Integrated Modeling for Optimization of Energy Systems

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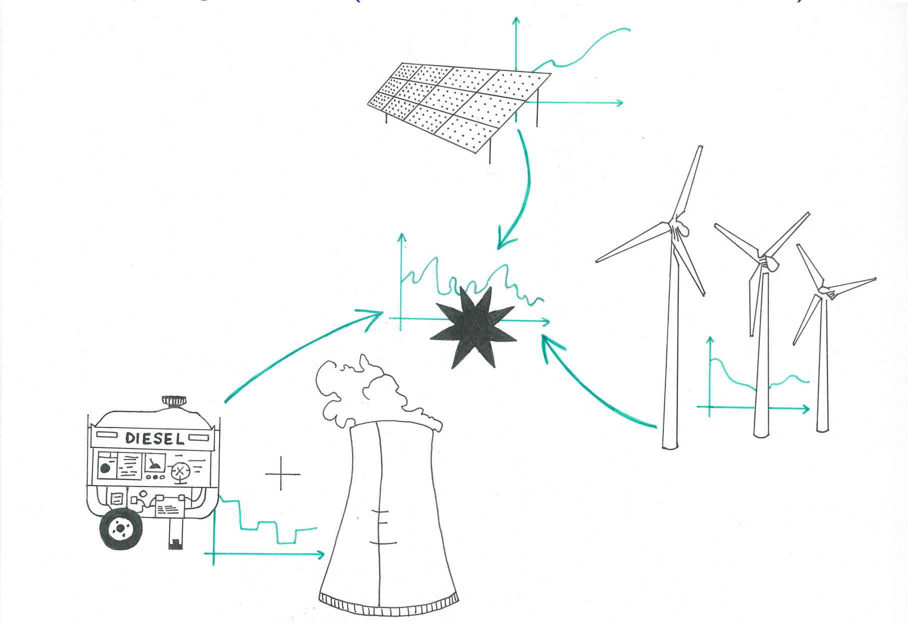
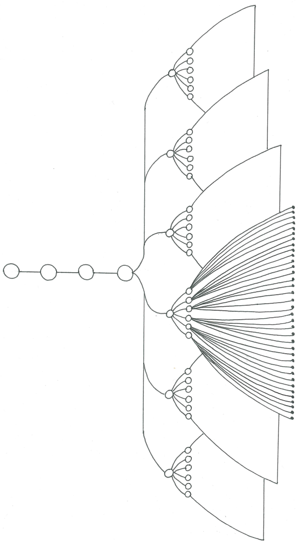
The setup: agents a =(solar, wind, diesel,consumer)



Variables and uncertainties

- Power distribution not modeled (single consumer location)
- Scenario tree is data
- T stages (use 6 here)
- Nodes $n \in \mathcal{N}$, n_+ successors
- Stagewise probabilities $\mu(m)$ to move to next stage $m \in n_+$
- Uncertain wind flow and cloud cover $\omega_a(n)$
- Actions u_a for each agent (dispatch, curtail, generate, shed), with costs C_a
- Recursive (nested) definition of expected cost-to-go $\theta(n)$

$t \in 0, 1, 2, 3, 4, 5, 6$



Model

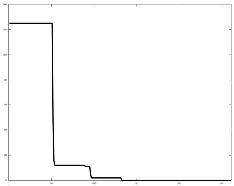
$$\text{SO:} \quad \min_{(\theta,u,x)\in\mathcal{F}(\omega)} \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0)$$

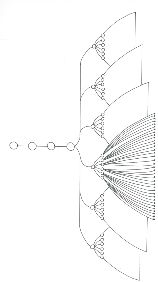
$$\text{s.t.} \quad \theta(n) \geq \sum_{m\in n_+} \mu(m) \left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0$$

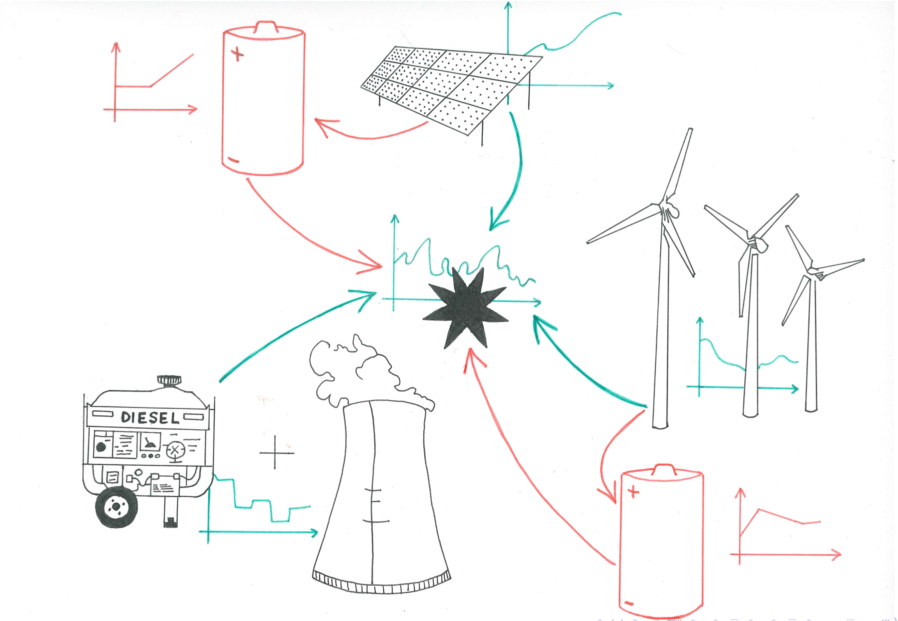
- g_a converts actions into energy.
- Solution (risk neutral, system optimal):
- consumer cost 1,308,201; probability of shortage 19.5%
- No transfer of energy across stages.

Prices π on energy constraint:





Add storage (smoother) to uncertain supply



Add storage

- Storage allows energy to be moved across stages (batteries, pump, compressed air, etc)
- Solution forcing use of battery consumer cost 1,228,357; probability of shortage 11.5%
- Solution allowing both options consumer cost 207,476; probability of shortage 1.1%

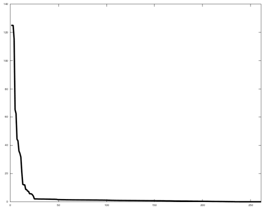
$$\min_{(\theta,u,x)\in\mathcal{F}} \quad \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0)$$

$$\text{s.t. } x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n)$$

$$\theta(n) \geq \sum_{m\in n_+} \mu(m) \left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0$$

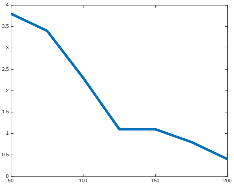
Prices π on energy constraint:



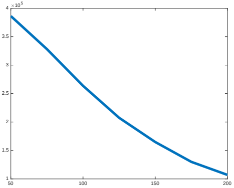
Investment planning: storage/generator capacity

Increasing battery capacity

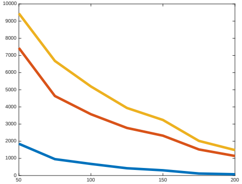
Shortage probability



Consumer cost

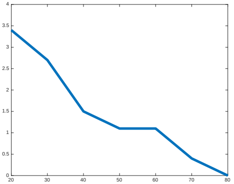


Agent profits

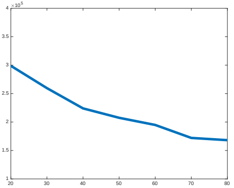


Increasing diesel generator capacity

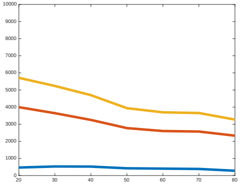
Shortage probability



Consumer cost



Agent profits



Decomposition by prices π

Split up θ into agent contributions θ_a and add weighted constraints into objective:

$$\min_{(\theta,u,x)\in\mathcal{F}} \quad \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta_a(0) - \pi^T \left(g_a(u_a(n))\right)$$

$$\text{s.t. } x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n)$$

$$\theta_a(n) \geq \sum_{m\in n_+} \mu(m)\left(C_a(u_a(m)) + \theta_a(m)\right)$$

Problem then decouples into multiple optimizations

$$\text{RA}(a,\pi): \min_{(\theta,u,x)\in\mathcal{F}} \quad Z_a(0) + \theta_a(0)$$

$$\text{s.t. } x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n)$$

$$\theta_a(n) \geq \sum_{m\in n_+} \mu(m)(Z_a(m) + \theta_a(m))$$

$$Z_a(n) = C_a(u_a(n)) - \pi(n)g_a(u_a(n))$$

SO equivalent to MOPEC (price takers)

- Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\{(u_a(n), \theta_a(n)), n \in \mathcal{N}\} \in \arg\min \mathrm{RA}(a, \pi)$$

and

$$0 \leq \sum_{a \in \mathcal{A}} g_a(u_a(n)) \perp \pi(n) \geq 0$$

- One optimization per agent, coupled together with solution of complementarity (equilibrium) constraint.
- Overall, this is a Nash Equilibrium problem, solvable as a large scale complementarity problem (replacing all the optimization problems by their KKT conditions) using the PATH solver.
- But in practice there is a gap between SO and MOPEC.
- How to explain?

Perfect competition

$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

Cournot: two agents (duopoly)

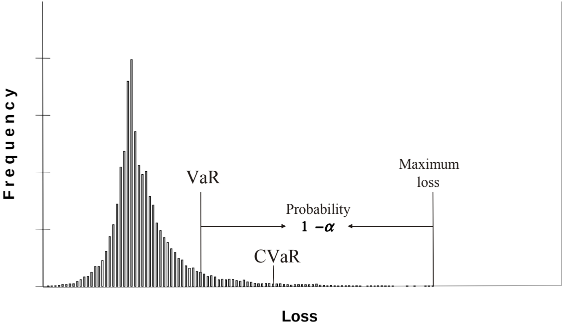
$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect π by choice of x_i
- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot, even Stackleberg)

Another explanation: risk

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \le \lambda_i \le p_i/(1-\alpha), \sum_i \lambda_i = 1\}$, then

$$\rho(Z) = \overline{CVaR}_\alpha(Z)$$

Risk averse equilibrium

Replace each agents problem by:

$$\text{RA}(a, \pi, \mathcal{D}_a): \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad Z_a(0) + \theta_a(0)$$

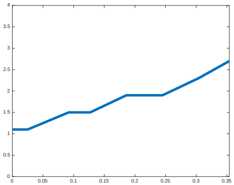
$$\begin{aligned} \text{s.t. } & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\ & \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n) \\ & Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) \end{aligned}$$

- $p_a^k(m)$ are extreme points of the agents risk set at m
- No longer system optimization
- Must solve using complementarity solver
- Need new techniques to treat stochastic optimization problems within equilibrium

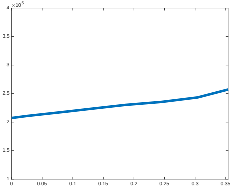
Computational results

Increasing risk aversion

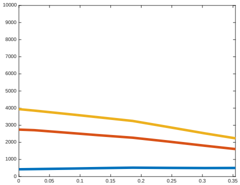
Shortage probability



Consumer cost

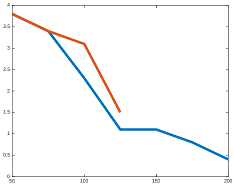


Agent profits

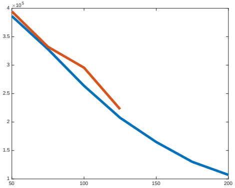


Increasing battery capacity

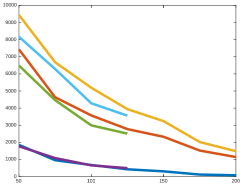
Shortage probability



Consumer cost



Agent profits



Equilibrium or optimization?

Theorem

If (u, θ) solves $SO(\mathcal{D}_s)$, then there is a probability distribution $(\sigma(n), n \in \mathcal{N})$ and prices $(\pi(n), n \in \mathcal{N})$ so that (u, π) solves $NE(\sigma)$. That is, the social plan is decomposable into a risk-neutral multi-stage stochastic optimization problem for each agent, with coupling via complementarity constraints.

(Observe that each agent must maximize their own expected profit using probabilities σ_k that are derived from identifying the worst outcomes as measured by SO. These will correspond to the worst outcomes for each agent only under very special circumstances)

- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Given any node n, an *Arrow-Debreu security* for node $m \in n_+$ is a contract that charges a price $\mu(m)$ in node $n \in \mathcal{N}$, to receive a payment of 1 in node $m \in n_+$.
- Conceptually allows to transfer money from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Such contracts complete the market

$$
\begin{aligned}
&\text{RAT}(a, \pi, \mu, \mathcal{D}_a): \\
&\min_{(\theta, Z, x, u, W) \in \mathcal{F}(\omega)} \; Z_a(0) + \theta_a(0) \\
&\quad \text{s.t.} \;\; \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m) - W_a(m)), k \in K(n) \\
&\qquad\quad\;\; Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) + \sum_{m \in n_+} \mu(m) W_a(m)
\end{aligned}
$$

Theorem

Consider agents $a \in \mathcal{A}$, with risk sets $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$. Now let (u, θ) be a solution to $SO(\mathcal{D}_s)$ with risk sets $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$. Suppose this gives rise to a probability measure $(\sigma(n), n \in \mathcal{N})$ and multipliers $(\pi(n)\sigma(n), n \in \mathcal{N})$ for energy constraints. The prices $(\pi(n), n \in \mathcal{N})$ and $(\mu_\sigma(n), n \in \mathcal{N} \setminus \{0\})$ and actions $u_a(\cdot)$, $\{W_a(n), n \in \mathcal{N} \setminus \{0\}\}$ form a multistage risk-trading equilibrium $RET(\mathcal{D}_\mathcal{A})$.

Conversely...

Theorem

Consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$. Suppose $(\bar{\pi}(n), n \in \mathcal{N})$ and $(\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\})$ form a multistage risk-trading equilibrium $RET(\mathcal{D}_{\mathcal{A}})$ in which agent a solves $RAT(a, \bar{\pi}, \bar{\mu}, \mathcal{D}_a)$ with a policy defined by $\bar{u}_a(\cdot)$ together with a policy of trading Arrow-Debreu securities defined by $\{\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}\}$. Then

(i) $(\bar{u}, \bar{\theta})$ *is a solution to* $SO(\mathcal{D}_s)$ *with* $D_s = \{\bar{\mu}\}$,

(ii) $\bar{\mu} \in \mathcal{D}_a$ *for all* $a \in \mathcal{A}$,

(iii) $(\bar{u}, \bar{\theta})$ *is a solution to* $SO(\mathcal{D}_s)$ *with risk sets* $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$,

where $\bar{\theta}$ is defined recursively (above) with $\mu_\sigma = \bar{\mu}$ and $u_a(n) = \bar{u}_a(n)$.

In battery problem can recover by trading the system optimal solution (and its properties) since the retailer/generator agent is risk neutral

A Simple Network Model

Load segments s represent electrical load at various instances

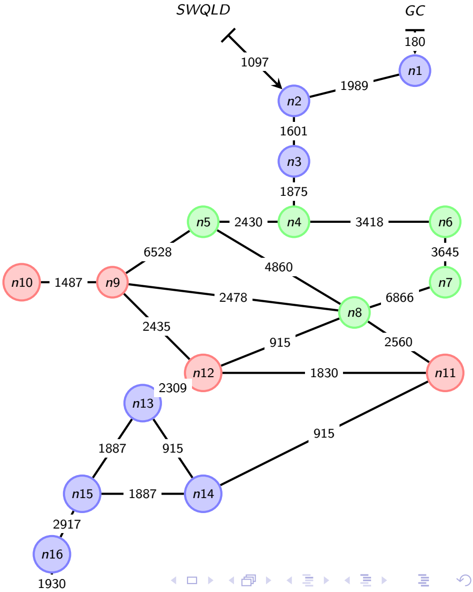
d_n^s Demand at node n in load segment s (MWe)

X_i^s Generation by unit i (MWe)

F_L^s Net electricity transmission on link L (MWe)

Y_n^s Net supply at node n (MWe)

π_n^s Wholesale price ($ per MWhe)



Nodes n, load segments s, generators i, Ψ is node-generator map

$$\begin{aligned}
\max_{X,F,d,Y} \quad & \sum_s \left(W(d^s(\lambda^s)) - \sum_i c_i(X_i^s) \right) \\
\text{s.t.} \quad & \Psi(X^s) - d^s(\lambda^s) = Y^s \\
& 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_s X_i^s \\
& Y \in \mathcal{X}
\end{aligned}$$

where the network is described using:

$$\mathcal{X} = \left\{ Y : \exists F, F^s = \mathcal{H} Y^s, -\overline{F}^s \le F^s \le \overline{F}^s, \sum_n Y_n^s \ge 0, \forall s \right\}$$

- Key issue: decompose. Introduce multiplier π^s on supply demand constraint (and use $\lambda^s := \pi^s$)
- How different approximations of $\mathcal{X}$ affect the overall solution

The Game: update red, blue and purple components

$$
\begin{aligned}
&\max_{d} \quad \sum_{s} \left(W(d^s(\lambda^s)) - \pi^s d^s(\lambda^s) \right) \\
&+ \max_{X} \quad \sum_{s} \left(\pi^s \Psi(X^s) - \sum_{i} c_i(X_i^s) \right) \\
&\quad \text{s.t.} \quad 0 \le X_i^s \le \overline{X}_i, \quad \overline{G}_i \ge \sum_{s} X_i^s \\
&+ \max_{Y} \quad \sum_{s} -\pi^s Y^s \\
&\quad \text{s.t.} \quad Y^s = \mathcal{A} F^s, -\overline{F}^s \le F^s \le \overline{F}^s
\end{aligned}
$$

$$
\pi^s \perp \Psi(X^s) - d^s(\lambda^s) - Y^s = 0
$$

Top down/bottom up

- $\lambda^s = \pi^s$ so use complementarity to expose (EMP: dualvar)
- Change interaction via new price mechanisms
- All network constraints encapsulated in (bottom up) NLP (or its approximation by dropping $\mathcal{L}F^s = 0$):

$$\begin{aligned} \max_{F,Y} \quad & \sum_s -\pi^s Y^s \\ \text{s.t.} \quad & Y^s = \mathcal{A}F^s, \mathcal{L}F^s = 0, -\overline{F}^s \leq F^s \leq \overline{F}^s \end{aligned}$$

- Could instead use the NLP over Y with $\mathcal{H}$
- Clear how to instrument different behavior or different policies in interactions (e.g. Cournot, etc) within EMP
- Can add additional detail into top level economic model describing consumers and producers
- Can solve iteratively using SELKIE

Pricing

Our implementation of the heterogeneous demand model incorporates three alternative pricing rules. The first is *average cost pricing*, defined by

$$P_{\mathrm{ACP}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{ACP}}} \sum_s q_{jns}}$$

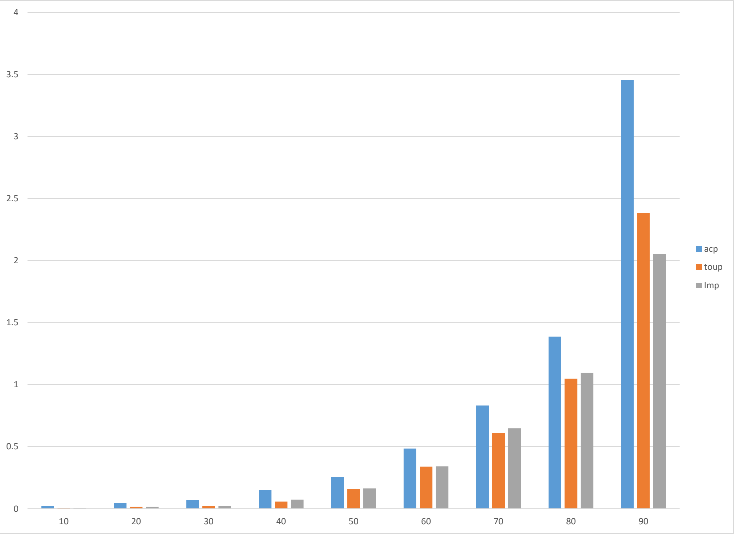
The second is *time of use pricing*, defined by:

$$P_s^{\mathrm{TOU}} = \frac{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} p_{jns} q_{jns}}{\sum_{jn \in \mathcal{R}_{\mathrm{TOU}}} q_{jns}}$$

The third is *location marginal pricing* corresponding to the wholesale prices denoted P_{ns} above. Prices for individual demand segments are then assigned:

$$p_{jns} = \begin{cases} P_{\mathrm{ACP}} & (jn) \in \mathcal{R}_{\mathrm{ACP}} \\ P_s^{\mathrm{TOU}} & (jn) \in \mathcal{R}_{\mathrm{TOU}} \\ P_{ns} & (jn) \in \mathcal{R}_{\mathrm{LMP}} \end{cases}$$

Smart Metering Lowers the Cost of Congestion



Contracts to mitigate risk

- Reserves: set aside operating capacity in future for possible dispatch under certain outcomes (2020 - can we improve uncertainty estimation to reduce amounts set aside)
- Contracts of differences and options on these (difference between promise and delivery)
- Contracts for guaranteed delivery of energy in future under certain outcomes (F/Wets)
- Arrow Debreu (pure) financial contracts under certain outcomes - trading risk (Philpott/F/Philpott)
- Localized storage as smoothers - transfer energy to future time at a given location (F/Philpott)

- Need market/equilibrium concept
- Need multiple period dynamic models and risk aversion

Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what
- It's available (in GAMS)
- Allows use and control of dual variables / prices
- MOPEC facilitates easy "behavior" description at model level
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- New decomposition algorithms available to modeler (Gauss Seidel, Randomized Sweeps, Gauss Southwell, Grouping of subproblems)
- Can evaluate effects of regulations and their implementation in a competitive environment
- Stochastic equilibria - clearing the market in each scenario
- Ability to trade risk using contracts