Optimization and Equilibrium in Energy Economics

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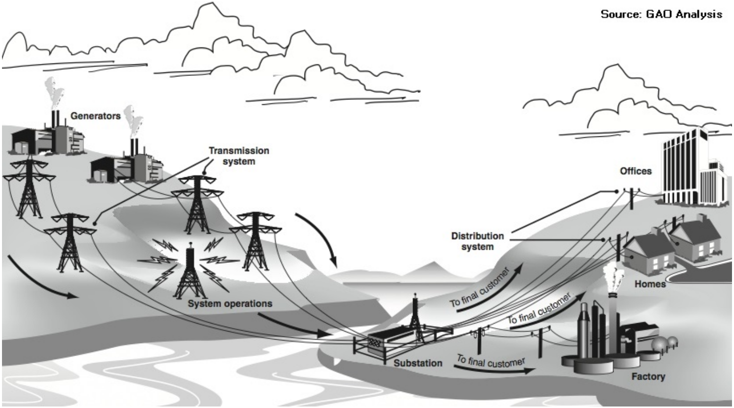
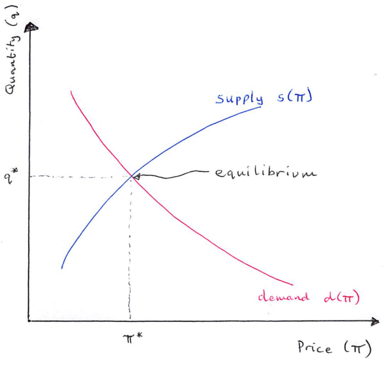
Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $\geq \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

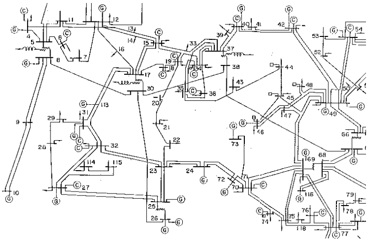
Single market, single good: equilibrium



Walras: $0 \leq s(\pi) - d(\pi) \perp \pi \geq 0$

Market design and rules to foster competitive behavior/efficiency

- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network



- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$\min_x \; c(x) \qquad \text{cost}$$

$$\text{s.t. } Ax \geq q \qquad \text{balance}$$

$$Bx = b, x \geq 0 \qquad \text{technical constr}$$

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$\begin{aligned} \min_x \; & c(x) && \text{cost} \\ \text{s.t. } & Ax \geq d(\pi) && \text{balance} \\ & Bx = b, x \geq 0 && \text{technical constr} \end{aligned}$$

- $q = d(\pi)$: issue is that π is the multiplier on the "balance" constraint
- Such multipliers (LMP's) are critical to operation of market
- Can try to solve the problem iteratively (shooting method):

$$\pi^{new} \in \text{multiplier}(OPF(d(\pi)))$$

Alternative: Form KKT of QP, exposing π to modeler

$$
\begin{aligned}
0 &\le Ax - d(\pi) && \perp \quad \mu \ge 0 \\
0 &= Bx - b && \perp \quad \lambda \\
0 &\le \nabla c(x) - A^T\mu - B^T\lambda && \perp \quad x \ge 0
\end{aligned}
$$

- empinfo: dualvar π balance
- Fixed point: replaces $\mu \equiv \pi$

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\end{aligned}
$$

- empinfo: dualvar π balance
- Fixed point: replaces $\mu \equiv \pi$
- LCP/MCP is then solvable using PATH

$$
z = \begin{bmatrix} \pi \\ \lambda \\ x \end{bmatrix}, \quad F(z) = \begin{bmatrix} & & A \\ & & B \\ -A^T & -B^T & \end{bmatrix} z + \begin{bmatrix} -d(\pi) \\ -b \\ \nabla c(x) \end{bmatrix}
$$

- Existence, uniqueness, stability from variational analysis
- EMP does this automatically from the annotations

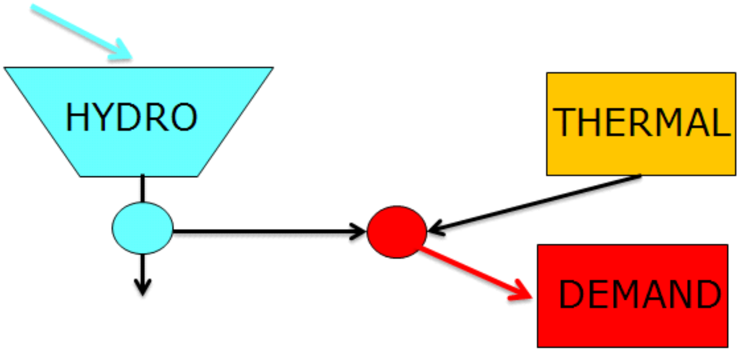
Other applications of complementarity

Complementarity can model fixed points and disjunctions

- Economics: Walrasian equilibrium (supply equals demand), taxes and tariffs, computable general equilibria, option pricing (electricity market), airline overbooking
- Transportation: Wardropian equilibrium (shortest paths), selfish routing, dynamic traffic assignment
- Applied mathematics: Free boundary problems
- Engineering: Optimal control (ELQP)
- Mechanics: Structure design, contact problems (with friction)
- Geology: Earthquake propogation

Good solvers exist for large-scale instances of Complementarity Problems

Hydro-Thermal System (Philpott/F./Wets)



- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities

Simple electricity "system optimization" problem

$$\begin{aligned} \text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\ & x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H} \end{aligned}$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

Decomposition by prices π

$$\max_{d_k, u_i, v_j, x_i \geq 0} \quad \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i)$$
$$+ \pi^T \left(\sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j - \sum_{k \in \mathcal{K}} d_k \right)$$
$$\text{s.t.} \quad x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}$$

Problem then decouples into multiple optimizations

$$\sum_{k \in \mathcal{K}} \max_{d_k \geq 0} \left(W_k(d_k) - \pi^T d_k \right) + \sum_{j \in \mathcal{T}} \max_{v_j \geq 0} \left(\pi^T v_j - C_j(v_j) \right)$$
$$+ \sum_{i \in \mathcal{H}} \max_{u_i, x_i \geq 0} \left(\pi^T U_i(u_i) + V_i(x_i) \right)$$
$$\text{s.t.} \ x_i = x_i^0 - u_i + h_i^1$$

SO equivalent to CE (price takers)

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\text{CE:} \quad \text{Consumers } k \in \mathcal{K} \text{ solve CP}(k) : \max_{d_k \geq 0} \; W_k(d_k) - \pi^T d_k$$

$$\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j) : \max_{v_j \geq 0} \; \pi^T v_j - C_j(v_j)$$

$$\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i) : \max_{u_i, x_i \geq 0} \; \pi^T U_i(u_i) + V_i(x_i)$$

$$\text{s.t. } x_i = x_i^0 - u_i + h_i^1$$

$$0 \leq \pi \perp \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.$$

But in practice there is a gap between SO and CE. How to explain?

MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h pi cons
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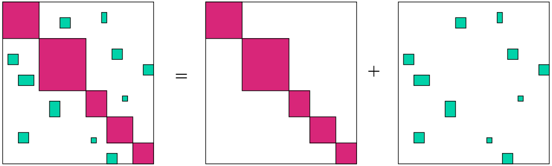
- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



Perfect competition

$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

Cournot: two agents (duopoly)

$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

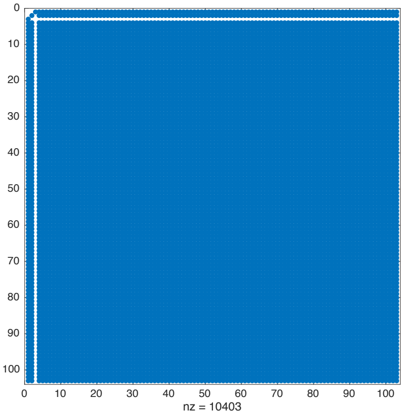
$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect π by choice of x_i
- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot)

Computational issue: PATH

- Cournot model: $|\mathcal{A}| = 5$
- Size $n = |\mathcal{A}| * N_a$

Size (n)	Time (secs)
1,000	35.4
2,500	294.8
5,000	1024.6

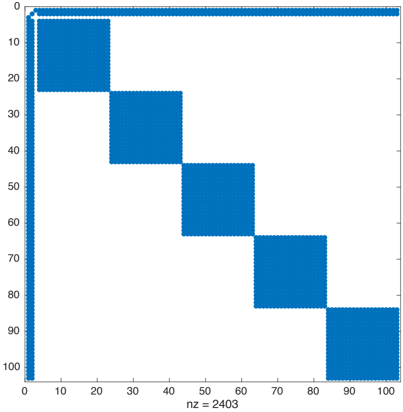


Jacobian nonzero pattern
$n = 100,\ N_a = 20$

Computation: implicit functions

- Use implicit fn: $z(x) = \sum_j x_j$
- Generalization to $F(z, x) = 0$ (via adjoints)
- empinfo: implicit z F

Size (n)	Time (secs)
1,000	2.0
2,500	8.7
5,000	38.8
10,000	$>$ 1080



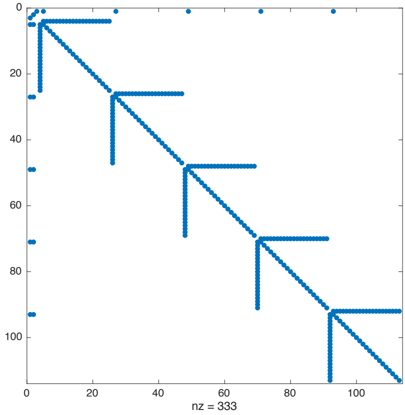
Jacobian nonzero pattern

$n = 100$, $N_a = 20$

Computation: implicit functions and local variables

- Use implicit fn: $z(x) = \sum_j x_j$ (and local aggregation)
- Generalization to $F(z, x) = 0$ (via adjoints)
- empinfo: implicit z F

Size (n)	Time (secs)
1,000	0.5
2,500	0.8
5,000	1.6
10,000	3.9
25,000	17.7
50,000	52.3



Jacobian nonzero pattern
$n = 100,\ N_a = 20$

Other specializations and extensions

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Shared constraints: some g_i's are known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Stochastic: Agents have recourse?

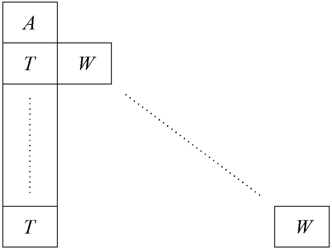
- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP:} \;\; \min \quad c(x^1) + \rho[q^T x^2]$$

$$\text{s.t.} \quad Ax^1 = b, \quad x^1 \geq 0,$$

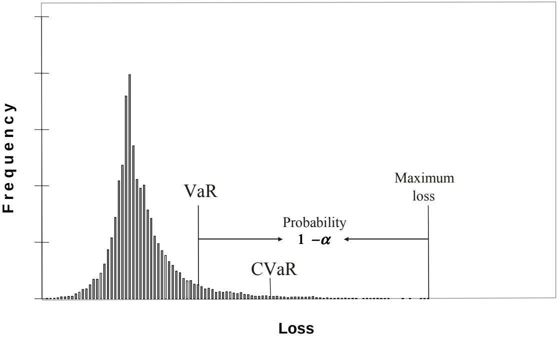
$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

$$\text{CP:} \quad \min_{d^1 \geq 0} \quad p^1 d^1 - W(d^1)$$

$$\text{TP:} \quad \min_{v^1 \geq 0} \quad C(v^1) - p^1 v^1$$

$$\text{HP:} \quad \min_{u^1, x^1 \geq 0} \quad - p^1 U(u^1)$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

Two stage stochastic MOPEC (1,1,1)

$$\text{CP:} \quad \min_{d^1, d^2_\omega \geq 0} \quad p^1 d^1 - W(d^1) + \rho_C \left[p^2_\omega d^2_\omega - W(d^2_\omega) \right]$$

$$\text{TP:} \quad \min_{v^1, v^2_\omega \geq 0} \quad C(v^1) - p^1 v^1 + \rho_T \left[C(v^2_\omega) - p^2_\omega v^2(\omega) \right]$$

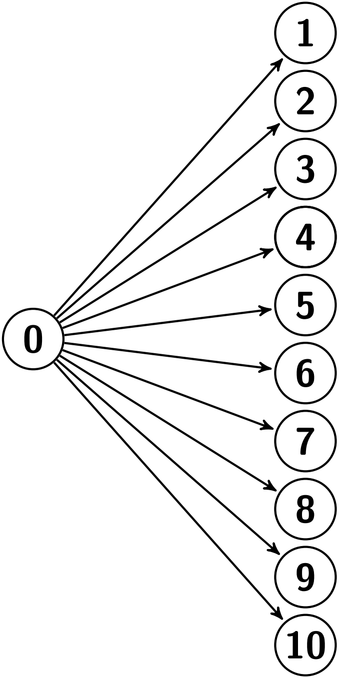
$$\text{HP:} \quad \min_{\substack{u^1, x^1 \geq 0 \\ u^2_\omega, x^2_\omega \geq 0}} \quad -p^1 U(u^1) + \rho_H \left[-p^2(\omega) U(u^2_\omega) - V(x^2_\omega) \right]$$

$$\text{s.t.} \quad x^1 = x^0 - u^1 + h^1,$$

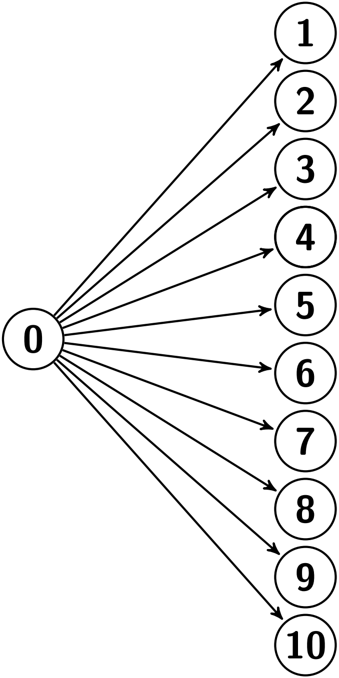
$$x^2_\omega = x^1 - u^2_\omega + h^2_\omega$$

$$0 \leq p^1 \perp U(u^1) + v^1 \geq d^1$$

$$0 \leq p^2_\omega \perp U(u^2_\omega) + v^2_\omega \geq d^2_\omega, \forall \omega$$



- Single hydro, thermal and representative consumer
- Initial storage 10, inflow of 4 to 0, equal prob random inflows of i to node i
- Risk neutral: SO equivalent to CE (key point is that each risk set is a singleton, and that is the same as the system risk set)



- Single hydro, thermal and representative consumer
- Initial storage 10, inflow of 4 to 0, equal prob random inflows of i to node i
- Risk neutral: SO equivalent to CE (key point is that each risk set is a singleton, and that is the same as the system risk set)

- Each agent has its own risk measure, e.g. $0.8EV + 0.2CVaR$
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

Equilibrium or optimization?

Theorem

If (d, v, u, x) solves (risk averse) SO, then there exists a probability distribution σ_k and prices p so that (d, v, u, x, p) solves (risk neutral) CE(σ)

(Observe that each agent must maximize their own expected profit using probabilities σ_k that are derived from identifying the worst outcomes as measured by SO. These will correspond to the worst outcomes for each agent only under very special circumstances)

- High initial storage level (15 units)
  - Worst case scenario is 1: lowest system cost, smallest profit for hydro
  - SO equivalent to CE
- Low initial storage level (10 units)
  - Different worst case scenarios
  - SO different to CE (for large range of demand elasticities)
- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

Reserves, interruptible load, demand response

- Generators set aside capacity for "contingencies" (reserves)
- Separate energy π_d and reserve π_r prices
- Consumers may also be able to reduce consumption for short periods
- Alternative to sharp price increases during peak periods
- Constraints linking energy "bids" and reserve "bids"

$$v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

- Multiple scenarios - linking constraints on bids require "bid curve to be monotone"

Price taking: model is MOPEC

Consumption d_k, demand response r_k, energy v_j, reserves u_j, prices π

$$\text{Consumer} \quad \max_{(d_k, r_k) \in \mathcal{C}} \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

$$\text{Generator} \quad \max_{(v_j, u_j) \in \mathcal{G}} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{s.t.} \quad v_j + u_j \leq \mathcal{U}_j, u_j \leq \mathcal{B}_j v_j$$

$$\text{Transmission} \quad \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

Market clearing

$$0 \leq \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A} f \geq 0$$

$$0 \leq \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \geq 0$$

Large consumer is price making: MPEC

Leader/follower

$$\text{Consumer} \;\; \max \text{utility}(d_k) - {\pi_d}^T d_k + \text{profit}(r_k, \pi_r)$$

with the constraints:

$$(d_k, r_k) \in \mathcal{C}$$

$$\text{Generator} \;\; \max_{(v_j, u_j) \in \mathcal{G}'} \text{profit}(v_j, \pi_d) + \text{profit}(u_j, \pi_r)$$

$$\text{Transmission} \;\; \max_{f \in \mathcal{F}} \text{congestion rates}(f, \pi_d)$$

$$0 \leq \pi_d \perp \sum_j v_j - \sum_k d_k - \mathcal{A}f \geq 0$$

$$0 \leq \pi_r \perp \sum_j u_j + \sum_k r_k - \mathcal{R} \geq 0$$

Solution and observations

- Formulate as MIP, add mononticity constraints and scenarios
- New Zealand (NZEM) data, large consumer at bottom of South Island
- Expected difference percentage between "wait and see" solutions versus model solution (evaluated post optimality with simulation)

Sample Size	1	2	4	6	8
Expected diff	31.34	17.83	9.22	7.35	9.26
Standard dev	22.86	9.62	4.86	7.69	6.59
Bound gap (%)	0	0	0	12.7	24.8

- More samples better(!)
- More research to model/solve more detailed problems

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- dualvar (use multipliers from one agent as variables for another)
- QS functions (both in objectives and constraints)
- implicit functions and shared constraints

- Currently available within GAMS
- Some solution algorithms implemented in modeling system - limitations on size, decomposition and advanced algorithms
- Can evaluate effects of regulations and their implementation in a competitive environment

Dual Representation of Risk Measures

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \le \lambda_i \le p_i/(1-\alpha), \sum_i \lambda_i = 1\}$, then

$$\rho(Z) = \overline{CVaR}_\alpha(Z)$$

- Special case of a Quadratic Support Function

$$\rho(y) = \sup_{u \in U} \langle u, By + b \rangle - \frac{1}{2} \langle u, Mu \rangle$$

- EMP allows any Quadratic Support Function to be defined and facilitates a model transformation to a tractable form for solution

Addition: compose equilibria with QS functions

- Add soft penalties to objectives and/or within constraints:

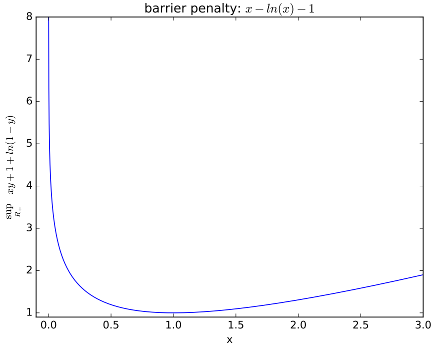
$$\min_x \theta(x) + \rho_O(F(x))$$
$$\text{s.t. } \rho_C(g(x)) \leq 0$$

```
QS g rhoC udef B M
...

QSF cvarup F rhoO theta p
```

- \$batinclude QSprimal modname using emp min obj
- Allow modeler to compose QS functions automatically

- Can solve using MCP or primal reformulations
- More general conjugate functions also possible:



The link to MOPEC

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\rho(y) = \sup_{u \in U} \langle u, y \rangle - \frac{1}{2} \langle u, Mu \rangle$$

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

$$0 \in \partial\theta(x) + \nabla F(x)^T u + N_X(x)$$

$$0 \in -u + \partial\rho(F(x)) \iff 0 \in -F(x) + Mu + N_U(u)$$

This is a MOPEC, and we have multiple copies of this for each agent