MOPEC, contracts, risk and stochastics

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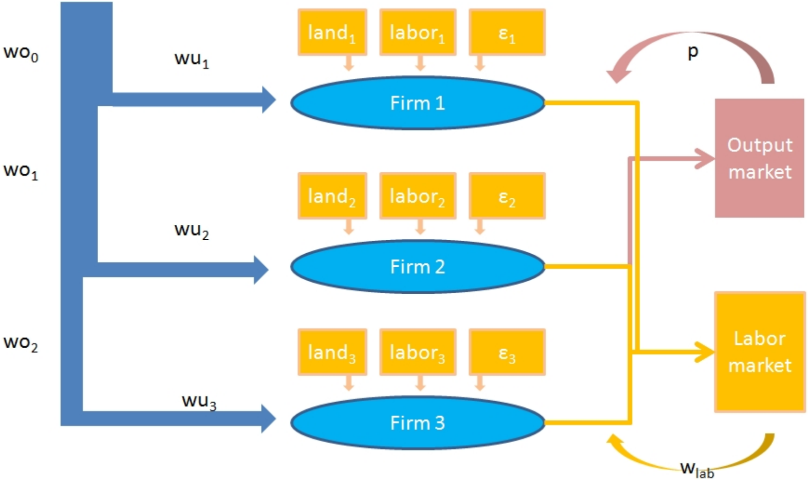
Water rights pricing (Britz/F./Kuhn)



Rare resources (Outrata/F./Cervinka/Outrata)

$$\min_{y_i \in A_i} c_i(y_i) + \pi(q_i(y_i) - e_i) - p(T)y_i$$

$$0 \leq \pi \perp (\Xi - \sum_{i=1}^{m} q_i(y_i)) \geq 0.$$

- Rare good needed for y_i's production: $q_i(y_i)$
- Inverse demand function $p(T)$
- Under (reasonable) assumptions (convexity, differentiability, etc) an equilibrium exists

- Solvable by equivalent complementarity problem, MPEC or bundle trust method

Theorem

Let $(\bar{\pi}, \bar{y})$ be a solution of above and assume that $\bar{\pi} > 0$ and $\bar{y}_i \in \operatorname{int} A_i$ for at least one $i \in \{1, 2, \ldots, m\}$. Then Ψ (MOPEC) is strongly metrically regular at $(\bar{\pi}, \bar{y}, 0_{\mathbb{R}^{m+1}})$, i.e., $(\Psi)^{-1}$ has a Lipschitz single-valued localization around $(0_{\mathbb{R}^{m+1}}, \bar{\pi}, \bar{y})$.

(M)OPEC

$$\min_{x} \theta(x, p) \text{ s.t. } g(x, p) \leq 0$$

$$0 \leq p \perp h(x, p) \geq 0$$

```
equilibrium
min theta x g
vi h p
```

- Solved concurrently

(M)OPEC

$$\frac{\min_{x} \theta(x,p) \text{ s.t. } g(x,p) \leq 0}{0 \leq p \perp h(x,p) \geq 0}$$

$$\begin{aligned} x \perp \nabla_x \theta(x,p) + \lambda^T \nabla_x g(x,p) \\ 0 \leq \lambda \perp -g(x,p) \geq 0 \\ 0 \leq p \perp h(x,p) \geq 0 \end{aligned}$$

```
equilibrium
min theta x g
vi h p
```

- Solved concurrently
- Requires global solutions of agents problems (or theory to guarantee KKT are equivalent)
- Theory of existence, uniqueness and stability based in variational analysis

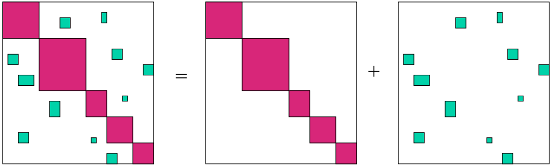
MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i$$

$$p \text{ solves VI}(h(x,\cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h p cons
```

- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



IAMs and Economic Theory

Integrated assessment models a stylized story of how markets work and the nature of agent interactions:

- Theory of the consumer (demand), including inter-temporal choice.
- Production and cost theory (supply), possibly based on (bottom-up) activity analysis engineering estimates of cost functions.
- The neoclassical paradigm: individual elements of the economy (consumers, firms, workers) are rational agents with objectives which can be expressed as quantitative functions to be optimized subject to constraints.

IAMs are typically used to provide logical implications of specific assumptions. Model results may provide the basis for normative conclusions.

Example of MOPEC models in policy analysis: data

- The latest GTAP database represents global production and trade for 113 country/regions, 57 commodities and 5 primary factors.
- Data characterizes intermediate demand and bilateral trade in 2007, including tax rates on imports/exports and other indirect taxes.
- The core GTAP model is a static, multi-regional model which tracks the production and distribution of goods in the global economy.
- In GTAP the world is divided into regions (typically representing individual countries), and each region's final demand structure is composed of public and private expenditure across goods.

The Model

The GTAP model (MOPEC) may be posed as a system of *nonsmooth equations*:

$$F_+(w, z; t) = 0$$

in which:

- w_r is a vector of regional *welfare* levels
- $z \in \mathrm{R}^N$ represents a vector of endogenous economic variables, e.g. prices and quantities, $z = \begin{pmatrix} P \\ Q \end{pmatrix}$.
- t represents matrices of trade tax instruments – import tariffs (t^M_{irs}) and export taxes (t^X_{irs}) for each commodity i and region r

Optimal Sanctions: scenario runs

Nash equilibrium (over trade tax instruments) between coalition states and Russia.

- NoRegrets: choose taxes so coalition members welfare is maximized
- MaxDamage: change objective so coalition members minimize Russian welfare
- SidePayments: allow compensatory payments within coalition while minimizing Russian welfare

Also consider only working with small number of tax instruments

Optimal Coalition Operation

Coalition member states strategically choose trade taxes which *minimize* Russian welfare:

$$\min_{t_r : r \in \mathcal{C}} w_{rus} + \gamma \left\| t_r \right\|_1$$

s.t.

$$F_+(w, z; t) = 0$$

$$t_r = \bar{t}_r \quad \forall r \notin \mathcal{C}$$

$$t^M_{i,rus,r} \leq \bar{t}^M_{i,r,rus} \quad \forall r \in \mathcal{C}$$

$$t^X_{i,r,rus} \leq \bar{t}^X_{i,rus,r} \quad \forall r \in \mathcal{C}$$

$$w_r \geq 0.98 * \bar{w}_r \quad \forall r \in \mathcal{C}$$

Optimal Retaliation

Russia choose trade taxes which *maximize* Russian welfare in response to the coalition actions:

$$\max_{t_{rus}} w_{rus}$$

s.t.

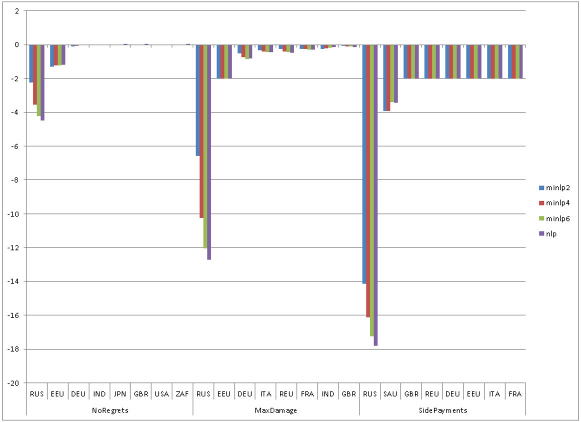
$$F_+(w, z; t) = 0$$

$$t_r = \begin{cases} \hat{t}_r & r \in \mathcal{C} \\ \bar{t}_r & r \notin \mathcal{C} \end{cases}$$

where $\hat{t}_r$ represents trade taxes for coalition countries ($r \in \mathcal{C}$) from the optimal sanction calculation.

Optimal Sanctions (Boehringer/F./Rutherford)

- GTAP global production/trade database: 113 countries, 57 goods, 5 factors
- Coalition members strategically choose trade taxes to *minimize* Russian welfare
- Russia chooses trade taxes to *maximize* Russian welfare in response
- Nash equilibrium

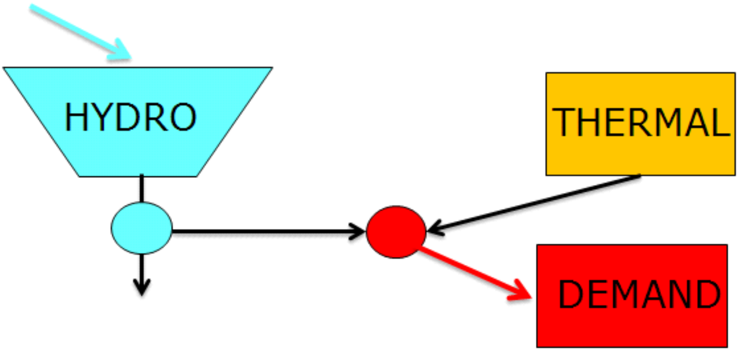


Resulting equilibrium with no regrets (coalition), maximize damage, side payments

In Defense of a Neoclassical Approach

1. Versatility. The basic model can be extended to take into account many aspects which are often assumed to be ignored: risk and uncertainty, technological details, expectations.
2. Can be either calibrated or estimated. Hence, it is possible to formulate a model which matches both current economic statistics (supply and demand) and historical evidence about the responsiveness of quantity to price.
3. Approach can be consistent with the principal of Occam's Razor: "A scientific theory should be as simple as possible, but no simpler."
4. Theoretical coherence provides a means of formulating models which perform better "out of sample".

Hydro-Thermal System (Philpott/F./Wets)



- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities

Simple electricity "system optimization" problem

$$
\begin{aligned}
\text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\
& x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}
$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE (price takers)

$$\begin{aligned}
\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k)\text{:} \quad & \max_{d_k \geq 0} && W_k(d_k) - p^T d_k \\
\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j)\text{:} \quad & \max_{v_j \geq 0} && p^T v_j - C_j(v_j) \\
\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i)\text{:} \quad & \max_{u_i, x_i \geq 0} && p^T U_i(u_i) + V_i(x_i) \\
& \text{s.t.} && x_i = x_i^0 - u_i + h_i^1
\end{aligned}$$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\begin{aligned}
\text{CE:} \quad d_k &\in \arg\max \text{CP}(k), && k \in \mathcal{K}, \\
v_j &\in \arg\max \text{TP}(j), && j \in \mathcal{T}, \\
u_i, x_i &\in \arg\max \text{HP}(i), && i \in \mathcal{H}, \\
0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) &+ \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.
\end{aligned}$$

Agents have stochastic recourse?

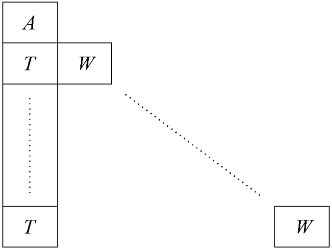
- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP: } \min \quad c^T x^1 + \rho[q^T x^2]$$

$$\text{s.t.} \quad Ax^1 = b, \quad x^1 \geq 0,$$

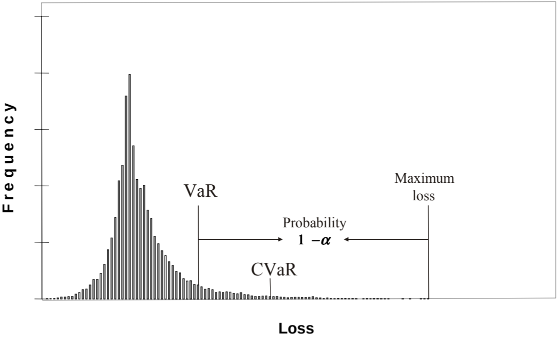
$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty
- Dual representation (of coherent r.m.) in terms of risk sets

Two stage stochastic MOPEC

$$\text{CP}(k): \quad \min_{d_k^1 \geq 0} \quad p^1 d_k^1 - W_k\left(d_k^1\right)$$

$$\text{TP}(j): \quad \min_{v_j^1 \geq 0} \quad C_j(v_j^1) - p^1 v_j^1$$

$$\text{HP}(i): \quad \min_{u_i^1, x_i^1 \geq 0} \quad - p^1 U_i(u_i^1)$$

$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

Two stage stochastic MOPEC

$$\text{CP}(k): \quad \min_{d_k^1, d_k^2(\omega) \geq 0} \quad p^1 d_k^1 - W_k\left(d_k^1\right) + \rho[p^2(\omega) d_k^2(\omega) - W_k\left(d_k^2(\omega)\right)]$$

$$\text{TP}(j): \quad \min_{v_j^1, v_j^2(\omega) \geq 0} \quad C_j(v_j^1) - p^1 v_j^1 + \rho[C_j\left(v_j^2(\omega)\right) - p^2(\omega) v_j^2(\omega)]$$

$$\text{HP}(i): \quad \min_{\substack{u_i^1, x_i^1 \geq 0 \\ u_i^2(\omega), x_i^2(\omega) \geq 0}} \quad - p^1 U_i(u_i^1) + \rho[-p^2(\omega) U_i(u_i^2(\omega)) - V_i(x_i^2(\omega))]$$

$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$x_i^2(\omega) = x_i^1 - u_i^2(\omega) + h_i^2(\omega)$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

Two stage stochastic MOPEC

$$\text{CP}(k): \quad \min_{d_k^1, d_k^2(\omega) \geq 0} \quad p^1 d_k^1 - W_k\left(d_k^1\right) + \rho[p^2(\omega) d_k^2(\omega) - W_k\left(d_k^2(\omega)\right)]$$

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$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$x_i^2(\omega) = x_i^1 - u_i^2(\omega) + h_i^2(\omega)$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

$$0 \leq p^2(\omega) \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^2(\omega)\right) + \sum_{j \in \mathcal{T}} v_j^2(\omega) \geq \sum_{k \in \mathcal{K}} d_k^2(\omega), \forall \omega$$

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

- Single hydro, thermal and representative consumer
- Random inflow scenarios (with $0.8EV + 0.2CVaR$)
- High initial storage level
  - Worst case scenario is 1: lowest total cost, smallest profit for hydro
  - SO equivalent to CE (risk averse set for social planner same as a modified risk neutral set for social planner)
- Low initial storage level
  - Different worst case scenarios
  - SO different to CE (for large range of demand elasticities)

Contracts in MOPEC (F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{aligned}
\min \quad & C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) & \\
\text{s.t.} \quad & p^1 x_i^1 + v y_i \le p^1 e_i^1 & \text{(budget time 1)} \\
& p^2(\omega) x_i^2(\omega) \le p^2(\omega)(D(\omega) y_i + e_i^2(\omega)) & \text{(budget time 2)}
\end{aligned}$$

$$0 \le v \perp -\sum_i y_i \ge 0 \qquad \text{(contract)}$$

$$0 \le p^1 \perp \sum_i \left(e_i^1 - x_i^1 \right) \ge 0 \qquad \text{(walras 1)}$$

$$0 \le p^2(\omega) \perp \sum_i \left(D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega) \right) \ge 0 \qquad \text{(walras 2)}$$

Example

- Low storage setting
- If thermal now uses EV, SO equivalent to CE
- If thermal is risk averse, then there is a CE, but different to original SO
- Trade risk to give minimum risk solutions for the sum of their positions
- Can compute an equivalent risk neutral set for which SO equivalent to this CE

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC
- Our contribution: apply in multistage setting over scenario tree

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Conclusions

- MOPEC problems capture complex interactions between optimizing agents
- Policy implications addressable using MOPEC
- MOPEC available to use within the GAMS modeling system
- Stochastic MOPEC enables modeling dynamic decision processes under uncertainty
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements