Stochastic Equilibrium Problems

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Risk Measures

Problem type

Objective function or Constraint

$$\min_{x \in X} \theta(x) + \rho(F(x)) \qquad \min_{x \in X} \theta(x) \text{ s.t. } \rho(F(x)) \le \alpha$$

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{y \in \mathcal{D}} \mathbb{E}_y[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{y \in [0, p/(1-\alpha)] : \langle \mathbf{1}, y \rangle = 1\}$, then $\rho(Z) = \overline{CVaR}_\alpha(Z)$
- Combinations - increasing risk aversion as λ increases

$$\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda \overline{CVaR}_\alpha(Z)$$

Minimax transformation to complementarity

$$\min_{x \in X} \theta(x) + \rho(F(x))$$
$$\text{where } \rho(u) = \sup_{y \in \mathcal{D}} \left\{ \langle y, u \rangle - \frac{1}{2} \langle y, My \rangle \right\}$$

optimality condition:

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

calculus ($\rho = (k + \delta_D)^*, k(y) = \frac{1}{2} \langle y, My \rangle$):

$$0 \in \partial\theta(x) + \nabla F(x)^T y + N_X(x)$$
$$0 \in -y + \partial\rho(F(x)) \iff 0 \in -F(x) + My + N_{\mathcal{D}}(y)$$

- This is a complementarity problem: opt conds in x coupled with opt conds in y - separated

Time results for PATHVI (LINLIB)

- PATHVI is a (nonsmooth) Newton method for solving variational inequalities via piecewise linearization

Model	MCP Size	PATHVI iterations	PATH iterations	PATHVI time (s)	PATH time (s)
cq9	23056	37026	>43820	93	>273
dbir2	46261	24527	>53737	263	>3095
deter1	21264	13073	15878	18	64
deter3	29424	18022	21761	40	182
osa-14	54797	1771	3329	29	35
p010	29090	3855	23624	9	161
south31	53846	34003	>14768	357	>600
stocfor3	32370	20805	>26445	53	>332
testbig	48836	14433	>21138	52	>600
woodw	9503	1853	15155	1	208

- ">" means either major iteration limit 50 is reached or time limit 600 is reached
- Iterations here are pivots
- PATH (VI) available in GAMS, AMPL, JuMP, AIMMS

PATH Time Results using Basis Package

Observations:

- HIGHS and CONOPT both outperform the default (LU-SOL) on update-heavy tasks.
- HIGHS factorization slower than other methods.
- UMPACK has good factorization.
- CONOPT has good performance in factorizing sparse matrices and has superior performance in update-heavy tasks.

Update Methods used by each basis method:

1. LUSOL: Bartels-Golub-Saunders Update
2. BLU-LUSOL: Block-LU (Eldersveld and Saunders)
3. UMFPACK: Multifrontal-LU (Davis)
4. HIGHS: Forrest-Tomlin
5. CONOPT: Bartels Golub-Reid Update (Drud)

Time Results using Basis Package

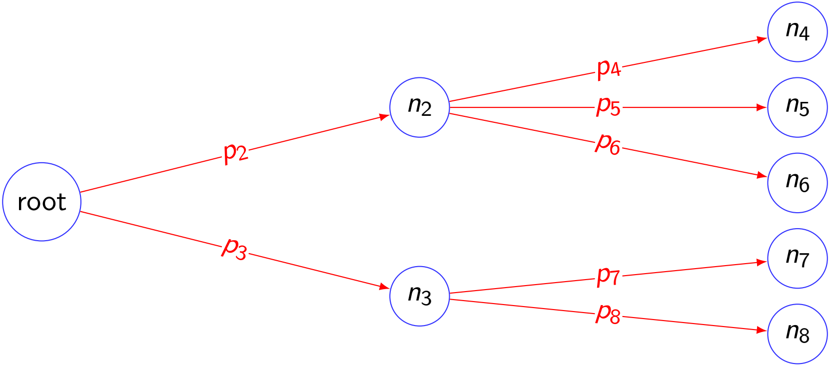
Model	Size	Density	LU_SOL	BLU_LUSOL	UMFPACK	HIGHS	CONOPT
bai_haung (pure crash)	2500	0.2%	0.01	0.01	0.01	0.03	0.01
	10000	0.05%	0.12	0.11	0.02	0.84	0.05
	22500	0.02%	0.33	0.31	0.04	4.86	1.18
	40000	0.01%	0.71	0.68	0.07	13.73	0.33
	62500	0.01%	1.40	1.33	0.13	81.27	0.57
bratu (pure crash)	5625	0.09%	0.22	0.23	0.10	1.03	0.28
	9801	0.05%	0.72	0.69	0.23	5.00	0.76
	22201	0.02%	3.53	3.40	0.84	45.6	3.09
dirkse2 (no crash)	15626	0.02%	19.89	14.79	9.98	12.74	3.16
	27001	0.01%	99.22	71.18	36.59	63.96	9.59
	42876	0.01%	388.87	270.33	111.38	249.38	24.30
opt_cont (no crash)	8032	0.21%	1.78	1.72	3.04	1.38	2.38
	16032	0.11%	9.92	8.41	12.57	7.36	8.20
	24032	0.07%	29.04	22.76	27.83	20.44	18.04
	32032	0.05%	61.15	47.99	49.50	43.24	30.31
genlibgnep1 (no crash)	9050	0.51%	6.69	5.16	2.59	5.40	2.13
genlibgnep2 (no crash)	27050	0.17%	215.36	149.59	71.10	170.76	39.15
genlibgnep3 (no crash)	27050	0.17%	262.92	178.33	92.54	207.21	44.70
genlibgnep4 (no crash)	27050	0.17%	224.20	155.68	84.01	177.93	34.96
genlibgnep5 (no crash)	27050	0.17%	1415.68	1099.50	676.56	1131.57	271.32
genlibgnep6 (no crash)	36080	0.16%	1474.09	1050.60	560.64	1005.24	178.59
genlibgnep7 (no crash)	3210	5.95%	40.44	18.92	9.67	5.68	21.63

A mathematical modelling approach to planning

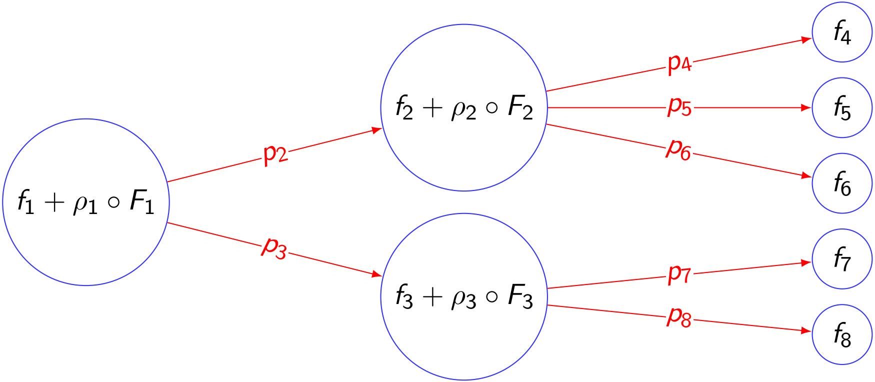
- Build and solve a social planning model that optimizes electricity design and operation with environmental constraints
- Social planning solution should be stochastic: i.e. account for future uncertainty
- Social planning solution should be risk-averse: because the industry is
- Approximate the outcomes of the social plan by a competitive equilibrium with risk-averse players
- Compensate for market failures from imperfect competition or incomplete markets

Need to be able to solve both stochastic optimization and equilibrium formulations

Shared data: (irregular) scenario tree



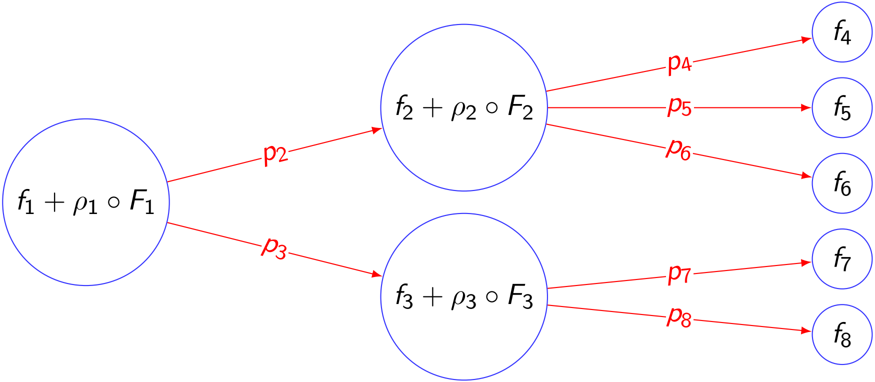
Shared data: (irregular) scenario tree



At leaf nodes:

$$\min_{x_\ell \in \mathcal{X}_\ell} \leftarrow f_\ell(x_\ell)$$

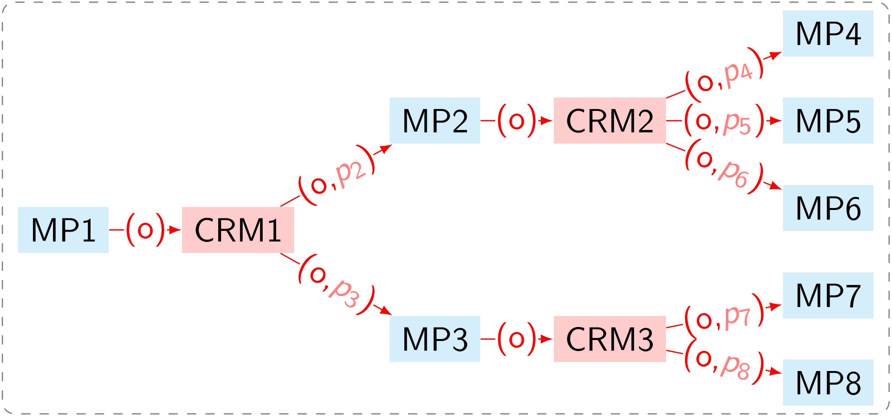
Shared data: (irregular) scenario tree



Recursively:

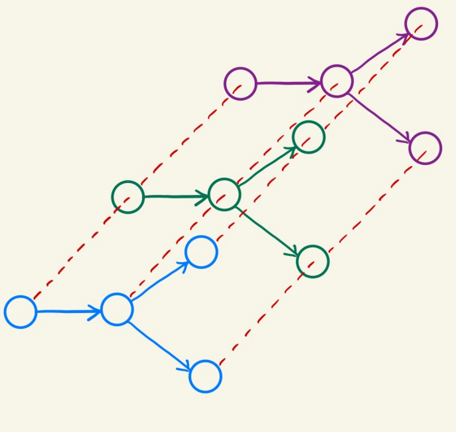
$$\min_{x_\cdot \in \mathcal{X}_0} f_1(x_1) + \rho_1([f_j(x_j) + \rho_j([f_\ell(x_\ell)]_{\ell \in j_+})]_{j \in 1_+})$$

EMPDAG



- Red nodes implement the coherent risk measure: in this case a max problem
- ReSHOP enables structured description of problem

Scenario trees linked across agents

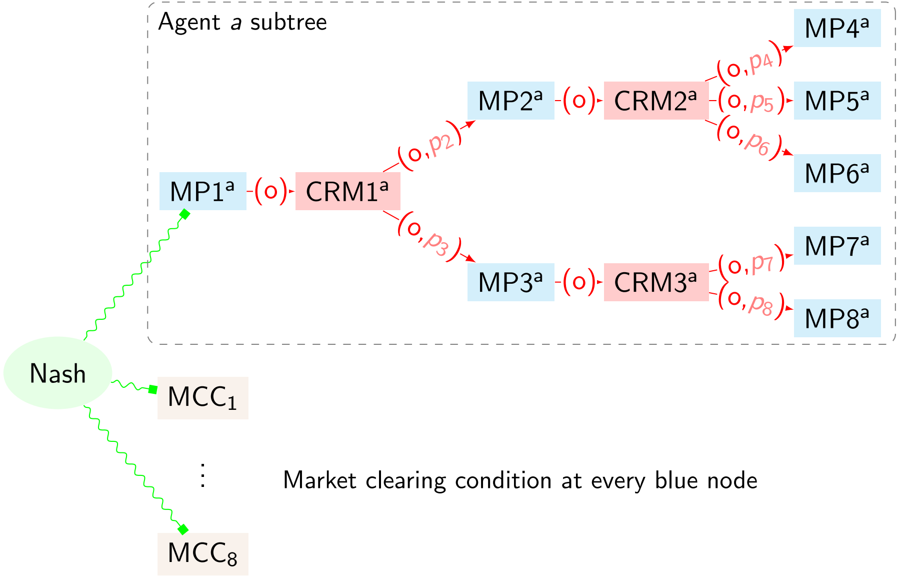


- Dynamics link over time
- Complementarity links nodes of scenario tree across agents

Three sources of difficulty:

1. Size: number of scenarios, agents, details
2. Non-convexity: Nash behavior
3. Risk aversion: Nonsmooth or Nonlinear (product of probabilities)

Stochastic equilibrium

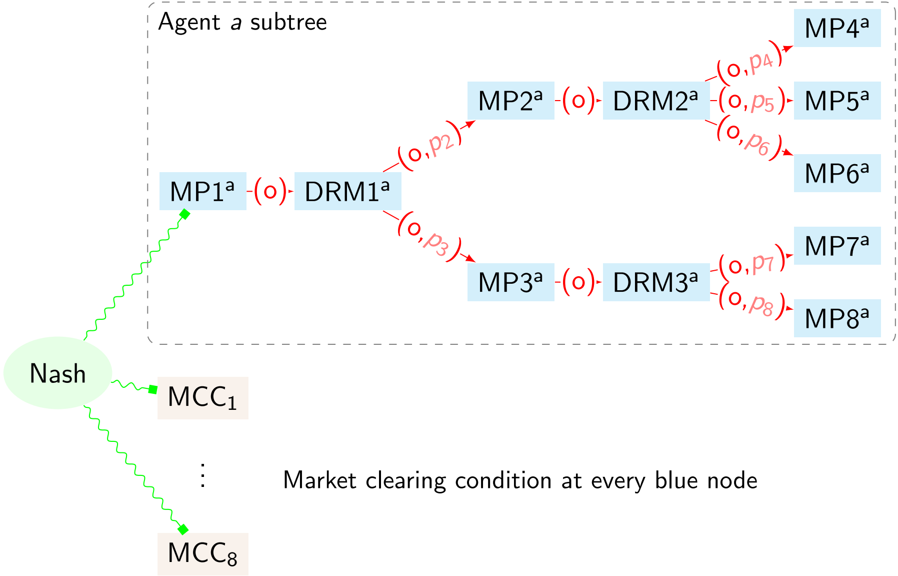


Market clearing condition at every blue node

Dynamic risked equilibria

- ReSHOP automatically generates model with (multi-stage) coherent risk measure
- Outstanding issues related to existence, uniqueness and stability of such solutions
- (F,Philpott 2021) show how such a market can be completed using Arrow/Debreu securities
- In practical situations, how far from system optimal are actual contracts?
- Need to solve both the system optimization and the (incomplete) stochastic equilibrium problem (with risk) to calculate the difference
- We consider here computational solution of such problems in practical settings

Stochastic equilibrium: Fenchel dual



Market clearing condition at every blue node

Stochastic Equilibrium as (extended) MOPEC

Interchange $\max_y \min_x$ to $\min_x \max_y$ and collect $\min_{x_i} \min_{x_{i+1}}$ together (recursively):

$$
\begin{aligned}
&\min_{x_{a1} \in \mathcal{X}_{a1}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \\
&\qquad \max_{y_{a1_+} \in \mathcal{D}_{a1}} \sum_{j \in 1_+} y_{aj} \Big(\min_{x_{aj} \in \mathcal{X}_{aj}} f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \\
&\qquad\qquad \max_{y_{aj_+} \in \mathcal{D}_{aj}} \sum_{\ell \in j_+} y_{a\ell} \min_{x_{a\ell} \in \mathcal{X}_{a\ell}} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \Big), \quad \forall a \in \mathcal{A} \\
&0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T}
\end{aligned}
$$

Then apply the equilibrium optimality condition to the minimax problem

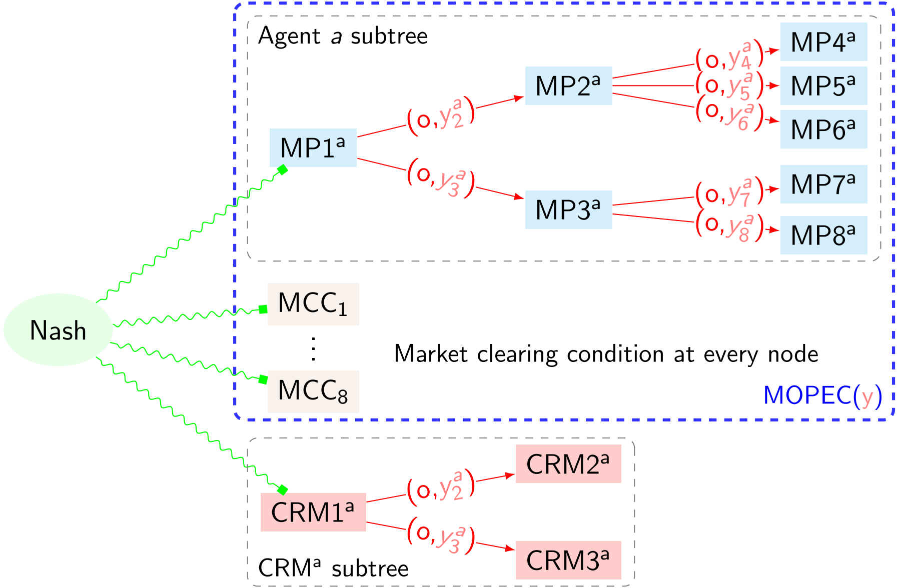
Stochastic Equilibrium as (extended) MOPEC

Interchange $\max_y \min_x$ to $\min_x \max_y$ and collect $\min_{x_i} \min_{x_{i+1}}$ together (recursively):

$$\begin{aligned} \min_{x_{a\cdot} \in \mathcal{X}_{a0}} & f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \\ & \max_{y_{a1_+} \in \mathcal{D}_{a1}} \max_{y_{aj_+} \in \mathcal{D}_{aj}} \sum_{j \in 1_+} y_{aj} \Big(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \\ & \qquad \sum_{\ell \in j_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \Big), \quad \forall a \in \mathcal{A} \\ 0 \in & H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T} \end{aligned}$$

Then apply the equilibrium optimality condition to the minimax problem

SMOPEC (telescope + equilibrium)



Stochastic Equilibrium as (extended) MOPEC

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \sum_{j \in 1_+} y_{aj} \left(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \sum_{\ell \in j_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \right), \quad \forall a \in \mathcal{A} \tag{1}$$

$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T} \tag{2}$$

$$\begin{aligned} r_{a1}(x,\pi) &= \max_{y_{a1_+} \in \mathcal{D}_{a1}} \sum_{j \in 1_+} y_{aj}(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + r_{aj}(x,\pi)) \\ r_{a2}(x,\pi) &= \max_{y_{a2_+} \in \mathcal{D}_{a2}} \sum_{\ell \in 2_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \\ r_{a3}(x,\pi) &= \max_{y_{a3_+} \in \mathcal{D}_{a3}} \sum_{\ell \in 3_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \end{aligned} \tag{3}$$

Algorithms and problems

- PATH: nonsmooth Newton method (defaults) (blue+black+red)
- PD (Primal-dual): iteratively blue+black then red
- PD-PTH (Primal-dual + PATH)
- PD-CC-PTH (Primal-dual + convex-comb(red) + PATH)
- Homot(λ) + Primal-dual + convex-comb(red) + PATH

- Multistage economic dispatch, capacity expansion, hydroelectric system
- 3 types of demand formulation (I,II and III)
- Two scenario trees (4 stages, 40 nodes) and (4 stages, 156 nodes)
- 32 data instances for each formulation
- Several modulus of convexity and risk aversion parameters

$$\rho(Z) = (1 - \lambda)\mathbb{E}[Z] + \lambda\overline{CVaR}_{\alpha}(Z)$$

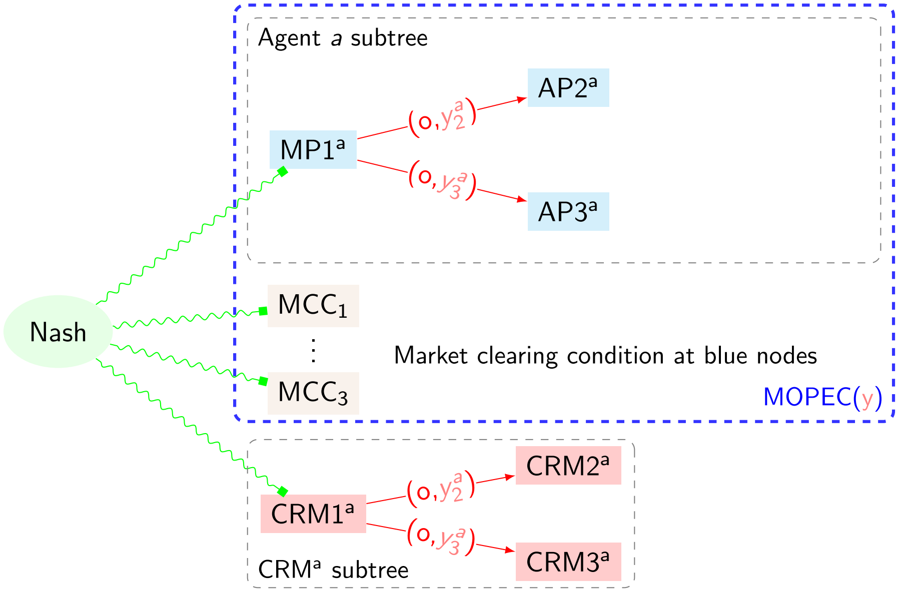
Dispatch example, large tree, type I

quad	λ	PATH	PD	PD-PTH	PD-CC-PTH	Homot
0.00	0.1	0.0	0.0	59.4	100.0	100.0
0.00	0.3	0.0	0.0	12.5	96.9	100.0
0.00	0.5	0.0	0.0	9.4	71.9	90.6
0.00	0.7	0.0	0.0	3.1	18.8	53.1
0.00	0.9	0.0	0.0	0.0	9.4	21.9
0.01	0.1	28.1	15.6	100.0	100.0	100.0
0.01	0.3	0.0	0.0	90.6	100.0	100.0
0.01	0.5	0.0	0.0	40.6	100.0	100.0
0.01	0.7	0.0	0.0	21.9	84.4	93.8
0.01	0.9	0.0	0.0	6.2	53.1	68.8
0.10	0.1	0.0	59.4	100.0	100.0	100.0
0.10	0.3	0.0	43.8	100.0	100.0	100.0
0.10	0.5	0.0	18.8	96.9	100.0	100.0
0.10	0.7	0.0	12.5	100.0	100.0	100.0
0.10	0.9	0.0	15.6	93.8	100.0	100.0

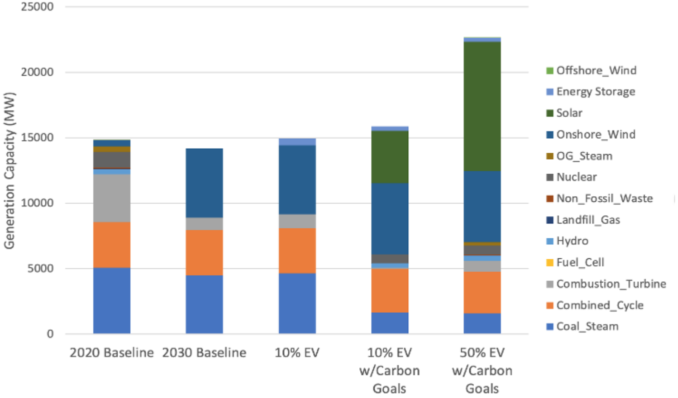
Remarks

- Primal model is large scale MOPEC; easier to solve since nonlinear complexity reduced.
- Model is not equivalent to a single optimization - agents have different y values.
- Solve forward model iteratively over stages or decompose by agent
  - Augmented Lagrangian needed for agent decomposition
  - Convex approximations of future costs may be poor
  - Parameterization (proximal or step-length) difficult to choose automatically

SMOPEC (approximation and iteration)

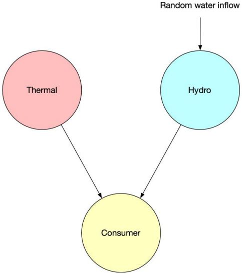


Impact of Electric Vehicles on Generator Investments

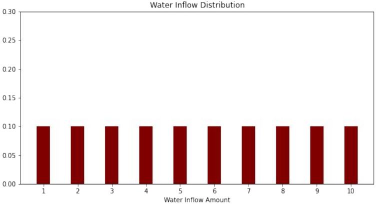


- Carbon Goals: 60% reduction on in-state carbon emissions
- Nuclear (low-carbon) used
- Coal steam generators shut down, supplanted by renewables
- Additional 180,000 MWh demand for EVs
- Storage investment needed
- Additional demand or carbon goals give more dramatic effects

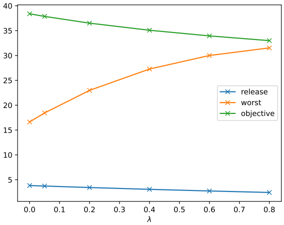
Simple example (3 agents, 2 stages, 10 scenarios)



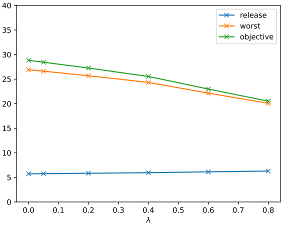
Second stage probabilities:



Low stage 1 inflow:



Higher stage 1 inflow:

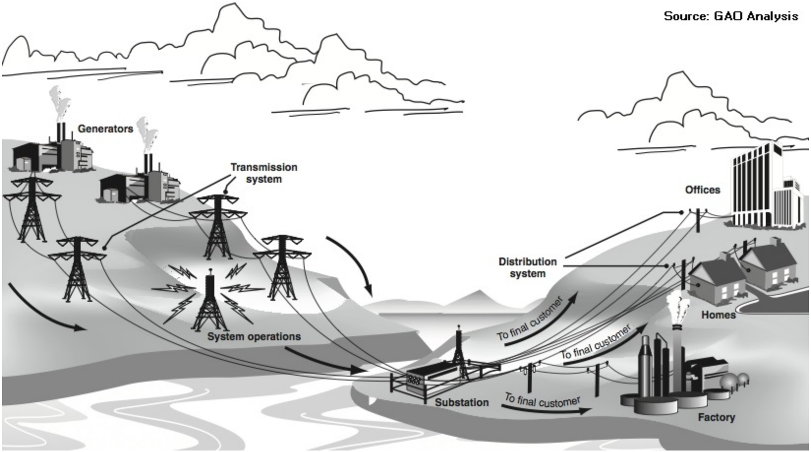


Large pumped storage investment: Lake Onslow

Technology	Without			With		
	SI	HAY	NI	SI	HAY	NI
ONSLOW	0.0	0.0	0.0	1000.0	0.0	0.0
SLOWBATT	500.0	500.0	500.0	0.0	500.0	500.0
WIND	0.0	2049.9	5000.0	0.0	1407.4	5000.0

- Worried about the effects of dry winters and excess wind capacity
- Pumped storage costs amortized over long period
- Economical if emissions constraint is strict enough (i.e. no more than 5% of 2017 levels)
- Remove large battery in SI, reduce wind capacity at HAY

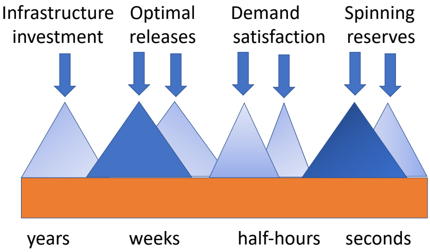
Engineering, Economics and Environment



- Determine generators' output to reliably/economically meet the load
- Power flows cannot exceed lines' transfer capacity
- Tradeoff: Impose environmental regulations/incentives

Uncertainty is experienced at different time scales

- Demand growth, technology change, capital costs are long-term uncertainties (years)
- Seasonal inflows to hydroelectric reservoirs are medium-term uncertainties (weeks)
- Levels of wind and solar generation are short-term uncertainties (half hours)
- Very short term effects from random variation in renewables and plant failures (seconds)



- Tradeoff: Uncertainty, cost and operability, regulations, security/robustness/resilience
- Needs modelling at finer time scales