Resilience, robustness and performance: tradeoffs in decision processes

Michael C. Ferris

Computer Sciences Department and Wisconsin Institute for Discovery,
University of Wisconsin, Madison

(Includes work with Olivier Huber, Cheng-Wei Lu, Jeff Linderoth, Andy Philpott and Jiajie Shen)

Industrial and Systems Engineering, University of Minnesota
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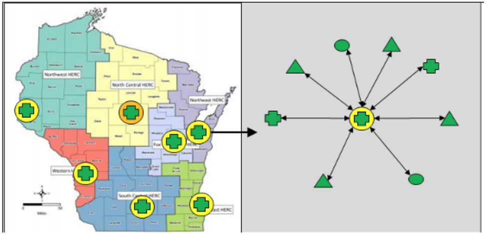
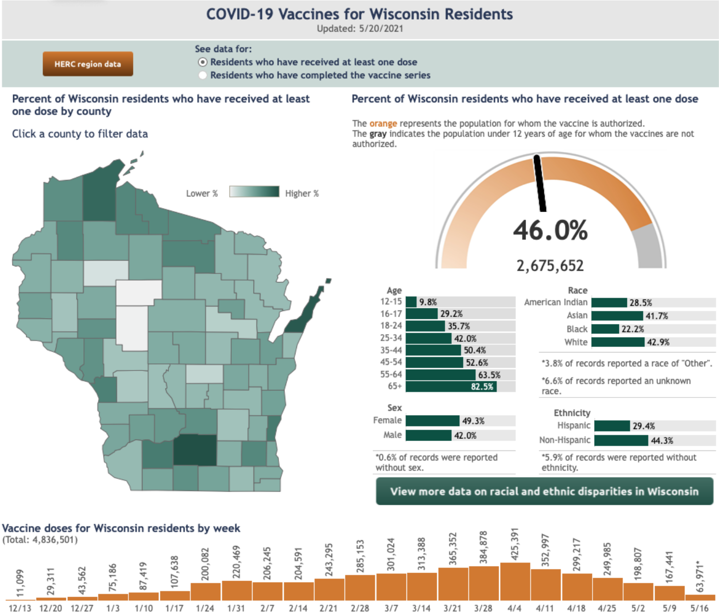
Overview

- Decision making with multiple agents, not necessarily aligned
- How can mathematical models impact or inform decision processes in such environments
  - Provide feasible/implementable solutions (design/engineering)
  - Demonstrate "value" of a given solution (verification of specific metrics)
  - Allow comparison of multiple solutions demonstrating tradeoffs
  - Argue for robustness and adaptability, not just resilience
- Three examples to tease out concepts

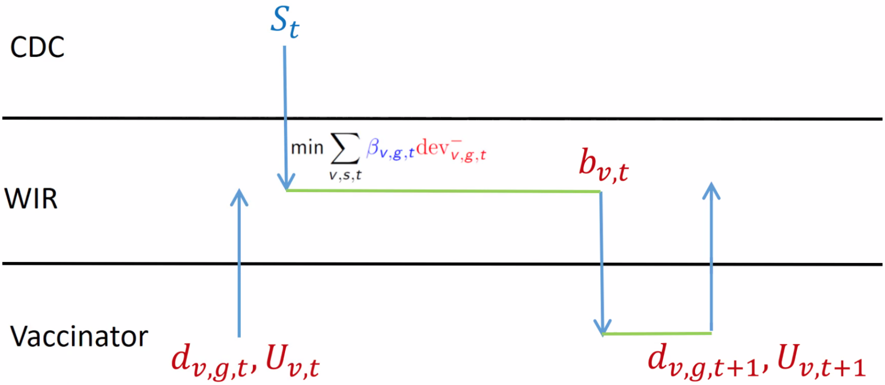
Covid vaccination distribution in Wisconsin



- players: CDC, State government, DHS (health services), National Guard, vaccinators
- motivation: politics, population health, logistics
- weekly cadence: get data (4 days), clean data (1 day), build and run model ($<$ 1 day), generate results, validate/update results (0.5 day), enter order (0.5 day)
- issues: fairness (svi, % vaccinated, age cohort, at-risk populations, etc), logistics (box size, transport, requests, repeated rejections, etc)

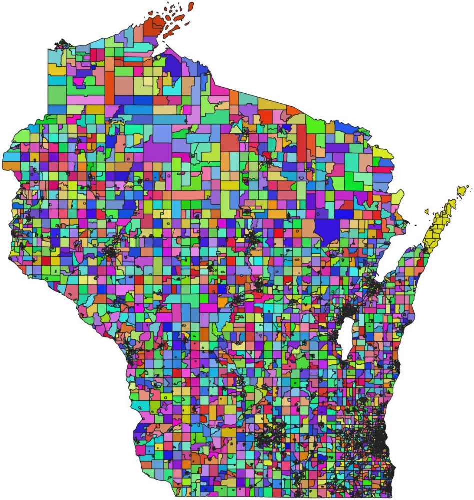


- https://www.dhs.wisconsin.gov/covid-19/vaccine-data.htm



- $U_{h,m,t}$: capacity of hub (storage size)
- $S_{m,t}$: number of vaccine boxes of type m delivered at t
- $N_{m,t}$: number of vaccines of type m in boxes delivered at t to WI
- two step process: convex optimization: fairness; mip: logistics
- in a crisis, logistics trumps fairness

Spatial Partitioning: Gerry Mandering



- Merge n precincts into d districts
- Ensure partitions are connected, compact, equi-population and have certain fairness properties e.g. vote distribution properties, diverse, robust to changing demographics and population
- Compactness could be described via average distance between "centers" of precincts

MIP approach [3]

- Generate random set of district centers
- Assign precincts to district centers to minimize total distance (compactness)
- Add flow constraints to guarantee connectedness
- Solution may take a long time

Heuristic: add properties to solution

Construction:

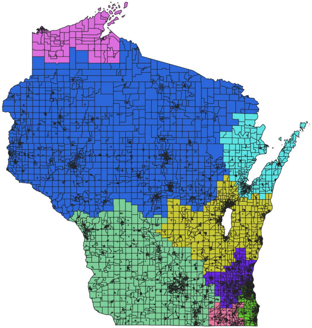
1. Randomly select one precinct, and assign its nearest adjacent precinct to it
2. Total number of precincts becomes $n - 1$ after one is merged to another precinct
3. Repeat until number of precincts is d

Solution is contiguous, but may not satisfy equi-population

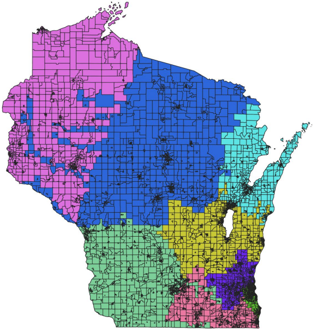
Improvement:

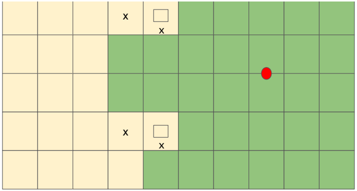
1. Choose a pair of adjacent districts that have the largest population disparity
2. Identify precincts from the overpopulated that can be assigned to the underpopulated without violating the contiguity
3. Choose the precinct whose reassignment will least impact compactness and assign it to the underpopulated
4. Repeat until termination bound achieved

Initial Solution

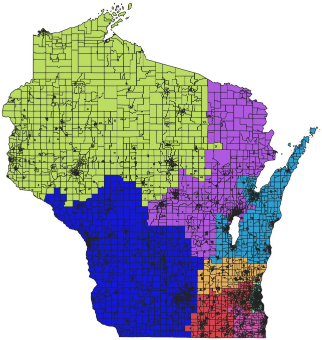
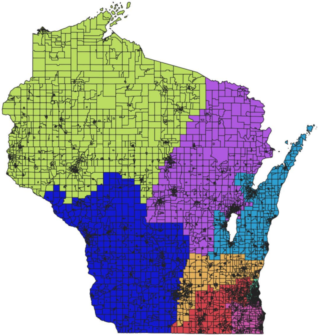


Final Solution



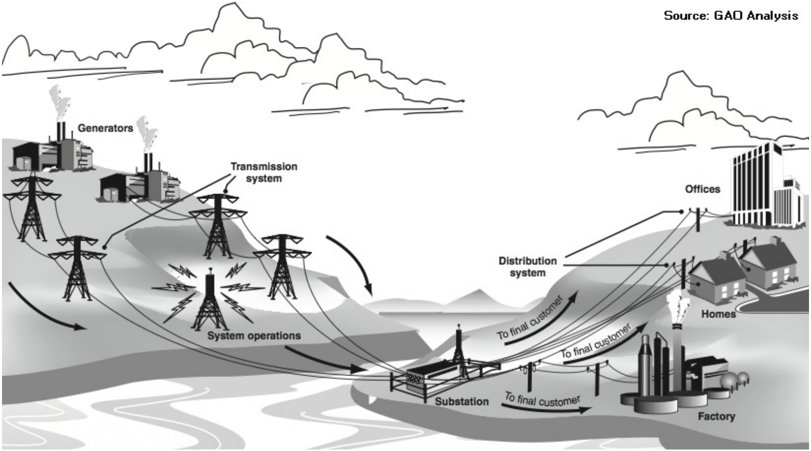


- Data issue: precincts inside precincts!
- Modify data to collapse into one change unit
- Decreases time and improves solution

- Political science approach [1]: "fairness" in terms of compactness and equi-population districts can lead to republican majorities due to population density
- Generate multiple feasible partitions
- Determine "metrics" over those solutions and allow decision makers to make trade-offs and select best
- MIRO application on NEOS; use in service area design, coverage, etc

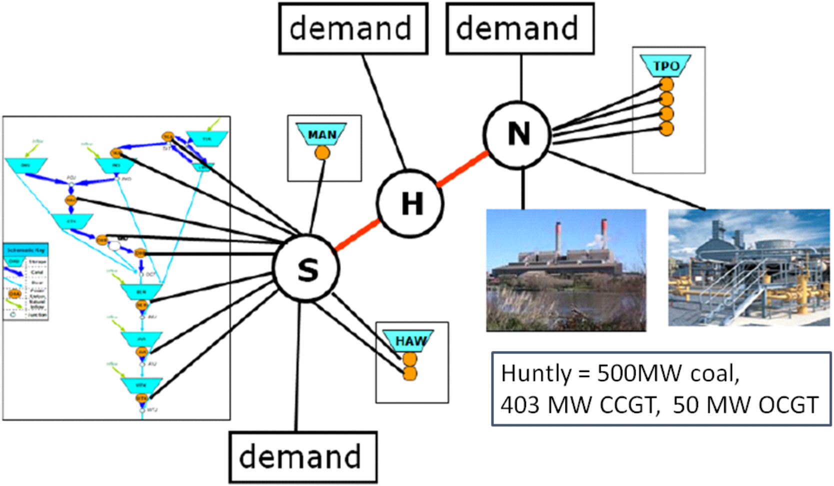
Engineering, Economics and Environment



- Determine generators' output to reliably/economically meet the load
- Feasibility: Power flows cannot exceed lines capacity
- Tradeoff: Impose environmental regulations/incentives

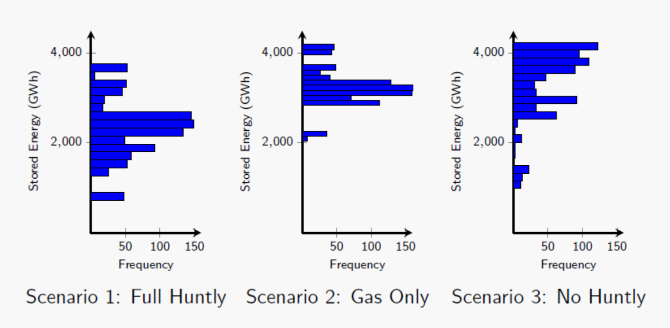
Jacinda - what does fully renewable in electricity mean?

- Permanently shutdown all thermal plants?
- Control GHG emissions from electricity generation?



Closing plants often increases average emissions (Fulton)

- Hydro can act as a giant battery [2]
- Simulation runs: Reduce plant capacity, store more water "in case of dry winter":



- With low nonrenewable plant capacity, can't wait till last minute and reservoir levels in summer need to be close to full just in case. Tradeoff: Burning fuel to achieve this increases emissions.

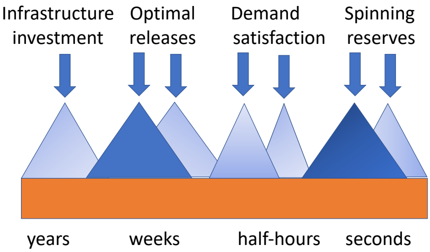
Many planning/market models

- Economic dispatch (day ahead and real time)
- Nash equilibrium (suppliers, transmission, consumers)
- System marginal costs provide locational, market-clearing, linear prices
- Contingency constraints
- Market design: bid based, security constrained, market power mitigation (fairness)
- Ancilliary services (regulation and reserves)
- Capacity markets and auctions
- Resource adequacy, and distributed energy resources (DER)

Systems in place aim to improve both robustness and resilience of system

Uncertainty is experienced at different time scales

- Demand growth, technology change, capital costs are long-term uncertainties (years)
- Seasonal inflows to hydroelectric reservoirs are medium-term uncertainties (weeks)
- Levels of wind and solar generation are short-term uncertainties (half hours)
- Very short term effects from random variation in renewables and plant failures (seconds)



- Tradeoff: Uncertainty, cost and operability, regulations, security/robustness/resilience
- Needs modelling at finer time scales

System properties

Resilience: the capacity to recover quickly from difficulties

Robustness: capable of performing without failure under a wide range of conditions (ability to withstand or survive known external shocks, to be stable in spite of uncertainty)

Reliability: the quality of performing consistently well

Sustainability: the ability to be maintained at a certain rate or level

Fairness: impartial and just treatment or behavior without favoritism or discrimination

If fairness is the goal, equality and equity are two processes through which we can achieve it. Equality simply means everyone is treated the same exact way, regardless of need or any other individual difference. Equity, on the other hand, means everyone is provided with what they need to succeed.

Value function

- Suppose you have a system that evolves with time, with an associated value function $v : [0, \infty) \mapsto \mathbb{R}$ that describes the "goodness" of the state of the system, or the outcome of that system at a given point in time. If $v(t) < 0$, then "things are bad" at time t, otherwise, "things are good."
- Typical value functions are the economic profit from operating the system, the efficiency of the system, the health of the (person or) system.
- Sometimes we measure properties such as errors in an prediction, or costs, or other bad system states and have to define v as a reciprocal or negative of that property.

Often we evaluate the performance of the system using its total value:

$$\text{Performance} := \int_{t=0}^{\infty} v(t)dt$$

Other ways, many of which change the perspective and hence the objective of the underlying design or optimization.

Resilience

- Popular buzz word
- Recover quickly from unforseen disturbances, e.g. disaster recovery
- Can plan for multiple given disturbances - that's robustness
- Goes back to "operating point" - no learning

A surrogate for "resilience" could be trying to mitigate the damage when things go bad. Thus, we would want

$$\textsc{Resilience} := \int_{t=0}^{\infty} v(t)\mathbf{1}_{v(t)<0}dt$$

to be large.

> *Argue that resilience is a desirable property, but hard to design a resilient system (a-priori). Instead, build or plan for an efficient recovery process.*

Robustness: planning for uncertainty

Key property: "reliability" – trying to keep things good as much as possible. Mathematically:

$$\textsc{Reliability} := \int_{t=0}^{\infty} \mathbf{1}_{v(t)\geq 0}\, dt$$

should be large.

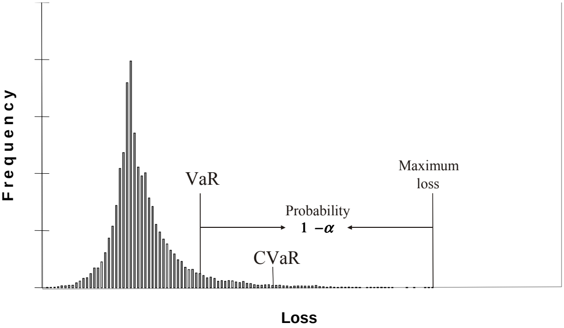
- Robust systems are designed to withstand known external shocks
- Dynamic models allow changes as uncertainties/shocks are realized
- Actions facilitate adaptability, not just recovery to a previous "operating point"
- Mitigation: the action of reducing the severity, seriousness, or painfulness of something (recourse)
- Hedging: the design to position so that effects are less severe, or mitigation is more effective

Learning: the process of acquiring new understanding, knowledge, behaviors, skills, values, attitudes, and preferences

Robustness facilitates learning and adaptation

Risk modeling

- Modern approach to modeling risk aversion uses concept of risk measures
- Considers not only the expected value of the uncertain quantities, but also more “extreme events”



- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)
- Dual representation (of coherent r.m.) in terms of risk sets: $\mathcal{D}$ [4]

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z]$$

- Different agents have different risk profiles

Risk Measures

Problem type

Objective function or Constraint

$$\min_{x\in X} \theta(x) + \rho(F(x)) \qquad \min_{x\in X} \theta(x) \text{ s.t. } \rho(F(x)) \le \alpha$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda \in [0, p/(1-\alpha)] : \langle \mathbf{1}, \lambda \rangle = 1\}$, then $\rho(Z) = \overline{CVaR}_\alpha(Z)$
- Popular examples include: expectation, Conditional Value at Risk, also known as expected shortfall, Average Value at Risk (AVaR), and expected tail loss (ETL), and mean-upper-absolute semideviation.

Using the algebra of support function, we can create new risk measures from existing ones: for instance

$$\lambda \mathbb{E} + (1-\lambda)\overline{CVaR}_\alpha$$

captures more realistic risk-averse behavior. For $\lambda > 0$, it is strictly monotone (desirable for time-consistency)

The transformation to complementarity

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\rho(y) = \sup_{u \in U} \left\{ \langle u, y \rangle - \frac{1}{2} \langle u, Mu \rangle \right\}$$

conjugate composite function:

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

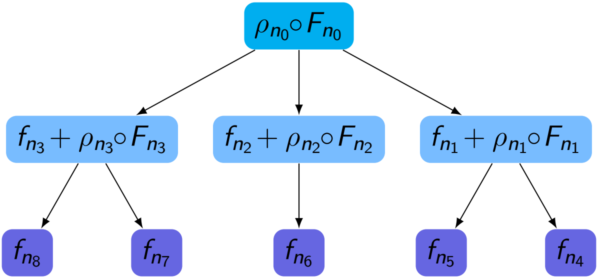
calculus:

$$0 \in \partial\theta(x) + \nabla F(x)^T u + N_X(x)$$

$$0 \in -u + \partial\rho(F(x)) \iff 0 \in -F(x) + Mu + N_U(u)$$

- This is a complementarity problem (solvable by PATH)
- Equilibrium formulation
- (Fenchel) duality formulation
- Extreme point formulation

Scenario tree with nodes $\mathcal{N} = \{0, 1, \ldots, 8\}$, and $T = 3$



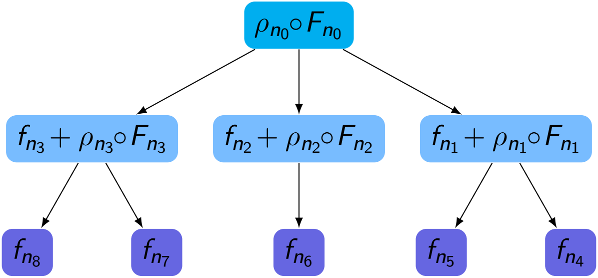
At leaf nodes:

$$\min_{x_{a\ell}} f_{a\ell}(x_{a\ell}; x_{-a\ell}, x_{\cdot\ell_-}, p_\ell) \quad \forall a \in \mathcal{A},$$

$$0 \in F_\ell(p_\ell; x_{\cdot\ell}) + N_{P_\ell}(p_\ell).$$

";" separates variables from parameters in function definition

Stochastic equilibrium (nested definition)



Recursing back to the root node:

$$
\begin{aligned}
\min_{x_{a\mathcal{S}(n_0)}} \; & f_{an_0}(x_{an_0}; x_{-an_0}, x_{\cdot n_{0-}}, p_{n_0}) \\
& + \mathcal{R}_{an_0}([f_{aj}(x_{aj}; x_{-aj}, x_{\cdot n_0}, p_j) \\
& \quad + \mathcal{R}_{aj}([f_{a\ell}(x_{a\ell}; x_{-a\ell}, x_{\cdot \ell_-}, p_\ell)]_{\ell \in j_+})]_{j \in n_{0+}}) \qquad \forall a \in \mathcal{A}, \\
0 \in & F_j(p_j; x_{\cdot j}) + N_{P_j}(p_j), \qquad \forall j \in \mathcal{S}(n_0).
\end{aligned}
$$

$\mathcal{S}(n)$ is the set of successor nodes of n, including n

Example: risk-averse stochastic equilibria

- market equilibrium: price defined by equilibrium constraints
- producers have a random upper bound on their production capacities and their ability to store goods from one stage to the other induces a coupling across stages
- objective function: revenue minus cost of production
- A, the scenario tree has 3 stages with 13 nodes, and there are 5 players in the market with 2 goods.
- B, the scenario tree has 4 stages with 30 nodes, and we have 2 players with 1 good.
- C has 5 stages, 121 nodes, 2 players and 1 good.

	Equilibrium			Duality			Conjugate		
	T (s)	vars	nnz	T (s)	vars	nnz	T (s)	vars	nnz
A	1.6	584	2775	5.2	644	2990	3.8	584	3530
B	9.0	455	2382	3.0	533	2774	Fail	455	2498
C	2.2	1400	8700	Fail	1640	10280	Fail	1400	7736

Different reformulations via option file

Conclusions

- Mathematical models help to provide feasible/implementable solutions
- Fairness is hard to quantify mathematically, many facets
- Decision makers are balancing complex tradeoffs - providing (implementable) alternatives with vectors of metrics is practically important
- Resiliency is desirable if robust design fails, hard to design (a-priori) due to unknown disturbances; precludes learning
- Alternative types and levels of robustness - use optimization to make robust designs, hedging strategies, etc
- Adaptability/learning is closely coupled to robustness; quantifiable disturbances for design optimization; relies heavily on predictions

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