Why use a modeling language: a view from optimization

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Models

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Why Model

1. To get an answer!
2. From building a model, we can gain insight.
3. We can "experiment" with a model.

Types of models

- Physical
  - Airline wing design
- Abstract
  - Statistical: Time Series, Regression, etc...
  - Simulation
  - Economic
  - **Optimization!**

Components of an Optimization Model

1. Decision variables
   - Variables representing the unknown quantities

Variables:

- x: Pounds of barley to purchase
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Constraints:

- $y \leq 2$
- $x \leq 8y$
- $z = 0.4x + 0.9y$

Components of an Optimization Model

1. Decision variables
   - Variables representing the unknown quantities
2. Constraints
   - Requirements that all solutions must satisfy, (expressed algebraically)
3. Objective
   - A quantity that you would like to make as small or as large as possible.

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- $y \leq 2$
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Objective:

- $\max z$

Solving

- Actually involves gathering and processing data: Turning your model into an instance.
  - Model : A structure containing (algebraic) relationships between entities.
  - Instance : A combination of data and model that can be solved. (i.e – it has "numbers")
  - Spreadsheet models "blur" this distinction, that's why I don't like them very much.

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  - Spreadsheet models "blur" this distinction, that's why I don't like them very much.
- Algebraic modeling languages are better!
  - Have hooks to solvers
  - Many have hooks to spreadsheets and databases

Data

```
set i /a, b, c, d/;
alias(i,j,k);
set orig(i) /a/;
set dest(i) /d/;
parameter demand /4000/;

set arcs(i,i) /a.(b*c),(b*c).d/;

table data(i,i,*)
            fixed\_time congestion\_factor
     a.b    45          0
     a.c    0           0.01
     b.d    0           0.01
     c.d    45          0
     c.b    eps         eps;

parameter fixed\_time(i,i) Fixed travel time on a link;
parameter congestion\_factor(i,i) Flow-dependent effects;

fixed\_time(arcs) = data(arcs,'fixed\_time');
congestion\_factor(arcs) = data(arcs,'congestion\_factor');
```

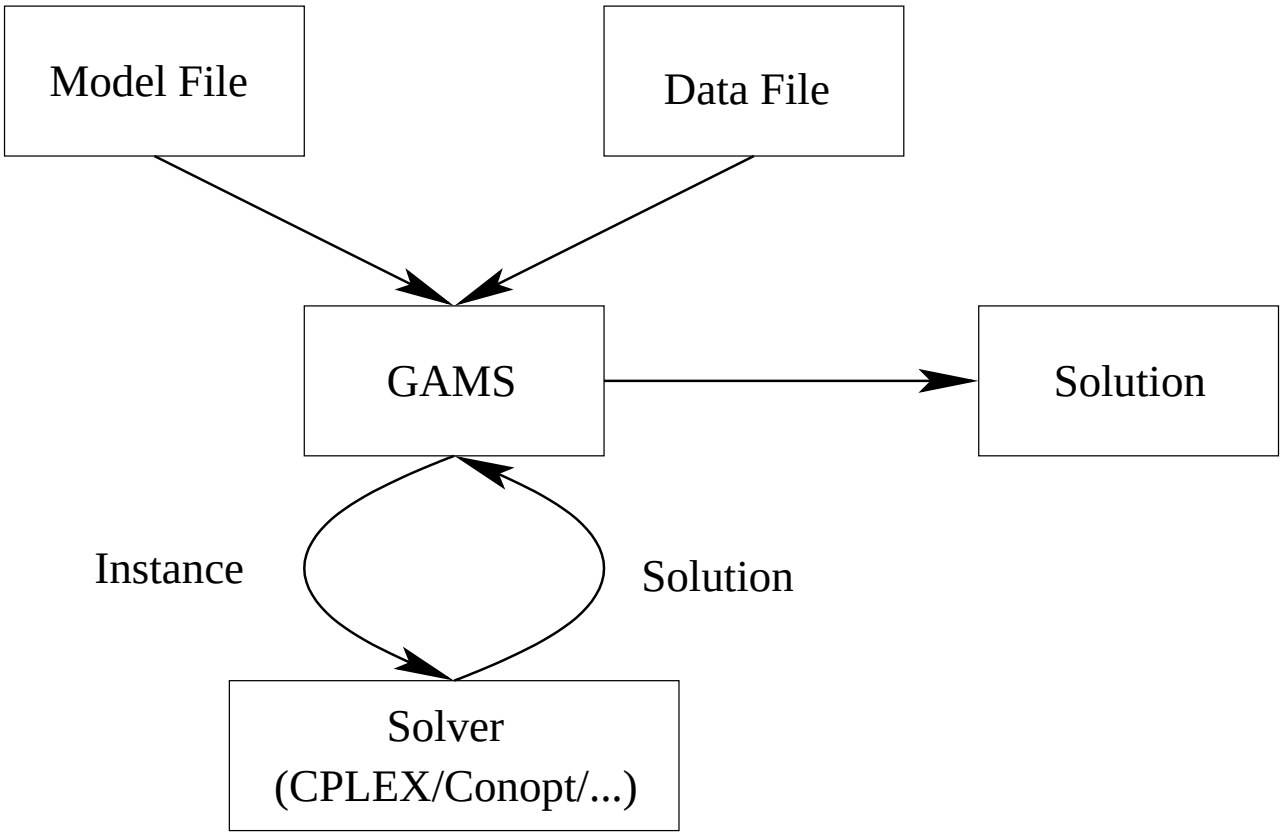
Model

```
positive variables Flow(i,j);
variables T(j);

equations Short(i,j), Balance(k);

Balance(k)..
  sum(arcs(i,k), Flow[i,k]) + demand$orig(k)
  =e= sum(arcs(k,j), Flow[k,j]) + demand$dest(k);

Short(i,j)$arcs(i,j)..
  fixed_time[i,j] + congestion_factor[i,j] * Flow[i,j]
  + T(j) =g= T(i);

model braess /Short.Flow,Balance.T/;
```

Solution

```
solve braess using mcp;

arcs('c','b') = yes;
fixed_time(arcs) = data(arcs,'fixed_time');
congestion_factor(arcs) = data(arcs,'congestion_factor');

solve braess using mcp;

fixed_time('c','b') = 2;
congestion_factor('c','b') = 0;

solve braess using mcp;
```

The Modeling Language Interface



This talk is not the real world

- We will often combine Model and Data file into one file, but this is not best practice

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The upshot!

(Optimization) models should be one tool in the decision making process.

Another View at Model Components

1. Inputs
   - Sets. Used typically for algebraic models.
     - e.g., P: Set of products, I: Set of locations
   - "Numbers". These are called parameters. The parameters may be indexed over sets.
     - e.g., u_p: The maximum amount of product p available

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3. Outputs
   - These may be optimal values of the decision variables, or a derived value, such as the objective function value
   - e.g.: $\sum_{i \in \text{Madison}} z_i$

Solution Analysis

Warning!

Optimization is extreme. If there is a mistake in your model, optimization will usually find it.

- Verification : Is the model correct?
  - Are physical laws being obeyed.
- Validation : Does the model give an accurate picture of reality?
  - Create instances for which you can expect a certain type of solution, and see if the model returns such a solution.
  - Careful! Maybe your intuition about a solution is wrong!
- Sensitivity Analysis
  - How much are extra resources worth to me?
  - This is "marginal information."
- What-if Analysis : Change the instance and re-run.

Categories of Optimization Models

- Linear vs. Nonlinear?
  - Are the functional relationships between decision variables linear functions or nonlinear functions?
- Convex vs. Nonconvex?
  - Are the functional relationships convex?
- Discrete vs. Continuous?
  - Must the decision variables take only discrete values?
- Deterministic vs. Stochastic?
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The upshot

- These categorizations have a significant impact on the tractability of an instance
- You should be able to categorize problem instances

GAMS Model Types

- LP: Linear Programming
- MIP: Mixed-Integer Programming
- NLP: Non-Linear Programming
- MCP: Mixed Complementarity Problems
- MPEC: Mathematical Programs with Equilibrium Constraints
- CNS: Constrained Nonlinear Systems
- DNLP: Non-Linear Programming with Discontinuous Derivatives
- MINLP: Mixed-Integer Non-Linear Programming
- QCP: Quadratically Constrained Programs
- MIQCP: Mixed Integer Quadratically Constrained Programs

Other Great Optimization Tools/Resources

- COIN-OR: `http://www.coin-or.org/`
- NEOS: `http://www.neos-guide.org/`

NEOS Case Studies

- `http://www.neos-guide.org/NEOS/index.php/Case_Studies`.
- A web-wiki description of a problem, understandable to the general public.
- A model and submission that would solve the problem and provide some form of output/visualization.

Installing Gams—An Interactive(?) Demo

1. Obtain GAMS. Two options
   1. Go to `http://www.gams.com/download/`, Download the appropriate executable for Win32 (or Win64) if your laptop is 64bit.
   2. There are also executables for Mac OS X (make sure to get right one), and Linux
2. Installation instructions at...
   - `http://www.gams.com/docs/gams/win-install.pdf`
   - `http://support.gams-software.com/doku.php?id=installation:how_do_i_install_the_gams_version_for_macintosh`
3. Interfaces to:
   - Matlab and R (via GDX files)