Extended Mathematical Programming: Competition and Stochasticity

Michael C. Ferris

Joint work with: Michael Bussieck, Steven Dirkse, and Alexander Meeraus

University of Wisconsin, Madison

Initiative for Computational Economics (ICE) Conference, University of Chicago, Chicago
July 27, 2012

The PIES Model (Hogan)

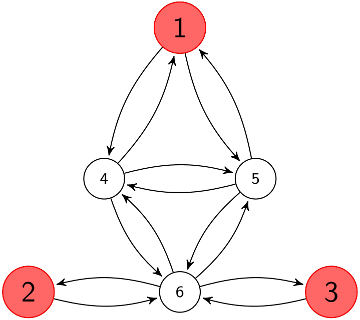
$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax = d(p) \\ & Bx = b \\ & x \geq 0 \end{aligned}$$

- Issue is that p is the multiplier on the “balance” constraint of LP
- Extended Mathematical Programming (EMP) facilitates annotations of models to describe additional structure
- empinfo: dualvar p balance
- Can solve the problem by writing down the KKT conditions of this LP, forming an LCP and exposing p to the model
- EMP does this automatically from the annotations

Model building with EMP

- Take one system of (nonlinear) equations and annotate them to:
  - form a simple nonlinear program (no annotations)
  - form a complementarity problem from an embedded optimization problem (nlp with side constraints outside of optimizers control)
  - form an equilibrium model consisting of optimality conditions of several nlp's along with equilibrium constraints (MOPEC)
  - form a bilevel program (an optimization problem with optimization problems as constraints)
  - Can assign multipliers (prices) from one sub-model as variables in another model
  - Can reformulate nonsmooth models using duality
  - Can introduce random variables into a model
- The annotations essentially detail who controls which equations and variables

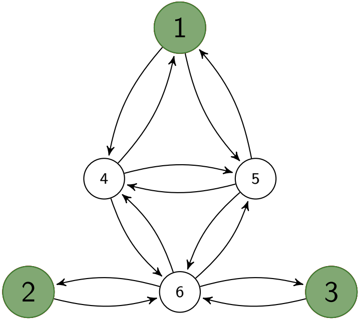
Spatial Price Equilibrium



$n \in \{1, 2, 3, 4, 5, 6\}$
$L \in \{1, 2, 3\}$

Supply quantity: S_L
Production cost: $\Psi(S_L) = ..$

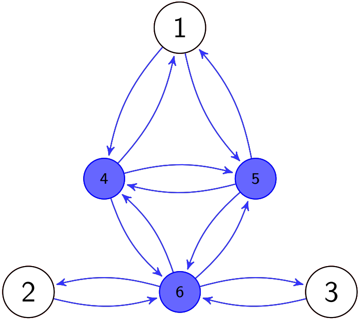
Spatial Price Equilibrium



$n \in \{1, 2, 3, 4, 5, 6\}$
$L \in \{1, 2, 3\}$

Supply quantity: S_L
Production cost: $\Psi(S_L) = ..$
Demand: D_L
Unit demand price: $\theta(D_L) = ..$

Spatial Price Equilibrium



$n \in \{1, 2, 3, 4, 5, 6\}$
$L \in \{1, 2, 3\}$

Supply quantity: S_L
Production cost: $\Psi(S_L) = ..$
Demand: D_L
Unit demand price: $\theta(D_L) = ..$
Transport: T_{ij}
Unit transport cost: $c_{ij}(T_{ij}) = ..$

One large system of equations and inequalities to describe this (GAMS).

Nonlinear Program Model (Monopolist)

- One producer controlling all regions
- Full knowledge of demand system
- Full knowledge of transportation system

$$
\begin{aligned}
\max_{(D,S,T)\in\mathcal{F}} \quad & \sum_{l\in L} \theta_l(D_l)D_l - \sum_{l\in L} \Psi_l(S_l) - \sum_{i,j} c_{ij}(T_{ij})T_{ij} \\
\text{s.t.} \quad & S_l + \sum_{i,l} T_{il} = D_l + \sum_{l,j} T_{lj}, \quad \forall l \in L
\end{aligned}
$$

EMP = NLP

2 agents: NLP + VI Model (Monopolist)

- One producer controlling all regions
- Full knowledge of demand system
- Price-taker in transportation system

$$\max_{(D,S,T)\in\mathcal{F}} \sum_{l\in L} \theta_l(D_l)D_l - \sum_{l\in L} \Psi_l(S_l) - \sum_{i,j} \overset{p_{ij}}{\cancel{c_{ij}(T_{ij})}} T_{ij} \qquad (1)$$

$$\text{s.t.} \quad S_l + \sum_{i,l} T_{il} = D_l + \sum_{l,j} T_{lj}, \quad \forall l \in L$$

$$p_{ij} = c_{ij}(T_{ij}) \qquad (2)$$

empinfo: vi tcDef tc

2 agents: NLP + VI Model (Monopolist)

- One producer controlling all regions
- Full knowledge of demand system
- Price-taker in transportation system

$$\max_{(D,S,T)\in\mathcal{F}} \quad \sum_{l\in L} \theta_l(D_l)D_l - \sum_{l\in L} \Psi_l(S_l) - \sum_{i,j} \overset{p_{ij}}{\cancel{c_{ij}(T_{ij})}} T_{ij} \tag{1}$$

$$\text{s.t.} \quad S_l + \sum_{i,l} T_{il} = D_l + \sum_{l,j} T_{lj}, \quad \forall l \in L$$

$$p_{ij} = c_{ij}(T_{ij}) \tag{2}$$

EMP = MOPEC $\implies$ MCP

Classic SPE Model (NLP + VI agents)

- One producer controlling all regions
- Price-taker in demand system
- Price-taker in transportation system

$$\max_{(D,S,T)\in\mathcal{F}} \quad \sum_{l\in L} \overset{\pi_l}{\cancel{\theta_l(D_l)}} D_l - \sum_{l\in L} \Psi_l(S_l) - \sum_{i,j} \overset{p_{ij}}{\cancel{c_{ij}(T_{ij})}} T_{ij} \qquad (1)$$

$$\text{s.t.} \quad S_l + \sum_{i,l} T_{il} = D_l + \sum_{l,j} T_{lj}, \quad \forall l \in L$$

$$p_{ij} = c_{ij}(T_{ij}) \qquad (2)$$

$$\pi_l = \theta_l(D_l) \qquad (3)$$

empinfo: vi tcDef tc
vi pricedef price

Classic SPE Model (NLP + VI agents)

- One producer controlling all regions
- Price-taker in demand system
- Price-taker in transportation system

$$\max_{(D,S,T)\in\mathcal{F}} \sum_{l\in L} \overset{\pi_l}{\cancel{\theta_l(D_l)}} D_l - \sum_{l\in L} \Psi_l(S_l) - \sum_{i,j} \overset{p_{ij}}{\cancel{c_{ij}(T_{ij})}} T_{ij} \qquad (1)$$

$$\text{s.t.} \quad S_l + \sum_{i,l} T_{il} = D_l + \sum_{l,j} T_{lj}, \quad \forall l \in L$$

$$p_{ij} = c_{ij}(T_{ij}) \qquad (2)$$

$$\pi_l = \theta_l(D_l) \qquad (3)$$

EMP = MOPEC $\Longrightarrow$ MCP

Cournot-Nash equilibrium (multiple agents)

Assumes that each agent (producer):

- Treats other agent decisions as fixed
- Is a price-taker in transport and demand

EMP info file

```
equilibrium
max obj('one') vars('one') eqns('one')
max obj('two') vars('two') eqns('two')
max obj('three') vars('three') eqns('three')
vi tcDef tc
vi pricedef price
```

EMP = MOPEC $\Longrightarrow$ MCP

Bilevel Program (Stackelberg)

- Assumes one leader firm, the rest follow
- Leader firm optimizes subject to expected follower behavior
- Follower firms act in a Nash manner
- All firms are price-takers in transport and demand

EMP info file

bilevel obj('one') vars('one') eqns('one')
max obj('two') vars('two') eqns('two')
max obj('three') vars('three') eqns('three')
vi tcDef tc
vi pricedef price

EMP = bilevel $\implies$ MPEC $\implies$ (via NLPEC) NLP(μ)

What is EMP?

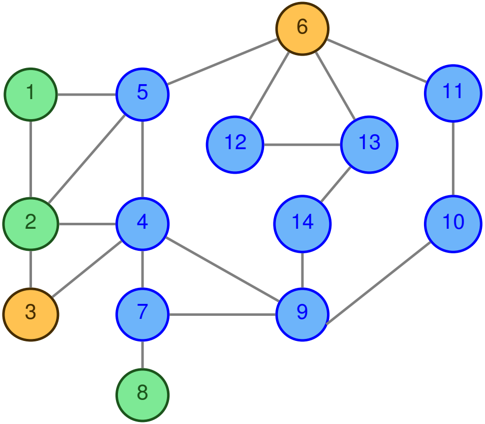
Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Power Systems: Economic Dispatch

$$\min_{(q,z,\theta)\in\mathcal{F}} \sum_k C(q_k) \text{ s.t. } q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k$$



- Independent System Operator (ISO) determines who generates what
- p_k: Locational marginal price (LMP) at k
- Volatile in "stressed" system
- Can we shed load from consumers to smooth prices?
- FERC (regulator) writes the rules - how to implement?

Understand: demand response and FERC Order No. 745

$$\min_{q,z,\theta,R,p} \sum_k p_k R_k$$

$$\text{s.t.}\; C_1 \geq \sum_k p_k d_k / \sum_k d_k$$

$$C_2 \geq \sum_k p_k (q_k + R_k) / \sum_k (d_k - R_k)$$

$$0 \leq R_k \leq u_k,$$

$$\text{and } (q, z, \theta) \text{ solves } \min_{(q,z,\theta)\in\mathcal{F}} \sum_k C(q_k)$$

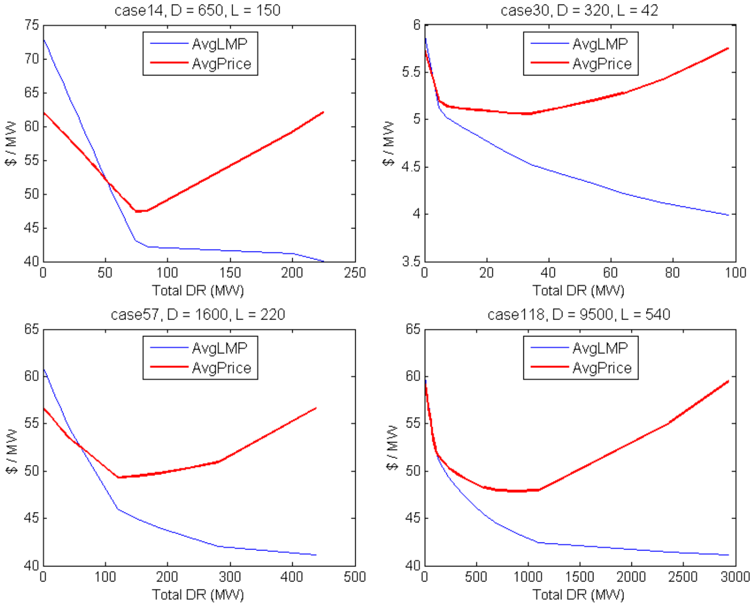
$$\text{s.t.}\quad q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k - R_k \qquad (1)$$

where p_k is the multiplier on constraint (1)

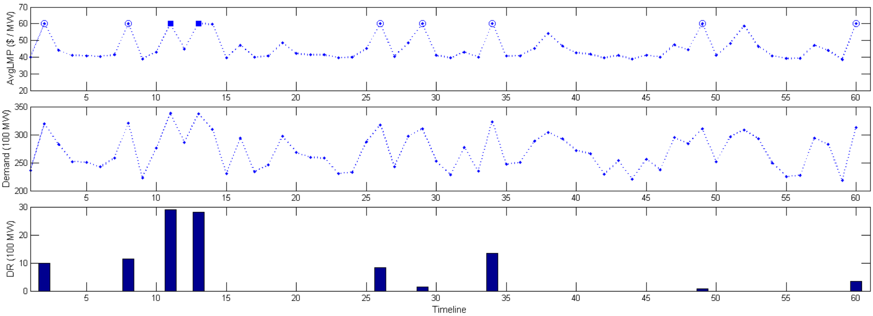
Solution Process (F./Liu)

- Bilevel program (hierarchical model)
- Upper level objective involves multipliers on lower level constraints
- Extended Mathematical Programming (EMP) annotates model to facilitate communicating structure to solver
  - dualvar p balance
  - bilevel R min cost q z θ balance ...
- Automatic reformulation as an MPEC (single optimization problem with equilibrium constraints)
- Model solved using NLPEC and Conopt
- bilevel $\Longrightarrow$ MPEC $\Longrightarrow$ NLP
- Potential for solution of "consumer level" demand response
- Challenge: devise robust algorithms to exploit this structure for fast solution

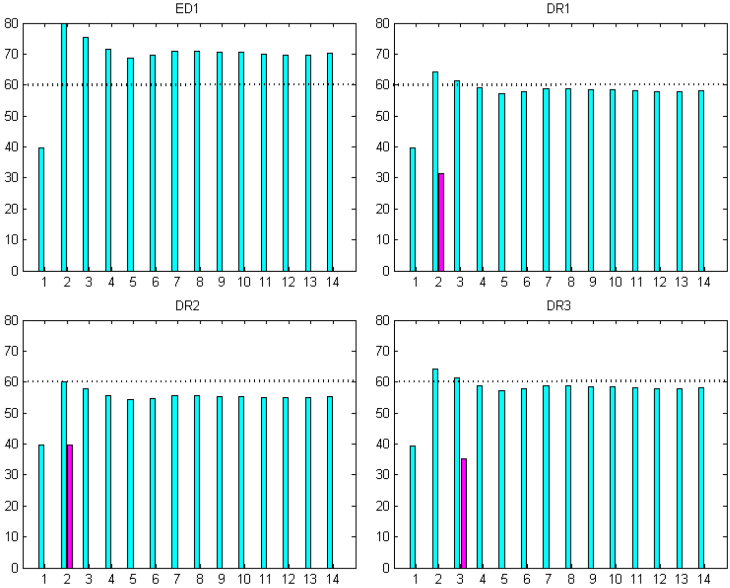
Stability and feasibility



Operational view: LMP, Demand, Response



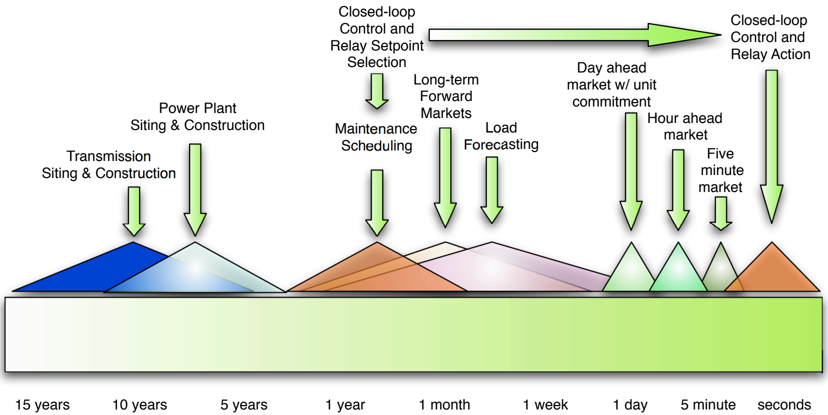
Alternative models: ED, avg, max, weighted avg



Extension: The smart grid

- The next generation electric grid will be more dynamic, flexible, constrained, and more complicated.
- Decision processes (in this environment) are predominantly hierarchical.
- Models to support such decision processes must also be layered or hierarchical.
- Optimization and computation facilitate adaptivity, control, treatment of uncertainties and understanding of interaction effects.
- Developing interfaces and exploiting hierarchical structure using computationally tractable algorithms will provide FLEXIBILITY, overall solution speed, understanding of localized effects, and value for the coupling of the system.

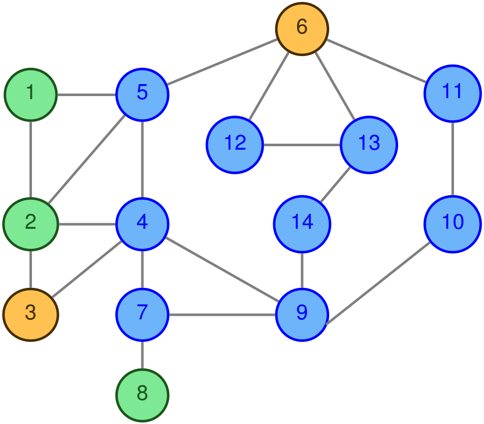
Representative decision-making timescales in electric power systems



A monster model is difficult to validate, inflexible, prone to errors.

Combine: Transmission Line Expansion Model (F./Tang)

$$\min_{x \in X} \sum_{\omega} \pi_\omega \sum_{i \in N} d_i^\omega p_i^\omega(x)$$



- Nonlinear system to describe power flows over (large) network
- Multiple time scales
- Dynamics (bidding, failures, ramping, etc)
- Uncertainty (demand, weather, expansion, etc)
- $p_i^\omega(x)$: Price (LMP) at i in scenario ω as a function of x
- Use other models to construct approximation of $p_i^\omega(x)$

Generator Expansion (2): $\forall f \in F$:

$$\min_{y_f} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j)$$

$$\text{s.t.} \sum_{j \in G_f} y_j \leq h_f, y_f \geq 0$$

G_f: Generators of firm $f \in F$
y_j: Investment in generator j
q_j^ω: Power generated at bus j in scenario ω
C_j: Cost function for generator j
r: Interest rate

Market Clearing Model (3): $\forall \omega$:

$$\min_{z,\theta,q^\omega} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \text{s.t.}$$

$$q_j^\omega - \sum_{i \in I(j)} z_{ij} = d_j^\omega \quad \forall j \in N(\perp p_j^\omega)$$

$$z_{ij} = \Omega_{ij}(\theta_i - \theta_j) \quad \forall (i,j) \in A$$

$$-b_{ij}(x) \leq z_{ij} \leq b_{ij}(x) \quad \forall (i,j) \in A$$

$$\underline{u}_j(y_j) \leq q_j^\omega \leq \overline{u}_j(y_j)$$

z_{ij}: Real power flowing along line ij
q_j^ω: Real power generated at bus j in scenario ω
θ_i: Voltage phase angle at bus i
Ω_{ij}: Susceptance of line ij
$b_{ij}(x)$: Line capacity as a function of x
$\underline{u}_j(y)$, $\overline{u}_j(y)$: Generator j limits as a function of y

Solution approach

- Use derivative free method for the upper level problem (1)
- Requires $p_i^\omega(x)$
- Construct these as multipliers on demand equation (per scenario) in an Economic Dispatch (market clearing) model
- But transmission line capacity expansion typically leads to generator expansion, which interacts directly with market clearing
- Interface blue and black models using Nash Equilibria (as EMP):

empinfo: equilibrium
forall f: min expcost(f) y(f) budget(f)
forall ω: min scencost(ω) q(ω) ...

Feasibility

$$\text{KKT of} \min_{y_f \in Y} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j) \quad \forall f \in F \quad (2)$$

$$\text{KKT of} \min_{(z,\theta,q^\omega) \in Z(x,y)} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \forall \omega \quad (3)$$

- Models (2) and (3) form a complementarity problem (CP via EMP)
- Solve (3) as NLP using global solver (actual $C_j(y_j, q_j^\omega)$ are not convex), per scenario (SNLP) this provides starting point for CP
- Solve (KKT(2) + KKT(3)) using EMP and PATH, then repeat
- Identifies CP solution whose components solve the scenario NLP's (3) to global optimality

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

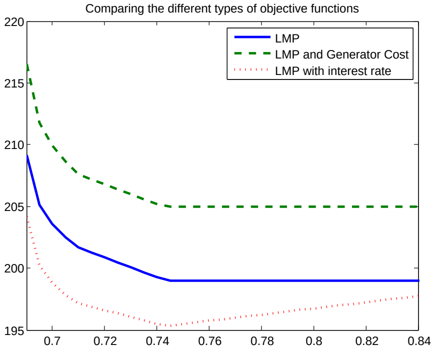
EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.34	4.62	5.01	3.99
ω_2		4.71	4.07	4.25	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	0.00	622.02			377.98
f_2			283.22	216.79	

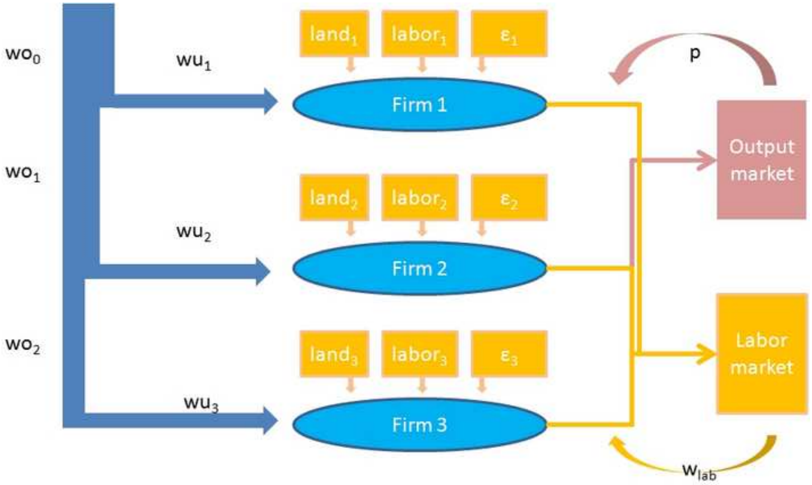
Observations

- But this is simply one function evaluation for the outer "transmission capacity expansion" problem
- Number of critical arcs typically very small
- But in this case, p_j^ω are very volatile
- Outer problem is small scale, objectives are open to debate, possibly ill conditioned



- Economic dispatch should use AC power flow model
- Structure of market open to debate
- Types of "generator expansion" also subject to debate
- Suite of tools is very effective in such situations

Water rights pricing (Britz/F./Kuhn)

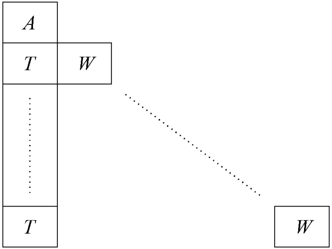


Models firms behavior with market to determine water rights

Agents have stochastic recourse?

- Two stage stochastic programming, x is here-and-now decision, recourse decisions y depend on realization of a random variable
- $\mathbb{R}$ is a risk measure (e.g. expectation, CVaR)

$$\text{SP: } \min \quad c^\top x + \mathbb{R}[q^\top y]$$
$$\text{s.t.} \quad Ax = b, \quad x \geq 0,$$
$$\forall \omega \in \Omega: \quad T(\omega)x + W(\omega)y(\omega) \leq d(\omega),$$
$$y(\omega) \geq 0.$$



Design: Stochastic competing agent models (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must “hedge” against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Each agent minimizes:

$$u_a = (\kappa - f(q_{a,0,*}))^2 + \sum_s \pi_s (\kappa - f(q_{a,s,*}))^2$$

Budget time 0: $\sum_i p_{0,i} q_{a,0,i} + \sum_j v_j y_{a,j} \leq \sum_i p_{0,i} e_{a,0,i}$

Budget time 1: $\sum_i p_{s,i} q_{a,s,i} \leq \sum_i p_{s,i} \sum_j D_{s,i,j} y_{a,j} + \sum_i p_{s,i} e_{a,s,i}$

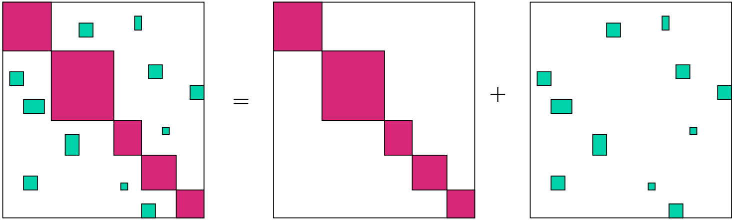
Additional constraints (complementarity) outside of control of agents:

$$\text{(contract) } 0 \leq -\sum_a y_{a,j} \perp v_j \geq 0$$

$$\text{(walras) } 0 \leq -\sum_a d_{a,s,i} \perp p_{s,i} \geq 0$$

Model and solve

- Can model financial instruments such as "financial transmission rights", "spot markets", "reactive power markets"
- Reduce effects of uncertainty, not simply quantify
- Use structure in preconditioners
  - Use nonsmooth Newton methods to formulate complementarity problem
  - Solve each "Newton" system using GMRES
  - Precondition using "individual optimization" with fixed externalities



The problem

A furniture maker can manufacture and sell four different dressers. Each dresser requires a certain number t of man-hours for carpentry, and a certain number t_{fj} of man-hours for finishing, $j = 1, \ldots, 4$. In each period, there are d_c man-hours available for carpentry, and d_f available for finishing. There is a (unit) profit $\bar{c}_j$ per dresser of type j that's manufactured. The owner's goal is to maximize total profit:

$$\max_{x \geq 0} \; 12x_1 + 25x_2 + 21x_3 + 40x_4 \qquad (\textit{profit})$$

subject to

$$4x_1 + 9x_2 + 7x_3 + 10x_4 \leq 6000 \qquad (\textit{carpentry})$$
$$x_1 + x_2 + 3x_3 + 40x_4 \leq 4000 \qquad (\textit{finishing})$$

Succinctly:

$$\max_{x} \; c^T x \text{ s.t. } Tx \leq d, x \geq 0$$

Is your time estimate that good?

- The time for carpentry and finishing for each dresser cannot be known with certainty
- Each entry in T takes on four possible values with probability $1/4$, independently
- 8 entries of T are random variables: $s = 65,536$ different T's each with same probability of occurring
- But decide "now" how many dressers x of each type to build
- Might have to pay for overtime (for carpentry and finishing)
- Can make different overtime decision y^s for each scenario s - recourse!

Models with explicit random variables

- Model transformation:
  - Write a core model as if the random variables are constants
  - Identify the random variables and decision variables and their staging
  - Specify the distributions of the random variables
  - emp.info: model transformation

```
randvar T('c','1') discrete .25  3.60 .25  3.90 .25  4.10 .25  4.40
randvar T('c','2') discrete .25  8.25 .25  8.75 .25  9.25 .25  9.75
randvar T('c','3') discrete .25  6.85 .25  6.95 .25  7.05 .25  7.15
randvar T('c','4') discrete .25  9.25 .25  9.75 .25 10.25 .25 10.75
randvar T('f','1') discrete .25  0.85 .25  0.95 .25  1.05 .25  1.15
randvar T('f','2') discrete .25  0.85 .25  0.95 .25  1.05 .25  1.15
randvar T('f','3') discrete .25  2.60 .25  2.90 .25  3.10 .25  3.40
randvar T('f','4') discrete .25 37.00 .25 39.00 .25 41.00 .25 43.00
randvar d('c')     discrete .25 5873. .25 5967. .25 6033. .25 6127.
randvar d('f')     discrete .25 3936. .25 3984. .25 4016. .25 4064.

stage 2 y t d cost cons obj
```

- Solver configuration:
  - Specify the manner of sampling from the distributions
  - Determine which algorithm (and parameter settings) to use
- Output handling:
  - Optionally, list the variables for which we want a scenario-by-scenario report

What do we learn?

- Deterministic solution: $x_d = (1333, 0, 0, 67)$
- Expected profit using this solution: \$16,942
- Expected (averaged) overtime costs: \$1,725
- Extensive form solution: $x_s = (257, 0, 666, 34)$ with expected profit \$18,051
- Deterministic solution is not optimal for stochastic program, but more significantly it isn't getting us on the right track!
- Stochastic solution suggests large number of "type 3" dressers, while deterministic solution has none!

Computation methods matter!

- Lindo solver defaults: 825 seconds
- Lindo solver barrier method: 382 seconds
- CPLEX solver barrier method: 4 seconds (8 threads)

1. May have multiple sources of uncertainty: e.g. man-hours d also can take on 4 values in each setting independently: $s = 1,048,576$
2. Generates extensive form problem with over 3 million rows and columns and 29 million nonzeros
3. Solves on 24 threaded cluster machine in 262 secs

Sampling methods

But what if the number of scenarios is too big (or the probability distribution is not discrete)? use sample average approximation (SAA)

- Take sample $\xi_1, \dots, \xi_N$ of N realizations of random vector ξ
  - viewed as historical data of N observations of ξ, or
  - generated via Monte Carlo sampling
- for any $x \in X$ estimate $f(x)$ by averaging values $F(x, \xi_j)$

$$\text{(SAA):} \ \min_{x \in X} \left\{ \hat{f}_N(x) := \frac{1}{N} \sum_{j=1}^{N} F(x, \xi_j) \right\}$$

- Nice theoretical asymptotic properties
- Can use standard optimization tools to solve the SAA problem
- EMP = SLP $\implies$ SAA $\implies$ (large scale) LP

Convergence

N	Time(s)	Soln	Profit
1000	0.6	(265,0,662,34)	18050
2000	1.0	(254,0,668,34)	18057
3000	1.6	(254,0,668,34)	18057
4000	2.3	(255,0,662,34)	18058
5000	3.1	(257,0,666,34)	18054
6000	3.9	(262,0,663,34)	18051
7000	5.0	(257,0,666,34)	18054
8000	6.1	(262,0,663,34)	18048
9000	7.3	(257,0,666,34)	18051
1m	262.0	(257,0,666,34)	18051

SAA can work well, but this is a 4 variable problem and distributions are discrete

Continuous distributions: News vendor problems (F./Liu)

N	Derand		SAA	
	Mean	Stdev	Mean	Stdev
2	16.85	2.185	16.94	3.615
5	14.84	1.369	14.92	2.791
10	14.23	1.127	14.57	2.248
20	14.03	0.797	14.18	1.635
100	14.01	0.100	14.48	0.745

1. As the sample size N increases, the optimal solutions obtained by both methods converge to the true solution, i.e. 14
2. For a given sample size N, new sampling method (derand) is always (slightly) closer to the true solution
3. But standard deviation of the optimal solutions obtained by derand is significantly smaller than the SAA method

Additional techniques requiring extensive computation

- Continuous distributions, sampling functions, density estimation
- Chance constraints: $Prob(T_i x + W_i y_i \geq h_i) \geq 1 - \alpha$ - can reformulate as MIP and adapt cuts (Luedtke) empinfo: chance E1 E2 0.95
- Use of discrete variables (in submodels) to capture logical or discrete choices (logmip - Grossmann et al)
- Robust or stochastic programming
- Decomposition approaches to exploit underlying structure identified by EMP
- Nonsmooth penalties and reformulation approaches to recast problems for existing or new solution methods (ENLP)
- Conic or semidefinite programs - alternative reformulations that capture features in a manner amenable to global computation

Conclusions

- Optimization helps understand what drives a system
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Extended Mathematical Programming available within the GAMS modeling system
- Uncertainty is present everywhere (the world is not "normal")
- We need not only to quantify it, but we need to hedge/control/ameliorate it
- Modeling, optimization, and computation embedded within the application domain is critical