Optimization approaches to combat Gerrymandering

Michael C. Ferris   Cheng-Wei Lu   Jim Luedtke

University of Wisconsin-Madison

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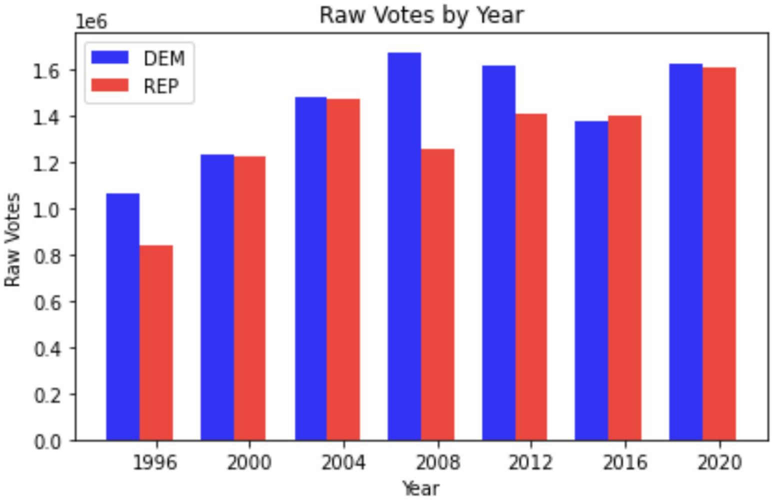
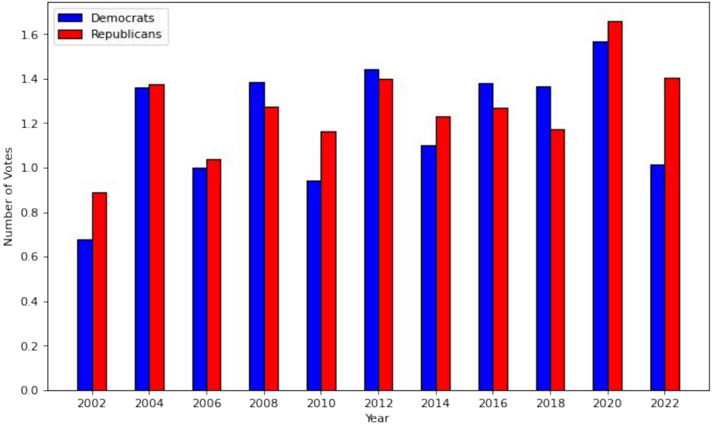
Historical Presidential Election in Wisconsin



Historical US House Election in Wisconsin



Ideal Seat assignment vs. real Seat assignment

$$\text{Ideal US House Seats for Republicans} = \text{round}(8 \times \frac{\text{Republican raw votes}}{\text{total raw votes}})$$

	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020	2022
Ideal	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:3 R:5
Real	D:4 R:4	D:4 R:4	D:5 R:3	D:5 R:3	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5	D:2 R:6

Map change from 2008 to 2010

Policies and Games (Recht 2023)

Finding what works

maximize outcome
policy

- For simple models, can be solved by optimization methods
- For games, everything rigid, can solve by brute force search
- For more complex models, can solve by DFO
  this is what we do in randomized controlled trials
- To understand people, what happens when the distribution changes?, when the moves change their meaning?, when the desired outcome changes?, when the outcome is unclear?, when outcomes are not comparable?…

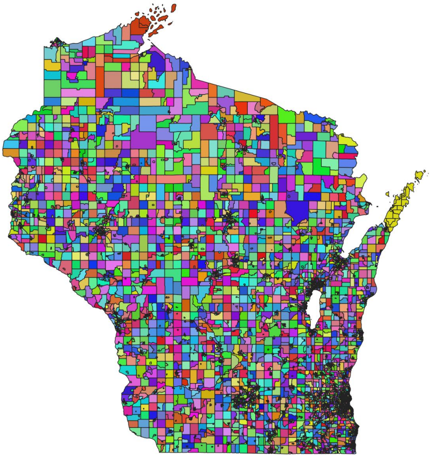
The rules

- Districts shall comply with the United States Constitution, the Voting Rights Act of 1965, and all applicable federal laws;
- Districts shall be drawn on the basis of inhabitants;
- Districts shall be geographically contiguous;
- Districts shall provide racial and language minorities with an equal opportunity to participate in the political process
- Districts shall respect the integrity of communities of interest to the extent practicable.
- Districts shall not split precincts and shall respect the geographic integrity of political subdivision boundaries to the extent that preceding criteria have been satisfied.
- The redistricting plan shall not, when considered on a statewide basis, unduly favor or disfavor any political party

Data used in our application

For our applications, we used ward-level district data. The state of Wisconsin is divided into 7078 wards in total.

1. Wisconsin shape data that contains the coordinated for each ward polygon
2. Wisconsin US House voting data for each ward
3. Wisconsin racial data for each ward



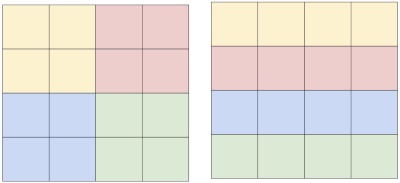
Wisconsin wards

Problem definition (rule interpretation)

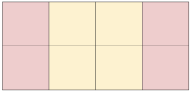
- How do we divide this map into 8 congressional districts?
- Redraw map boundaries to do this fairly.

1. Assign each ward to a district
2. Satisfying Contiguous Constraint : a district should not be broken into multiple parts.
3. Satisfying Population Constraint : the population of every district should satisfy some predefined bounds.
4. Trying to Maximize the Compactness of Districts : prefer districts with more compact shape instead of long and thin districts.

Problem definition



Assume each grid is a ward and each unique color means an assignment to a district. The strategy on the left is more compact than the one on the right.



In the above scenario, the strategy does not satisfy contiguous constraint since the red district is separated.

Methodology of Gerrymandering

Objective:

$$\min \quad \sum_{i \in V} \sum_{j \in V} w_{ij} x_{ij} \tag{1a}$$

Constraints:

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \tag{1b}$$

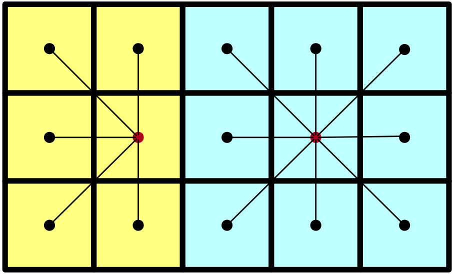
$$\sum_{j \in V} x_{jj} = k \tag{1c}$$

$$L x_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq U x_{jj} \qquad \forall j \in V \tag{1d}$$

$$x_{ij} \leq x_{jj} \qquad \forall i, j \in V \tag{1e}$$

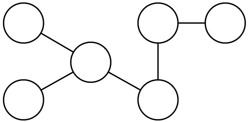
$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in V \tag{1f}$$

contiguity constraints (1g)

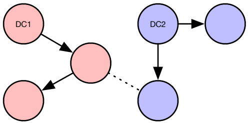


Objective function for the optimal MIP model. Suppose there are two districts. One in blue and the other in yellow, and the district centers are units that contain the red centroid point. The objective value is the sum of the distance between each unit and its assigned district center's centroid; i.e., the sum of thin black lines length in the figure.

Methodology of Gerrymandering - The SHIR Model

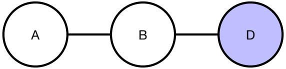


We can represent the shape data as a node edge graph with each node representing a ward and each edge representing the adjacency of two wards.



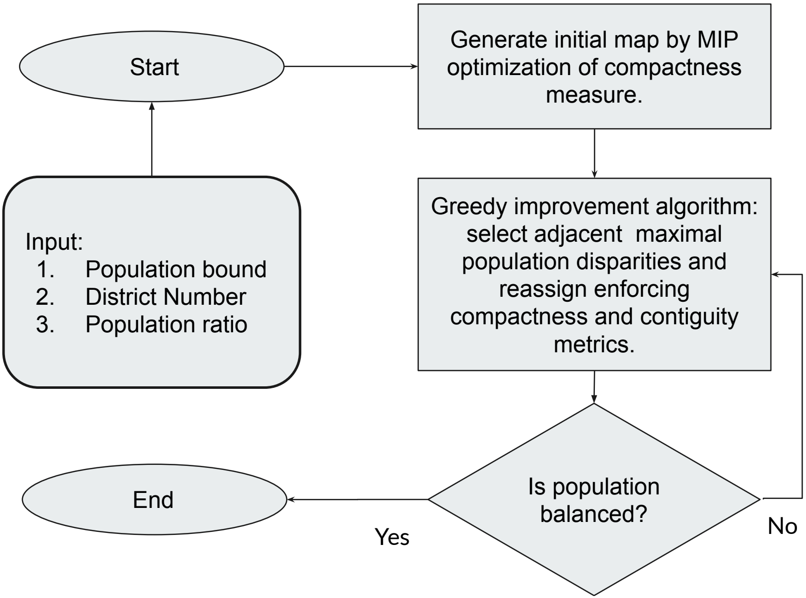
The SHIR model uses network flow constraints to maintain contiguity.

Methodology of Gerrymandering - The Closer Unit Model

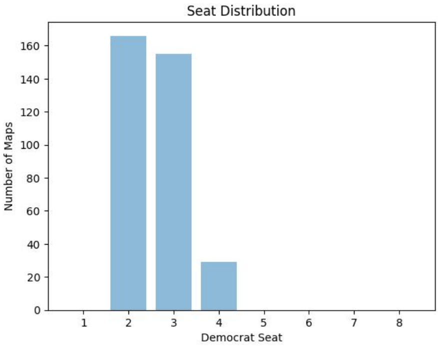


Suppose D is the district center, then ward A can be assigned to district center D only if one of its adjacent wards that is closer to D (which is B in the above picture) is assigned to D.

Sampling compact maps - generation algorithm



Fair maps are not “fair”



The bar chart on the left is the distribution of Democrat congressional Seats in 2020 US House election. We’ve generated 300 hundred maps based on the population of 2020 presidential elections. Note the ideal congressional seats for Democrats are 4.

Policy change: multi-representative districts

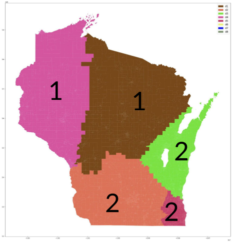
District number	Population ratio
5	2:2:2:1:1
6	2:2:1:1:1:1
7	2:1:1:1:1:1:1
8	1:1:1:1:1:1:1:1

Voting Calculation Policy	
Winner takes all All congressional Seats are assigned to the winning party in a district.	**Assigned by weights** Congressional Seats are assigned to the parties according to the voting ratio in a district.

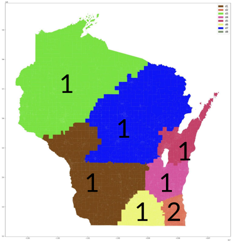
We generate 300 maps for every district number, and then do experiments on the two allocation policies.

Examples of multi-representative maps

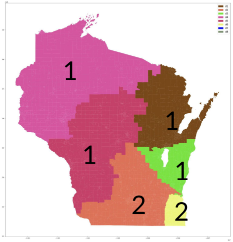
5 districts



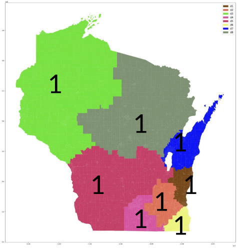
7 districts



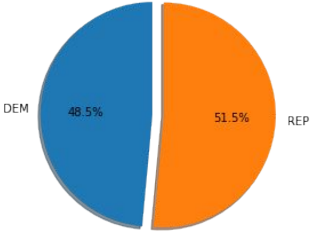
6 districts



8 districts

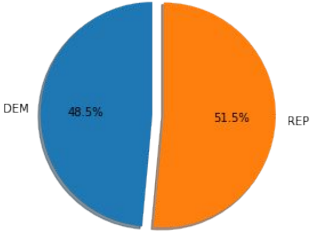


Democratic Seats Won: winner-takes-all (2020 UHS)



number of districts	1 seat	2 seats	3 seats	4 seats	5 seats
5	4	7	73	197	19
6	3	15	121	159	2
7	0	37	160	103	0
8	0	116	155	29	0

Democratic Seats Won: proportional (2020 UHS)



number of districts	1 seat	2 seats	3 seats	4 seats	5 seats
5	0	0	179	96	25
6	0	69	125	105	1
7	0	130	134	30	6
8	0	116	155	29	0

Use Stochastic Programming to Counter Adversary

Cost Risk

$$\min \quad c(x) + \lambda t + (1-\lambda)\{\sum_{i=1}^{n} y_i + \frac{1}{1-\alpha}\sum_{s\in S} P(s) u_{is}\}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$

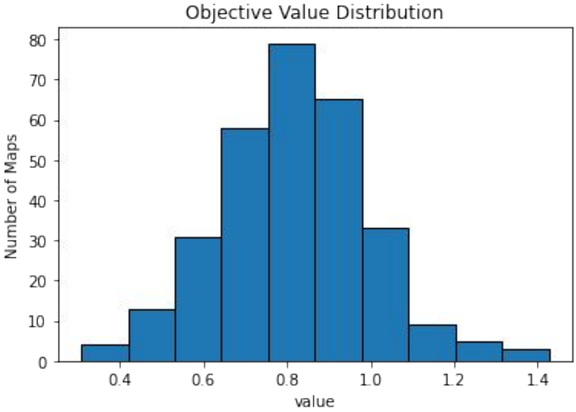
$$t \geq x_i \sum_{s\in S} P(s) Deviation(i,s) \quad \forall i = 1,\ldots,n$$

$$u_{is} \geq Deviation(i,s) x_i - y_i \quad \forall s \in S, i = 1,\ldots,n$$

$$x \in \{0,1\}^n$$

Cost Term	Risk Term
1. Compactness Measures 2. Differences between the current map and the proposed map	1. Deviation Measures (Fairness) 2. Population Imbalance

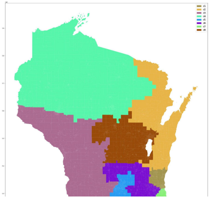
Objective over scenarios



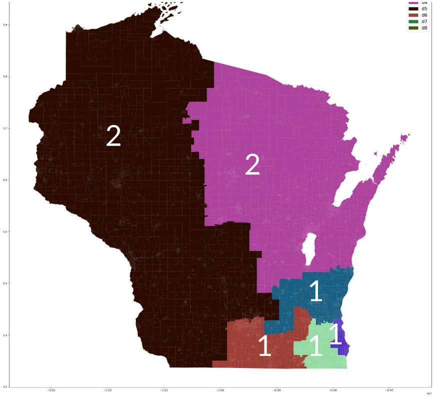
8 district setting

	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
Ideal	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4
Chosen Map	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:3 R:5	D:4 R:4	D:3 R:5	D:3 R:5	D:4 R:4	D:4 R:4
Real Map	D:4 R:4	D:4 R:4	D:5 R:3	D:5 R:3	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5	D:3 R:5

Chosen Map



SP Selected Map (6 Districts)



Countering adversaries

Winner-take-all

	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
Ideal	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4
Chosen Map	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:2 R:6

Proportional

	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
Ideal	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4
Chosen Map	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4

Average

	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
Ideal	D:3 R:5	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4
Chosen Map	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:4 R:4	D:3 R:5

Conclusion and Outlook

- Fair maps are not necessarily "fair"
- Adversarial approach (policy and demographic changes) leads to fairer solutions
- Forward versus backward model
- Causality, visualization, clarity of model is key for policy impact
- Currently interacting with state representatives to see how to further discussion.