Stochastic Equilibria: Data and Applications

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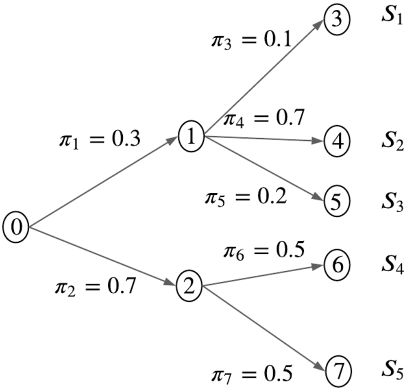
Olvi Mangasarian

- Scholar
- Leader
- Friend
- Impacted areas and researchers beyond his core interests

Scenario Tree



- Treat uncertainty more robustly
- Decomposition
- Parallel computation
- Forward/backward passes

Risk Measures

Problem type

Objective function	or	Constraint
$\min_{x \in X} \theta(x) + \rho(F(x))$		$\min_{x \in X} \theta(x) \text{ s.t. } \rho(F(x)) \leq \alpha$

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda \in [0, p/(1-\alpha)] : \langle \mathbb{1}, \lambda \rangle = 1\}$, then $\rho(Z) = \overline{AVaR}_\alpha(Z)$

The transformation to complementarity

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\rho(y) = \sup_{u \in U} \left\{ \langle u, y \rangle - \frac{1}{2} \langle u, Mu \rangle \right\}$$

optimal value function:

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

calculus:

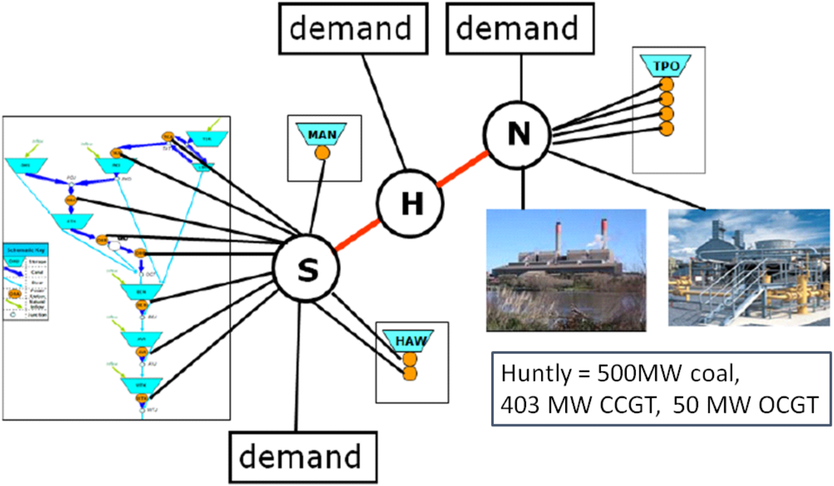
$$0 \in \partial\theta(x) + \nabla F(x)^T u + N_X(x)$$

$$0 \in -u + \partial\rho(F(x)) \iff 0 \in -F(x) + Mu + N_U(u)$$

- This is a complementarity problem

What does fully renewable in electricity mean?

- Permanently shutdown all thermal plants?
- Control GHG emissions from electricity generation?



Trading risk

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0, t^C} \quad \sigma t^C + p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) - t^C_\omega \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0, t^T} \quad \sigma t^T + C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2_\omega - t^T_\omega \right]$$

$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0, t^H}} \quad \sigma t^H - p^1 U(u^1) + \rho_H \left[-p^2_\omega U(u^2_\omega) - V(x^2_\omega) - t^H_\omega \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$

$$0 \leq \sigma_\omega \perp t^C_\omega + t^T_\omega + t^H_\omega \geq 0, \forall \omega \quad \sigma = (\sigma_\omega)$$

The EMP framework

- Each agent solves a multi-stage stochastic optimization problem
- Tie them together as a MOPEC

$$
\begin{aligned}
\underset{x_i}{\text{minimize}} \quad & f_i(x_i, x_{-i}, p), \\
\text{subject to} \quad & g_i(x_i, x_{-i}, p) \leq 0, \\
& h_i(x_i, x_{-i}, p) = 0, \\
& \text{for } i = 1, \ldots, N, \\
\hline
p \quad & \in \mathrm{SOL}(K, F).
\end{aligned}
$$

- The time structure of $x_i = x_i(t_0, t_1, \ldots, t_N)$ is not explicit above, neither is the way risk is evaluated
- This complicated things!

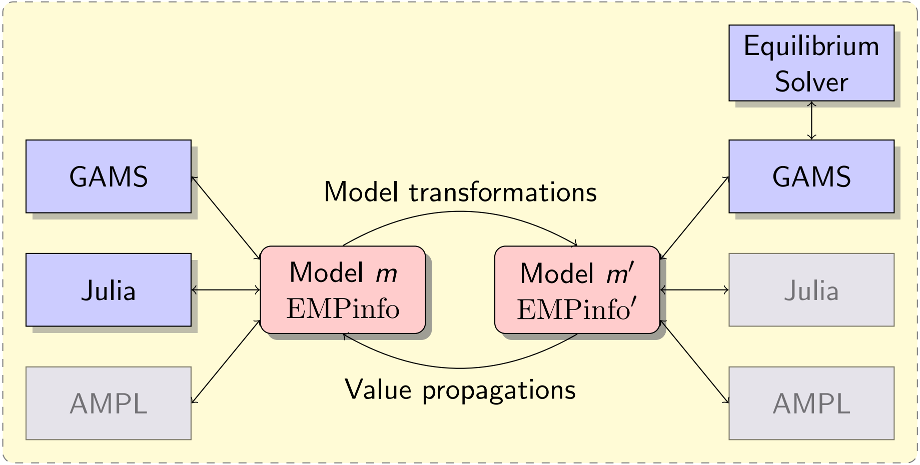
The EMP framework

- Each agent solves a multi-stage stochastic optimization problem
- Tie them together as a MOPEC
- Annotate equations and variables in an `empinfo` file.
- The framework automatically transforms the problem into another computationally more tractable form.

$$
\begin{aligned}
\underset{x_i}{\text{minimize}} \quad & f_i(x_i, x_{-i}, p), \\
\text{subject to} \quad & g_i(x_i, x_{-i}, p) \leq 0, \\
& h_i(x_i, x_{-i}, p) = 0, \\
& \text{for } i = 1, \ldots, N, \\
\hline
p \quad & \in \mathrm{SOL}(K, F).
\end{aligned}
$$

```
equilibrium
min f('1') x('1') g('1') h('1')
...
min f('N') x('N') g('N') h('N')
vi F pi K
```

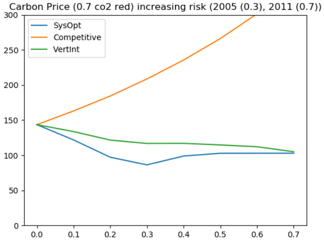
EMP framework



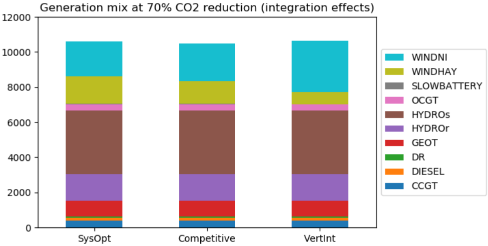
The model representation inside the EMP solver is independent of any model language

Increasing risk aversion: carbon price and investment

- $\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda \text{AVaR}_{0.90}(Z)$
- Same price risk neutral
- Competitive equilibrium: increased price
- VertInt: co-ownership of wind/thermal results in more wind closer to existing thermal



(a) Carbon prices with increasing λ



(b) Ownership at $\lambda = 0.3$

Conclusion

- Using optimization in new and innovative ways to solve problems of critical importance
- Scholar
- Leader
- Friend
- Thank you, Olvi