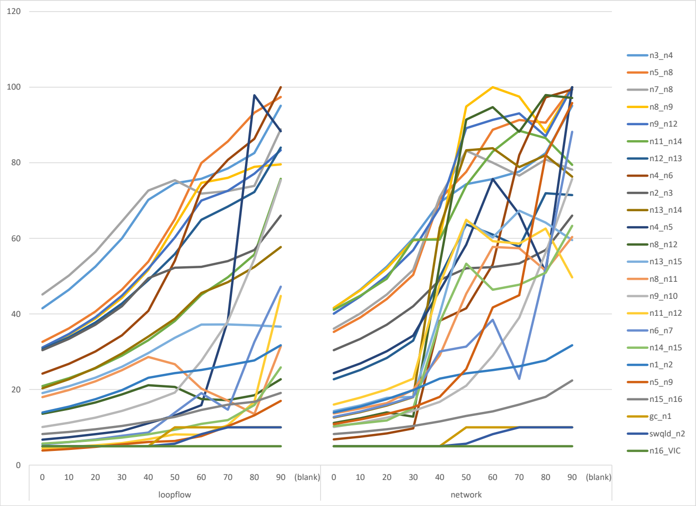
Comparing Network and Loopflow: Generation

Likewise, generation is similar in the two models:



Comparing Network and Loopflow: Transmission

Network transmission levels reveal that the two models are quite different:



Top down/bottom up

- $\lambda^s = \pi^s$ so use complementarity to expose (EMP: dualvar)
- All network constraints encapsulated in (bottom up) NLP (or its approximation by dropping $\mathcal{L}F^s = 0$):

$$\max_{F,Y} \quad \sum_s -\pi^s Y^s$$
$$\text{s.t.} \quad Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s$$

- Could instead use the NLP over Y with $\mathcal{H}$
- Can add additional detail into top level economic model describing consumers and producers
- Change interaction via new price mechanisms
- Clear how to instrument different behavior or different policies in interactions (e.g. Cournot, etc) within EMP

Loop flow model: update red, blue and purple components

$$
\begin{aligned}
\max_{d} \quad & \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s)\right) \\
+ \max_{X} \quad & \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s)\right) \\
\text{s.t.} \quad & 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_{s} X_i^s \\
+ \max_{Y} \quad & \sum_{s} -\pi^s Y^s \\
\text{s.t.} \quad & \sum_{i} Y_i^s \ge 0, -\overline{F}^s \le \mathcal{H} Y^s \le \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$

A Heterogeneous Demand Model

Electricity demand is defined by demand segment (j), node (n), and load segment (s). The demand functions are defined by reference price-quantity pairs ($\bar{p}_{jns}, \bar{q}_{jns}$), an elasticity of substitution across load segments (σ_j) and an elasticity of aggregate demand by segment and node (ϵ_j):

$$d_n^s = \sum_j q_{jns} = \sum_j \bar{q}_{jns} \left(\frac{P_{jn} \bar{p}_{jns}}{p_{jns}} \right)^{\sigma_j} (P_{jn})^{-\epsilon_j}$$

where

$$P_{jn} = \left(\sum_s \theta_{jns} \left(\frac{p_{jns}}{\bar{p}_{jns}} \right)^{1-\sigma_j} \right)^{1/(1-\sigma_j)}$$

Demand Response and Substitution

For concreteness, we might define the set of demand segments j as consisting of WASHING, HEATING, STREETLIGHTS, INDUSTRY and COMMERCIAL.

The nested constant elasticity of substitution (CES) model defined across nodes, demand segments, and load segments accommodates demand responsiveness both due to changes in aggregate electricity use as well as substitution across load segments.

For example, peak load pricing can induce households to run washing machines at night while still doing the same number of loads per week ($\sigma_{\text{WASHING}} \gg 0$,$\epsilon_{\text{WASHING}} \approx 0$). Conversely there may be no scope for shifting street lights use across load segments, and the only way to curtail demand is to reduce lighting ($\sigma_{\text{STREETLIGHTS}} = 0$, $\epsilon_{\text{STREETLIGHTS}} > 0$).

Pricing

Our implementation of the heterogeneous demand model incorporates three alternative pricing rules. The first is *average cost pricing*, defined by

$$P_{\mathrm{ACP}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s q_{jns}}$$

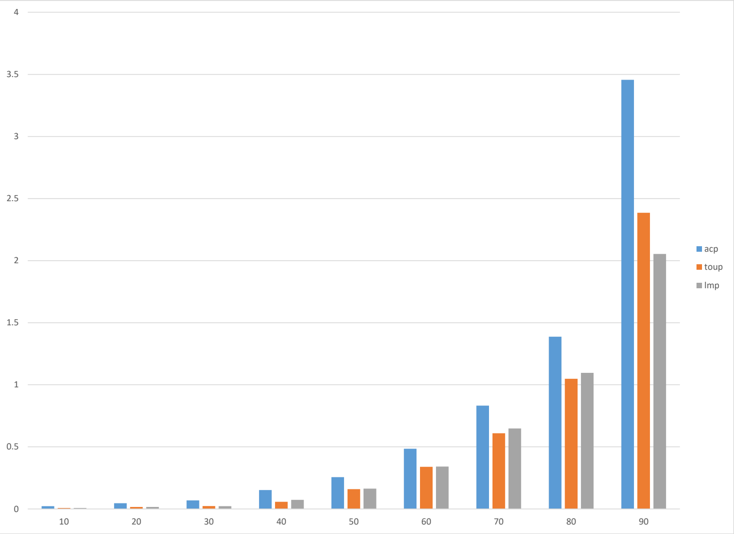
The second is *time of use pricing*, defined by:

$$P_s^{\mathrm{TOU}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} q_{jns}}$$

The third is *location marginal pricing* corresponding to the wholesale prices denoted P_{ns} above. Prices for individual demand segments are then assigned:

$$p_{jns} = \begin{cases} P_{\mathrm{ACP}} & (jn) \in \mathcal{R}_{\mathrm{ACP}} \\ P_s^{\mathrm{TOU}} & (jn) \in \mathcal{R}_{\mathrm{TOU}} \\ P_{ns} & (jn) \in \mathcal{R}_{\mathrm{LMP}} \end{cases}$$

Smart Metering Lowers the Cost of Congestion



The Derived Demand Model: Short Run

Following the previous model, we now expand the electricity model to account for downstream sectors. Electricity demand in segment j at node n is modelled as an intermediate input to the production function of j (q_{jns}), aggregate across load segments within the production function:

$$Z_{jn} = f_j(q_{jns}, \ell_{jn}, \bar{K}_{jn}, M_{jn})$$

In this expression ℓ_{jn} represents employment, $\bar{K}_{jn}$ is (fixed) capital stock and M_{jn} represents intermediate inputs other than electricity.
f_j exhibits constant returns to scale in the inputs. In the short run, capital is fixed and the competitive sector j firm is modeled by the profit maximization problem, taking p^Z as given:

$$\max_{\ell_{jn}, q_{jns}} p^Z f_j(q_{jns}, \ell_{jn}, \bar{K}_{jn}, M_{jn}) - \sum_s p_{jns} q_{jns} - w_{jn}\ell_{jn} - p_M M_{jn},$$

hence, electricity market reforms affect electricity prices, demand segment output and employment.

The Derived Demand Model: Long Run

In the long run, capital is a decision variable and the competitive sector j firm is modeled by the cost minimization problem:

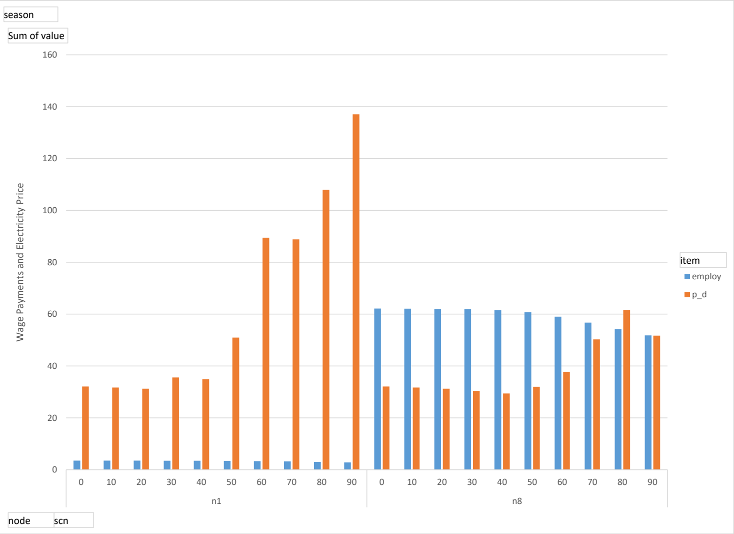
$$\min_{q,\ell,K,M} \sum_s p_{jns} q_{jns} + w_{jn}\ell_{jn} + r_K K_{jn} + p_M M_{jn}$$

subject to the constraint:

$$f_j(q_{jns}, \ell_{jn}, \bar{K}_{jn}, M_{jn}) = \bar{Z}_{jn}$$

where $\bar{Z}_{jn}$ is treated as a constant in solving the producer's problem, but is determined endogenously through a downward sloping demand curve. (Firms in each demand segment are atomistic and competitive.)

Electricity Prices May or May Not Affect Employment



What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

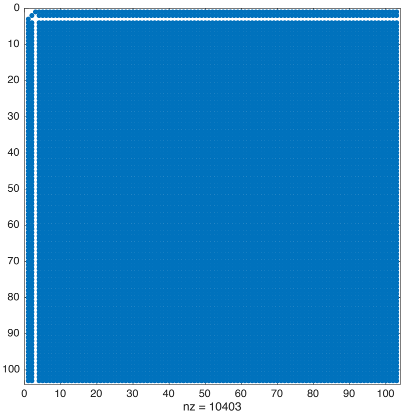
- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- dualvar (use multipliers from one agent as variables for another)
- QS functions (both in objectives and constraints)
- implicit functions and shared constraints

- Currently available within GAMS
- Some solution algorithms implemented in modeling system - limitations on size, decomposition and advanced algorithms
- Can evaluate effects of regulations and their implementation in a competitive environment

Computational issue: PATH

- Cournot model: $|\mathcal{A}| = 5$
- Size $n = |\mathcal{A}| * N_a$

Size (n)	Time (secs)
1,000	35.4
2,500	294.8
5,000	1024.6

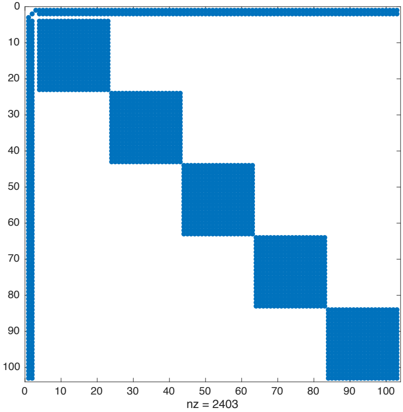


Jacobian nonzero pattern
$n = 100, \ N_a = 20$

Computation: implicit functions

- Use implicit fn: $z(x) = \sum_j x_j$
- Generalization to $F(z,x) = 0$ (via adjoints)
- empinfo: implicit z F

Size (n)	Time (secs)
1,000	2.0
2,500	8.7
5,000	38.8
10,000	> 1080

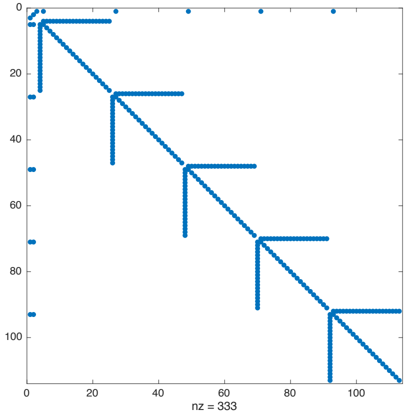


Jacobian nonzero pattern
$n = 100,\ N_a = 20$

Computation: implicit functions and local variables

- Use implicit fn: $z(x) = \sum_j x_j$ (and local aggregation)
- Generalization to $F(z, x) = 0$ (via adjoints)
- empinfo: implicit z F

Size (n)	Time (secs)
1,000	0.5
2,500	0.8
5,000	1.6
10,000	3.9
25,000	17.7
50,000	52.3



Jacobian nonzero pattern
$n = 100,\ N_a = 20$

Reserves, interruptible load, demand response

- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Consumers may also be able to reduce consumption for short periods
- Alternative to sharp price increases during peak periods
- Constraints linking energy "bids" and reserve "bids"

$$v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

- Multiple scenarios - linking constraints on bids require "bid curve to be monotone"

Price taking: model is MOPEC

Consumption d_k, demand response r_k, energy v_j, reserves u_j, prices π

$$\text{Consumer} \quad \max_{(d_k,r_k)\in\mathcal{C}} \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

$$\text{Generator} \quad \max_{(v_j,u_j)\in\mathcal{G}} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{s.t. } v_j + u_j \le \mathcal{U}_j, u_j \le \mathcal{B}_j v_j$$

$$\text{Transmission} \quad \max_{f\in\mathcal{F}} \text{congestion rates}(f, \pi_d)$$

Market clearing

$$0 \le \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A}f \ge 0$$

$$0 \le \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \ge 0$$

Large consumer is price making: MPEC

Leader/follower

$$\text{Consumer} \;\; \max \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

with the constraints:

$$(d_k, r_k) \in \mathcal{C}$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}'} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{Transmission} \;\; \max_{f \in \mathcal{F}} \text{ congestion rates}(f, \pi_d)$$

$$0 \le \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A} f \ge 0$$

$$0 \le \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \ge 0$$