Modeling and Optimization of Electricity Markets

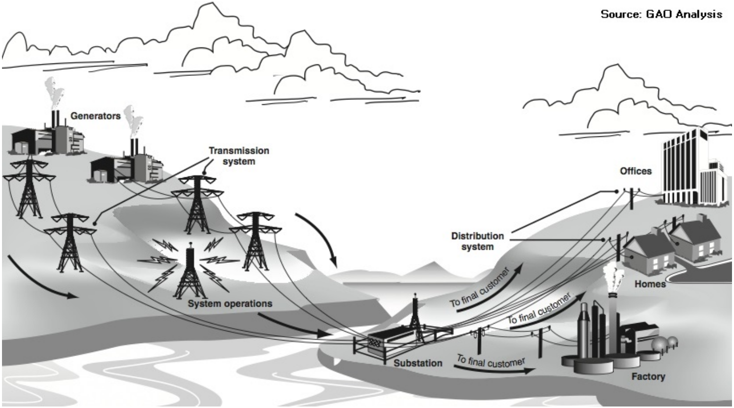
Michael C. Ferris

Joint work with: Andy Philpott, Roger Wets, Yanchao Liu, Jesse Holzer and Lisa Tang

University of Wisconsin, Madison

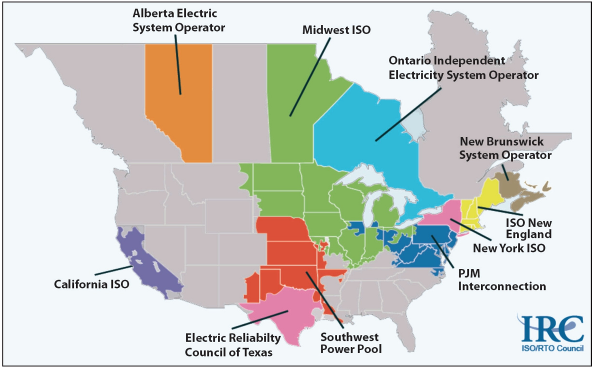
TWCCC, Madison, Wisconsin
October 7, 2014

Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $= \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

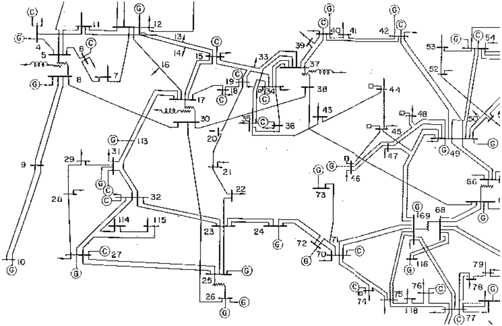
Managing the Grid

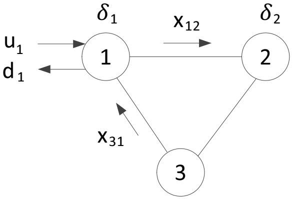


- Independent System Operator (ISO)[1]
- 10 ISOs in N. America, serving 2/3 of all electricity customers in the U.S.
- U.S. daily generation in 2013: 11 million MWh[2]
- Average wholesale price: \$30 - \$80/MWh

[1] Another name is Regional Transmission Organization (RTO)
[2] Information from www.eia.gov

Economic dispatch (a linear program)





Nodal power balance:

$$u_1 - x_{12} + x_{31} = d_1$$

Flow definition:

$$x_{12} = B_{12}\,(\delta_2 - \delta_1)$$

Variables: Generators' output u; Power flows on lines x; Bus voltage angle δ

Objective: Minimize the total generation cost, $c^T u$

Constraints:

- Kirchhoff's laws: $g(x,u) = 0$, where g is a linear function, including:
  - Nodal balance equations, line flow equations.
- Variable bounds: $h(x,u) \leq 0$, including:
  - Line limit: $-\bar{x} \leq x \leq \bar{x}$; Generator capacity: $0 \leq u \leq \bar{u}$

The PIES Model (Hogan)

$$\begin{aligned} \min_x \quad & c^T x & \text{cost} \\ \text{s.t.} \quad & Ax \geq d & \text{balance} \\ & Bx = b & \text{technical constr} \\ & x \geq 0 \end{aligned}$$

The PIES Model (Hogan)

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax \geq d(p) = \bar{d} - p \\ & Bx = b \\ & x \geq 0 \end{aligned}$$

cost

balance

technical constr

- Issue is that p is the multiplier on the "balance" constraint of LP
- Such multipliers (LMP's - locational marginal prices) are critical to operation of market
- Can solve the problem by writing down the KKT conditions of this LP, forming an LCP and exposing p to the model
- EMP does this automatically from the annotations

Reformulation details

$$
\begin{array}{lcl}
0 \le Ax - d(p) & \perp & \mu \ge 0 \\
0 = Bx - b & \perp & \lambda \\
0 \le -A^T\mu - B^T\lambda + c & \perp & x \ge 0
\end{array}
$$

- empinfo: dualvar p balance
- replaces $\mu \equiv p$
- LCP/MCP is then solvable using PATH

$$
z = \begin{bmatrix} p \\ \lambda \\ x \end{bmatrix}, \quad F(z) = \begin{bmatrix} & & A \\ & & B \\ -A^T & -B^T & \end{bmatrix} z + \begin{bmatrix} -d(p) \\ -b \\ c \end{bmatrix}
$$

Extension: maximizing profit

$$\begin{aligned} \max_x \quad & p^T x - c^T x && \text{profit} \\ \text{s.t.} \quad & Ax \geq d(p) && \text{balance} \\ & Bx = b && \text{technical constr} \\ & x \geq 0 \end{aligned}$$

- Issue is that there are multiple producers i
- The price is now determined by total production

$$\begin{aligned} \max_{x_i} \quad & p(\textstyle\sum_j x_j)^T x_i - c_i^T x_i && \text{profit} \\ \text{s.t.} \quad & B_i x_i = b_i && \text{technical constr} \\ & x_i \geq 0 \end{aligned}$$

and

$$0 \leq d(p) - \sum_i x_i \perp p \geq 0$$

Special case: many agents

$$\max_{x_i} \quad p\cancel{\left(\sum_j x_j\right)}^T x_i - c_i^T x_i \qquad \text{profit}$$

$$\text{s.t.} \quad B_i x_i = b_i \qquad \text{technical constr}$$

$$x_i \geq 0$$

and

$$0 \leq (\bar{d} - p) - \sum_i x_i \perp p \geq 0$$

- When there are many agents, assume none can affect p by themselves
- Each agent is a price taker
- Two agents, $\bar{d} = 24$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $p = 2$

Special case: two agents (duopoly)

$$\max_{x_i} \quad (\bar{d} - \sum_j x_j)^T x_i - c_i^T x_i \qquad \text{profit}$$

$$\text{s.t.} \quad B_i x_i = b_i \qquad \text{technical constr}$$

$$x_i \geq 0$$

- Cournot: assume each can affect p by choice of x_i
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $p = 29/3$
- Exercise of market power (some price takers, some Cournot)

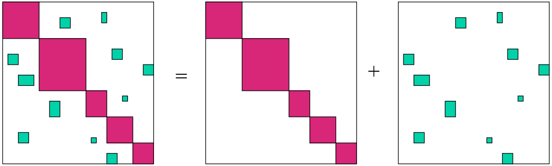
MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i$$

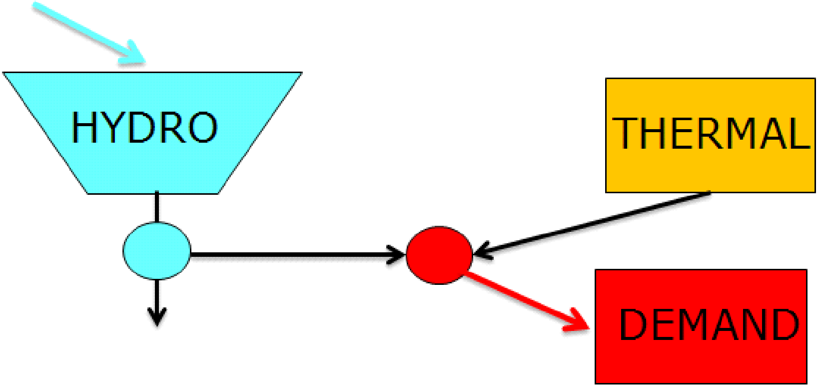
$$p \text{ solves VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h p cons
```

- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve complementarity problem
- Precondition using "individual optimization" with fixed externalities



Hydro-Thermal System (Philpott/F./Wets)



Simple electricity "system optimization" problem

$$
\begin{aligned}
\text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\
& x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}
$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE

$$
\begin{aligned}
\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k)\text{:} \quad & \max_{d_k \geq 0} \quad W_k(d_k) - p^T d_k \\
\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j)\text{:} \quad & \max_{v_j \geq 0} \quad p^T v_j - C_j(v_j) \\
\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i)\text{:} \quad & \max_{u_i, x_i \geq 0} \quad p^T U_i(u_i) + V_i(x_i) \\
& \text{s.t.} \quad x_i = x_i^0 - u_i + h_i^1
\end{aligned}
$$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$
\begin{aligned}
\text{CE:} \qquad d_k &\in \arg\max \text{CP}(k), & k \in \mathcal{K}, \\
v_j &\in \arg\max \text{TP}(j), & j \in \mathcal{T}, \\
u_i, x_i &\in \arg\max \text{HP}(i), & i \in \mathcal{H}, \\
0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) &+ \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.
\end{aligned}
$$

General Equilibrium models

$$(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } p^T x_k \le i_k(y,p)$$

$$(I): i_k(y,p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j)$$

$$(P): \max_{y_j \in Y_j} p^T g_j(y_j)$$

$$(M): \max_{p \ge 0} p^T \left(\sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l p_l = 1$$

This is an example of a MOPEC

Nash Equilibria

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, q), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.

- Quantities q given exogenously, or via complementarity:

$$0 \leq H(x, q) \;\perp\; q \geq 0$$

- empinfo: equilibrium
  min loss(i) x(i) cons(i)
  vi H q

- Applications: Discrete-Time Finite-State Stochastic Games. Specifically, the Ericson & Pakes (1995) model of dynamic competition in an oligopolistic industry.

Key point: models generated correctly solve quickly

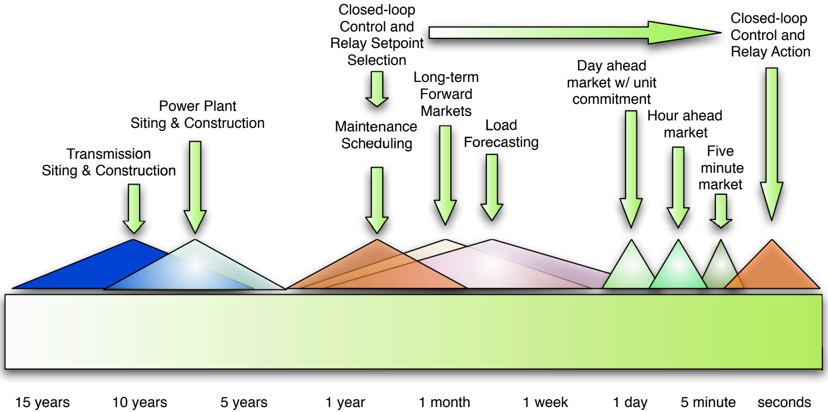
Here S is mesh spacing parameter

S	Var	rows	non-zero	dense(%)	Steps	RT (m:s)
20	2400	2568	31536	0.48	5	0 : 03
50	15000	15408	195816	0.08	5	0 : 19
100	60000	60808	781616	0.02	5	1 : 16
200	240000	241608	3123216	0.01	5	5 : 12

Convergence for $S = 200$ (with new basis extensions in PATH)

Iteration	Residual
0	$1.56(+4)$
1	$1.06(+1)$
2	1.34
3	$2.04(-2)$
4	$1.74(-5)$
5	$2.97(-11)$

Representative decision-making timescales in electric power systems



Many interacting levels, with different time scaled decisions at each level - collections of models needed.

Complications and myriad of acronyms

- Size/integrity
  - AC/DC models, reactive power, new devices, design/operation
  - Multi-period, demand response, load shedding, demand bidding
  - Day ahead, reserves, regulation, FTR's, co-optimization
- Integer:
  - Unit commitment (DAUC, RUC, RT)
  - Minimum up and down time
  - Transmission line switching
- Stochastic
  - Security constraints (SCED/SCUC)
  - Stochastic demand, dynamic
  - Renewables/storage

Bilevel Program (Stackelberg)

- Assumes one leader firm, the rest follow
- Leader firm optimizes subject to expected follower behavior
- Follower firms act in a Nash manner
- Bilevel programs:

$$\begin{array}{ll} \min_{x^*,y^*} & f(x^*,y^*) \\ \text{s.t.} & g(x^*,y^*) \le 0, \\ & y^* \text{ solves } \min_{y} v(x^*,y) \text{ s.t. } h(x^*,y) \le 0 \end{array}$$

- model bilev /deff,defg,defv,defh/;
  empinfo: bilevel min v y defv defh
- EMP tool automatically creates the MPCC

$$\begin{array}{lll} \min_{x^*,y^*,\lambda} & f(x^*,y^*) & \\ \text{s.t.} & g(x^*,y^*) \le 0, & \\ & 0 \le \nabla v(x^*,y^*) + \lambda^T \nabla h(x^*,y^*) & \perp y^* \ge 0 \\ & 0 \le -h(x^*,y^*) & \perp \lambda \ge 0 \end{array}$$

EMP(ii): MPCC: complementarity constraints

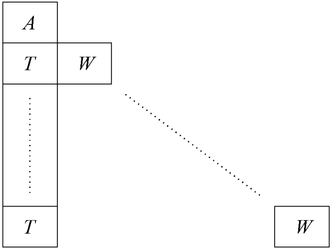
$$\begin{array}{ll} \min\limits_{x,s} & f(x,s) \\ \text{s.t.} & g(x,s) \leq 0, \\ & 0 \leq s \perp h(x,s) \geq 0 \end{array}$$

- g, h model “engineering” expertise: finite elements, etc
- $\perp$ models complementarity, disjunctions
- Complementarity “$\perp$” constraints available in AIMMS, AMPL and GAMS
- NLPEC: use the convert tool to automatically reformulate as a parameteric sequence of NLP’s
- Solution by repeated use of standard NLP software
  - Problems solvable, local solutions, hard

Agents have stochastic recourse?

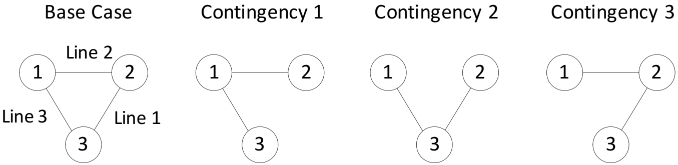
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- $\mathbb{R}$ is a risk measure (e.g. expectation, CVaR)

$$
\begin{aligned}
\text{SP:}\ \max \quad & c^T x^1 + \mathbb{R}[q^T x^2] \\
\text{s.t.} \quad & Ax^1 = b, \quad x^1 \geq 0, \\
& T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega), \\
& x^2(\omega) \geq 0, \forall \omega \in \Omega.
\end{aligned}
$$



EMP/SP extensions to facilitate these models

Contingency: a single line failure



- A network with N lines can have up to N contingencies
- Each contingency case:
  - Corresponds to a different network topology
  - Requires a different set of equations g and h
  - E.g., equations g_k and h_k for the k-th contingency.

Control v.s. State variables

- Generator output u is a CONTROL variable:
  - System operator can directly set/adjust its level
  - No abrupt change, i.e., it takes time to ramp up/down a generator
- Line flow x is a STATE variable:
  - The level depends on u and the network topology
  - Automatically jumps to a new level when topology changes, e.g., when a line suddenly fails
- **Security requirement:** When a line fails, other lines should not overload.
- Change "base" state and control variables to achieve this.

Security-constrained Economic Dispatch

- Base-case network topology g_0 and line flow x_0.
- If the k-th line fails, line flow jumps to x_k in new topology g_k.
- Ensure that x_k is within limit, for all k.
- SCED model:

$$
\begin{array}{lll}
\min\limits_{u,x_0,\ldots,x_k} & c^T u & \triangleright \text{Total cost} \\
\text{s.t.} & 0 \leq u \leq \bar{u} & \triangleright \text{GEN capacity const.} \\
& g_0(x_0, u) = 0 & \triangleright \text{Base-case network eqn.} \\
& -\bar{x} \leq x_0 \leq \bar{x} & \triangleright \text{Base-case flow limit} \\
& g_k(x_k, u) = 0, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy network eqn.} \\
& -\bar{x} \leq x_k \leq \bar{x}, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy flow limit}
\end{array}
$$

Model structure

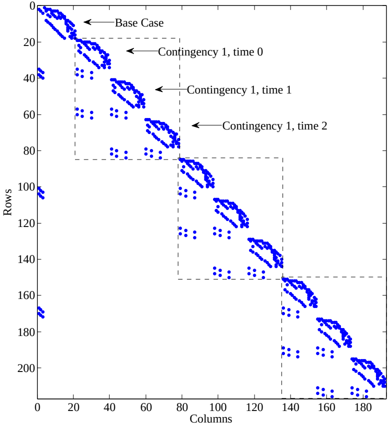


Figure : Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

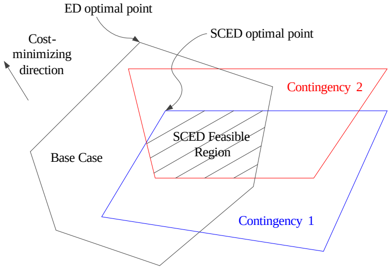


Figure : On the u_0 plane, the feasible region of a SCED is the intersection of K+1 polyhedra.

Contracts in MOPEC (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{aligned}
\min \quad & C(x_i^1) + \sum_\omega \pi_\omega C(x_i^2(\omega)) \\
\text{s.t.} \quad & p^1 x_i^1 + v y_i \le p^1 e_i^1 && \text{(budget time 1)} \\
& p^2(\omega) x_i^2(\omega) \le p^2(\omega)(D(\omega) y_i + e_i^2(\omega)) && \text{(budget time 2)}
\end{aligned}$$

$$0 \le v \perp -\sum_i y_i \ge 0 \qquad \text{(contract)}$$

$$0 \le p^1 \perp \sum_i (e_i^1 - x_i^1) \ge 0 \qquad \text{(walras 1)}$$

$$0 \le p^2(\omega) \perp \sum_i \left(D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega)\right) \ge 0 \qquad \text{(walras 2)}$$

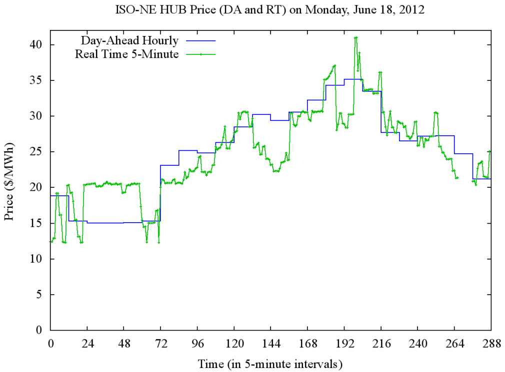
Observations

- Examples from literature solved using homotopy continuation seem incorrect - need transaction costs to guarantee solution
- Solution possible via disaggregation only seems possible in special cases
  - When problem is block diagonally dominant
  - When overall (complementarity) problem is monotone
  - (Pang): when problem is a potential game
- Progressive hedging possible to decompose in these settings by agent and scenario
- Can do multi-stage models via stochastic process over scenario tree
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

PJM buy/sell dynamic model

- Storage transfers energy over time (horizon $= T$).
- PJM: given price path p_t, determine charge q_t^+ and discharge q_t^-:

$$
\begin{aligned}
\max_{h_t, q_t^+, q_t^-} \quad & \sum_{t=0}^{T} p_t (q_t^- - q_t^+) \\
\text{s.t.} \quad & \partial h_t = e q_t^+ - q_t^- \\
& 0 \le h_t \le \mathcal{S} \\
& 0 \le q_t^+ \le \mathcal{Q} \\
& 0 \le q_t^- \le \mathcal{Q} \\
& h_0, h_T \text{ fixed}
\end{aligned}
$$



- Uses: price shaving, load shifting, transmission line deferral
- What about real-time storage, or different storage technologies?

Stochastic price paths (day ahead market)

$$
\begin{aligned}
\min_{x,h,q^+,q^-} \; & c^1(x) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^2(q^+_{\omega t} + q^-_{\omega t}) \right] \\
\text{s.t. } & \partial h_{\omega t} = e q^+_{\omega t} - q^-_{\omega t} \\
& 0 \le h_{\omega t} \le \mathcal{S} x \\
& 0 \le q^+_{\omega t}, q^-_{\omega t} \le \mathcal{Q} x \\
& h_{\omega 0}, h_{\omega T} \text{ fixed}
\end{aligned}
$$

- First stage decision x: amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty

Distribution of (multiple) storage types

Determine storage facilities x_k to build, given distribution of price paths: no entry barriers into market, etc. MOPEC: for all k solve a two stage SP

$$
\begin{aligned}
\forall k: \quad \min_{x_k, h_k, q_k^+, q_k^-} \; & c_k^1(x_k) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q_{\omega k t}^+ - q_{\omega k t}^-) + c_k^2(q_{\omega k t}^+ + q_{\omega k t}^-) \right] \\
\text{s.t. } & \partial h_{\omega k t} = e q_{\omega k t}^+ - q_{\omega k t}^- \\
& 0 \le h_{\omega k t} \le \mathcal{S} x_k \\
& 0 \le q_{\omega k t}^+, q_{\omega k t}^- \le \mathcal{Q} x_k \\
& h_{\omega k 0}, h_{\omega k T} \text{ fixed}
\end{aligned}
$$

$$
p_{\omega t} = f\left(\gamma, \mathcal{D}_{\omega t} + \sum_k (q_{\omega k t}^+ - q_{\omega k t}^-) \right)
$$

Parametric function (γ) determined by regression. Storage operators react to shift in demand.

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC models capture behavioral effects (as an EMP)
- Extended Mathematical Programming available within the GAMS modeling system
- Modeling, optimization, statistics and computation embedded within the application domain is critical