Optimization and Equilibrium in Energy Economics

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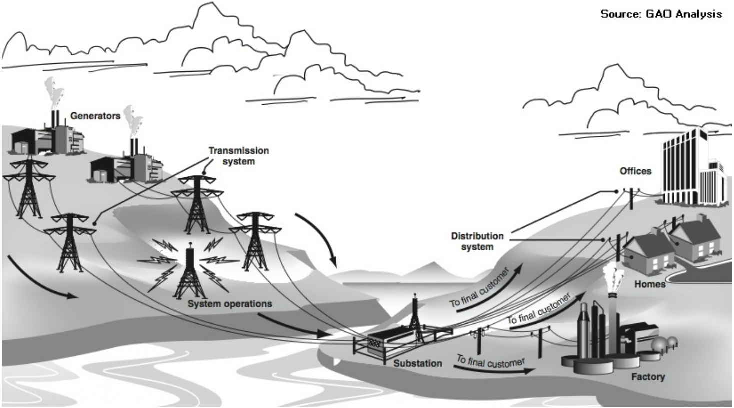
Welcome and thanks

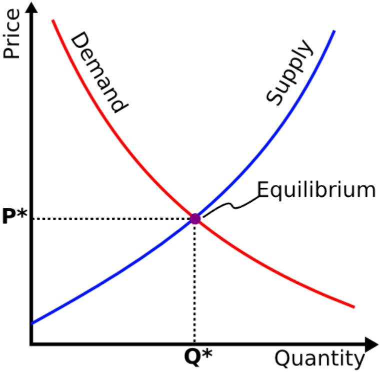
- What: Physical system + markets + stochastics + modeling + computation
- Who: power system engineers, economists, mathematicians, operations researchers, and computer scientists
- Why: create more dialogue between researchers in this area with different expertize
- How?
  - IPAM: Christian Ratsch and Roland McFarland
  - Organizing committee: Benjamin Hobbs, Antonio Conejo, Andrew Philpott, Claudia Sagastizabal
  - All of you for attending and preparing presentations
- Outcomes: new ideas and models, white paper summary
- This tutorial aims to provide some context and vocabulary for the meeting

Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $\geq \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

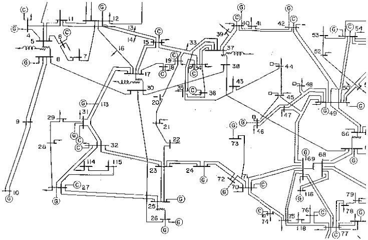
Single market, single good: equilibrium



Walras: $0 \leq s(\pi) - d(\pi) \perp \pi \geq 0$

Market design and rules to foster competitive behavior/efficiency

- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network



- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$\begin{aligned} &\min_x \; c(x) && \text{cost} \\ &\text{s.t. } Ax \geq q && \text{balance} \\ &\quad\;\; Bx = b, x \geq 0 && \text{technical constr} \end{aligned}$$

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$
\begin{aligned}
&\min_x \; c(x) && \text{cost} \\
&\text{s.t. } Ax \geq d(\pi) && \text{balance} \\
&\qquad Bx = b, x \geq 0 && \text{technical constr}
\end{aligned}
$$

- $q = d(\pi)$: issue is that π is the multiplier on the "balance" constraint
- Such multipliers (LMP's) are critical to operation of market
- Can solve the problem iteratively or by writing down the KKT conditions of this QP, forming an LCP and exposing π to the model
- Existence, uniqueness, stability from variational analysis
- EMP does this automatically from the annotations

Reformulation details

$$
\begin{aligned}
0 &\leq Ax - d(\pi) & \perp \quad & \mu \geq 0 \\
0 &= Bx - b & \perp \quad & \lambda \\
0 &\leq \nabla c(x) - A^T\mu - B^T\lambda & \perp \quad & x \geq 0
\end{aligned}
$$

- empinfo: dualvar π balance
- replaces $\mu \equiv \pi$

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\end{aligned}
$$

- empinfo: dualvar π balance
- replaces $\mu \equiv \pi$
- LCP/MCP is then solvable using PATH

$$
z = \begin{bmatrix} \pi \\ \lambda \\ x \end{bmatrix}, \quad F(z) = \begin{bmatrix} & & A \\ & & B \\ -A^T & -B^T & \end{bmatrix} z + \begin{bmatrix} -d(\pi) \\ -b \\ \nabla c(x) \end{bmatrix}
$$

Reformulation details (VI formulation)

$$
\begin{array}{rlcl}
 & 0 \le Ax - q & \perp & \pi \ge 0 \\
 & 0 = Bx - b & \perp & \lambda \\
 & 0 \le \nabla c(x) - A^T\pi - B^T\lambda & \perp & x \ge 0 \\
\cancel{q = d(\pi)} & 0 = -p(q) + \pi & \perp & q
\end{array}
$$

- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- $(x, q) \in C$, $0 \in \begin{bmatrix} \nabla c(x) \\ -p(q) \end{bmatrix} + N_c(x, q)$
- (x, q) solves $VI(F, C)$, $F(x, q) = (\nabla c(x), -p(q))^T$
- New solvers for VI: PATHVI, decomposition, distributed solution
- Straightforward to extend to more general production functions and cost functions

Extensions: maximizing profit and multiple agents

$$\begin{aligned} \max_x \ & \pi^T x - c(x) && \text{profit} \\ \text{s.t. } & Ax \geq d(\pi) && \text{balance} \\ & Bx = b, x \geq 0 && \text{technical constr} \end{aligned}$$

- Issue is that there are multiple producers i
- The price is now determined by total production

$$\begin{aligned} \max_{x_i} \ & p(\sum_j x_j)^T x_i - c_i(x_i) && \text{profit} \\ \text{s.t. } & B_i x_i = b_i, x_i \geq 0 && \text{technical constr} \end{aligned}$$

Special case: perfect competition

$$\max_{x_i} \pi \cancel{p(\sum_j x_j)}^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \sum_i x_i - d(\pi) \perp \pi \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = \bar{d} - \pi$, $\bar{d} = 24$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

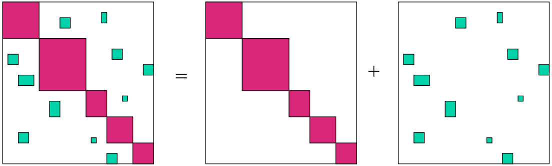
MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i$$

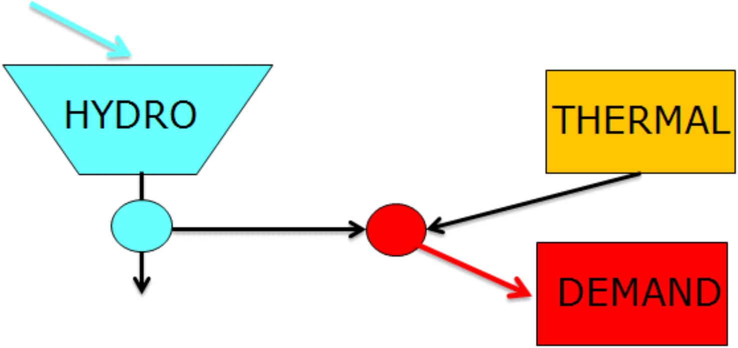
$$p \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h p cons
```

- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using “individual optimizations”?



Hydro-Thermal System (Philpott/F./Wets)



- Competing agents (consumers, or generators in energy market)
- Each agent maximizes objective independently (profit)
- Market prices are function of all agents activities

Simple electricity "system optimization" problem

$$\begin{aligned} \text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\ & x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H} \end{aligned}$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE (price takers)

$$\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k)\text{:} \quad \max_{d_k \geq 0} \quad W_k(d_k) - \pi^T d_k$$

$$\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j)\text{:} \quad \max_{v_j \geq 0} \quad \pi^T v_j - C_j(v_j)$$

$$\begin{aligned} \text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i)\text{:} \quad \max_{u_i, x_i \geq 0} \quad & \pi^T U_i(u_i) + V_i(x_i) \\ \text{s.t.} \quad & x_i = x_i^0 - u_i + h_i^1 \end{aligned}$$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\begin{aligned} \text{CE:} \qquad d_k &\in \arg\max \text{CP}(k), && k \in \mathcal{K}, \\ v_j &\in \arg\max \text{TP}(j), && j \in \mathcal{T}, \\ u_i, x_i &\in \arg\max \text{HP}(i), && i \in \mathcal{H}, \\ 0 \leq \pi \perp \sum_{i \in \mathcal{H}} U_i(u_i) &+ \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k. \end{aligned}$$

General Equilibrium models (static)

$$(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } \pi^T x_k \leq i_k(y, \pi)$$

$$(I): i_k(y, \pi) = \pi^T \omega_k + \sum_j \alpha_{kj} \pi^T g_j(y_j)$$

$$(P): \max_{y_j \in Y_j} \pi^T g_j(y_j)$$

$$(M): \max_{\pi \geq 0} \pi^T \left(\sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l \pi_l = 1$$

- This is an example of a MOPEC: strategic, top-down, policy analyses
- Can extend these models in several ways: more goods (not just energy), more agents (refineries, farmers), different behavior patterns: who is driving the bus?

Bus or Taxi: two agents (duopoly)

$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect p by choice of x_i
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot)

UBER: Bilevel Program (Stackelberg)

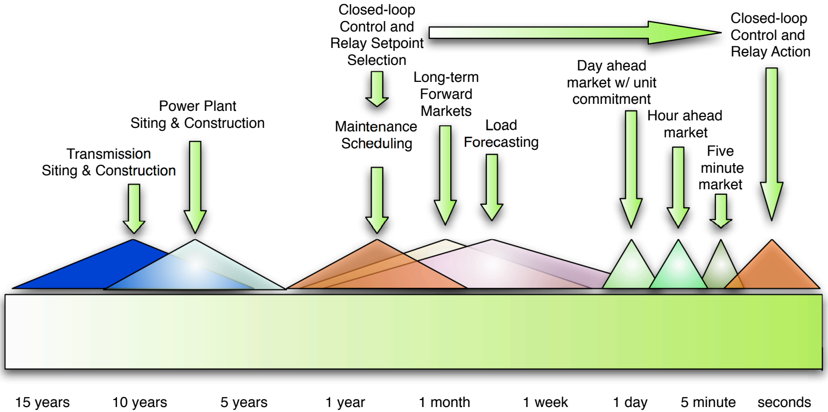
- Assumes one leader firm, the rest follow
- Leader firm optimizes subject to expected follower behavior
- Follower firms act in a competitive Nash manner
- Bilevel programs:

$$\begin{array}{ll} \min\limits_{x^*,y^*} & f(x^*,y^*) \\ \text{s.t.} & g(x^*,y^*) \leq 0, \\ & y^* \text{ solves } \min\limits_{y} v(x^*,y) \text{ s.t. } h(x^*,y) \leq 0 \end{array}$$

- model bilev /deff,defg,defv,defh/;
  empinfo: bilevel min v y defv defh
- EMP tool automatically creates the MPCC

$$\begin{array}{lll} \min\limits_{x^*,y^*,\lambda} & f(x^*,y^*) & \\ \text{s.t.} & g(x^*,y^*) \leq 0, & \\ & 0 \leq \nabla v(x^*,y^*) + \lambda^T \nabla h(x^*,y^*) & \perp y^* \geq 0 \\ & 0 \leq -h(x^*,y^*) & \perp \lambda \geq 0 \end{array}$$

Representative decision-making timescales in electric power systems

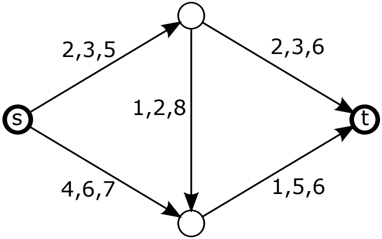


Many interacting levels/hierarchies, with different time scaled decisions at each level - collections of models needed.

Complications and myriad of acronyms

- Size/integrity:
  - AC/DC models, reactive power, new devices
  - Day ahead, regulation, FTR's, co-optimization
  - Semidefinite programming (global), EPEC's
- Discrete:
  - Unit commitment (DAUC, RUC, RT)
  - Topology optimization (e.g. Transmission line switching, siting)
  - Design of flexible computing systems
  - How do we price discrete decisions? (make-good, fix ints, ...)
- Dynamic:
  - Design/operation
  - Multi-period, unit commitment, minimum up and down time
  - Demand response, load shedding, demand bidding
- Stochastic:
  - Security constraints - failures/reserves (SCED/SCUC)
  - Stochastic demand
  - Renewables/storage/feed-in prices

Discrete: Totally Unimodular Congestion Games



$N = 3$, $m = 5$. Each player chooses a path from s to t. Costs for one, two or three players using arc are given by triplets.

The goal is to find a pure Nash equilibrium, i.e. a state $x = (x^1, \ldots, x^N) \in X$ such that, for each player i and $\bar{x}^i \in X^i$:

$$c^i(x^1, \ldots, x^i, \ldots, x^N) \leq c^i(x^1, \ldots, \bar{x}^i, \ldots, x^N).$$

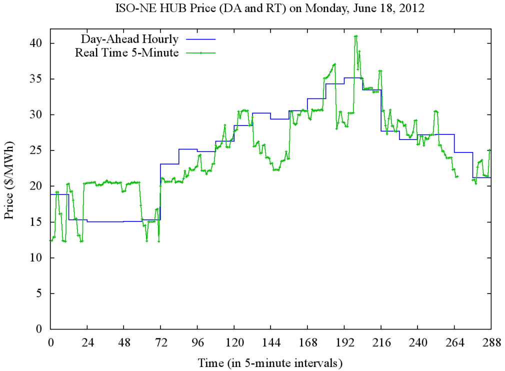
Theorem (DelPia/F./Micini)

There is a strongly polynomial-time algorithm for finding a pure Nash equilibrium in symmetric TU congestion games.

Dynamic: PJM buy/sell storage

- Storage transfers energy over time (horizon $= T$).
- PJM: given price path p_t, determine charge q_t^+ and discharge q_t^-:

$$
\begin{aligned}
\max_{h_t, q_t^+, q_t^-} \quad & \sum_{t=0}^{T} p_t (q_t^- - q_t^+) \\
\text{s.t.} \quad & \partial h_t = e q_t^+ - q_t^- \\
& 0 \le h_t \le \mathcal{S} \\
& 0 \le q_t^+ \le \mathcal{Q} \\
& 0 \le q_t^- \le \mathcal{Q} \\
& h_0, h_T \text{ fixed}
\end{aligned}
$$



- Uses: price shaving, load shifting, transmission line deferral
- What about real-time storage, or different storage technologies?

Stochastic: Agents have recourse?

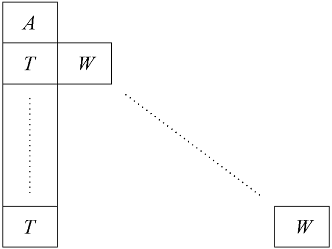
- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP:}\ \min \quad c^T x^1 + \rho[q^T x^2]$$

$$\text{s.t.} \quad Ax^1 = b, \quad x^1 \geq 0,$$

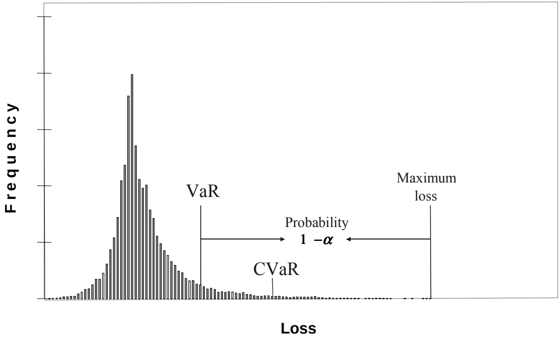
$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



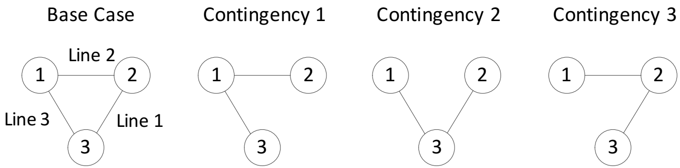
- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Stochastic price paths (day ahead market)

$$
\begin{aligned}
\min_{x,h,q^+,q^-} \quad & c^1(x) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^2(q^+_{\omega t} + q^-_{\omega t}) \right] \\
\text{s.t.} \quad & \partial h_{\omega t} = e q^+_{\omega t} - q^-_{\omega t} \\
& 0 \le h_{\omega t} \le \mathcal{S} x \\
& 0 \le q^+_{\omega t}, q^-_{\omega t} \le \mathcal{Q} x \\
& h_{\omega 0}, h_{\omega T} \text{ fixed}
\end{aligned}
$$

- First stage decision x: amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty

Contingency: a single line failure



- A network with N lines can have up to N contingencies
- Each contingency case:
  - Corresponds to a different network topology
  - Requires a different set of equations g and h
  - E.g., equations g_k and h_k for the k-th contingency.

Security-constrained Economic Dispatch

- Base-case network topology g_0 and line flow x_0.
- If the k-th line fails, line flow jumps to x_k in new topology g_k.
- Ensure that x_k is within limit, for all k.
- SCED model:

$$
\begin{array}{llr}
\min\limits_{u,x_0,\ldots,x_k} & c^T u & \triangleright \text{ Total cost} \\
\text{s.t.} & 0 \leq u \leq \bar{u} & \triangleright \text{ GEN capacity const.} \\
& g_0(x_0, u) = 0 & \triangleright \text{Base-case network eqn.} \\
& -\bar{x} \leq x_0 \leq \bar{x} & \triangleright \text{Base-case flow limit} \\
& g_k(x_k, u) = 0, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy network eqn.} \\
& -\bar{x} \leq x_k \leq \bar{x}, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy flow limit}
\end{array}
$$

Model structure

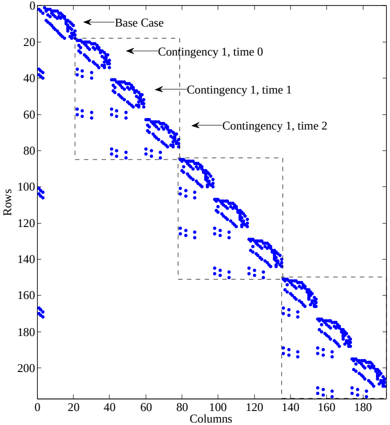


Figure : Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

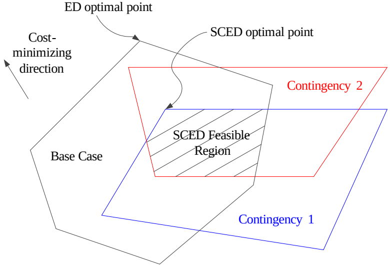


Figure : On the u_0 plane, the feasible region of a SCED is the intersection of K+1 polyhedra.

Decomposition approaches allow solution of realistic sized models with many contingencies in minutes

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{array}{lll} \min & C(x_i^1) + \rho_i\left(C(x_i^2(\omega))\right) & \\ \text{s.t.} & p^1 x_i^1 + v y_i \le p^1 e_i^1 & \text{(budget time 1)} \\ & p^2(\omega)x_i^2(\omega) \le p^2(\omega)(D(\omega)y_i + e_i^2(\omega)) & \text{(budget time 2)} \end{array}$$

$$0 \le v \perp -\sum_i y_i \ge 0 \qquad \text{(contract)}$$

$$0 \le p^1 \perp \sum_i \left(e_i^1 - x_i^1\right) \ge 0 \qquad \text{(walras 1)}$$

$$0 \le p^2(\omega) \perp \sum_i \left(D(\omega)y_i + e_i^2(\omega) - x_i^2(\omega)\right) \ge 0 \qquad \text{(walras 2)}$$

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC enables modeling dynamic decision processes under uncertainty
- Modeling, optimization, statistics and computation embedded within the application domain is critical
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS