From Complementarity to Risk-averse stochastic equilibria: models and algorithms

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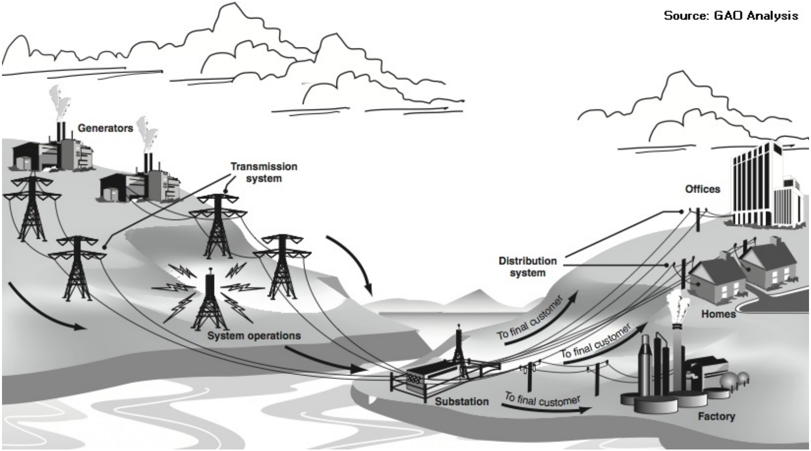
SIAM Optimization Meeting
June 1, 2023

Engineering, Economics and Environment



- Determine generators' output to reliably/economically meet the load
- Power flows cannot exceed lines' transfer capacity
- Tradeoff: Impose environmental regulations/incentives

Perfect competition (MOPEC)

$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

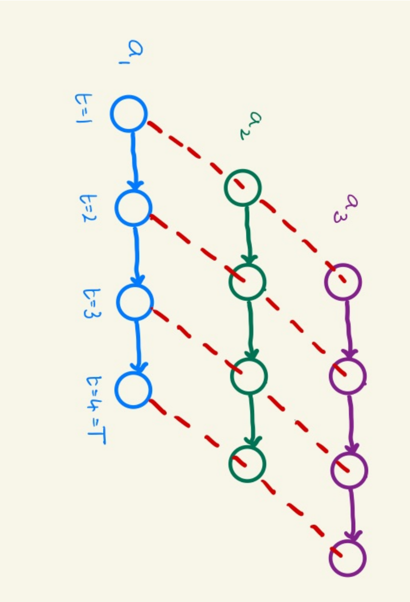
Duopoly: two agents (Cournot)

$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect π by choice of x_i
- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot), or maybe agent hedging

Simple dynamics (discrete time, finite horizon)

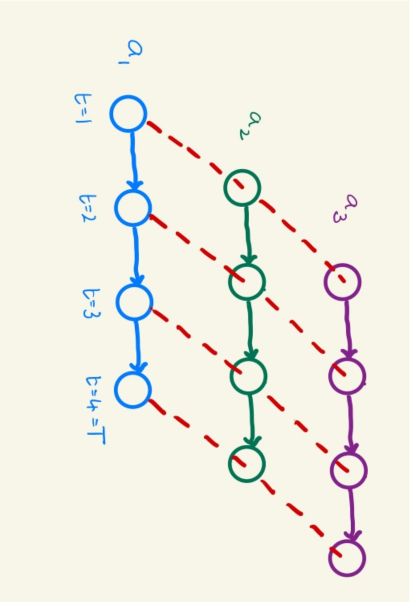


$\forall a \in \mathcal{A}$:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; \cdot, \cdot) + f_{a2}(x_{a2}; \cdot, \cdot) + \cdots + f_{aT}(x_{aT}; \cdot, \cdot)$$

- Dynamics link over time

Simple dynamics (discrete time, finite horizon)



$\forall a \in \mathcal{A}$:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + f_{a2}(x_{a2}; x_{-a2}, \pi_2)$$

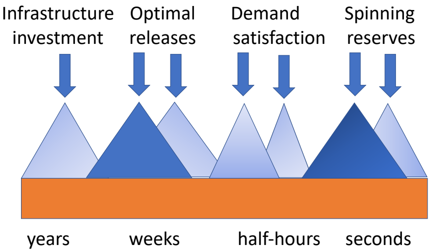
$$+ \cdots + f_{aT}(x_{aT}; x_{-aT}, \pi_T)$$

$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j),$$

- Dynamics link over time
- Complementarity links nodes across agents

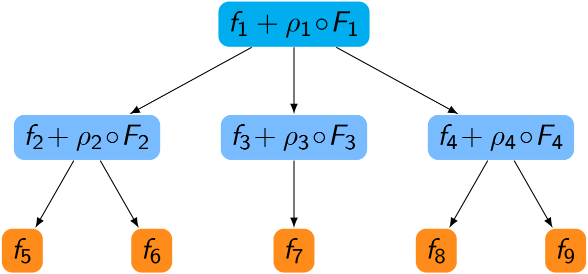
Uncertainty is experienced at different time scales

- Demand growth, technology change, capital costs are long-term uncertainties (years)
- Seasonal inflows to hydroelectric reservoirs are medium-term uncertainties (weeks)
- Levels of wind and solar generation are short-term uncertainties (half hours)
- Very short term effects from random variation in renewables and plant failures (seconds)



- Tradeoff: Uncertainty, cost and operability, regulations, security/robustness/resilience
- Needs modelling at finer time scales

Scenario tree with nodes $\mathcal{N} = \{1, 2, \ldots, 9\}$, and $T = 3$



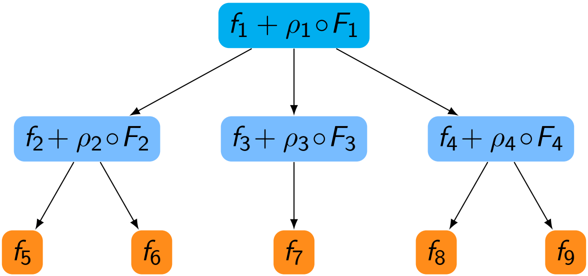
At leaf nodes:

$$\min_{x_{a\ell} \in \mathcal{X}_{a\ell}} \leftarrow f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \quad \forall a \in \mathcal{A},$$

$$0 \in H_\ell(\pi_\ell; x_{\cdot\ell}) + N_{P_\ell}(\pi_\ell)$$

";" separates variables from parameters in function definition

Stochastic equilibrium (MOPEC)

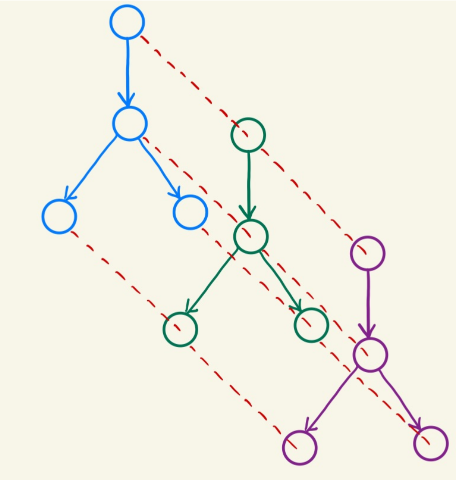


Agents solve problem at root node, linking at all nodes:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \rho_{a1}\left(\left[f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \rho_{aj}\left(\left[f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell)\right]_{\ell \in j_+}\right)\right]_{j \in 1_+}\right) \quad \forall a \in \mathcal{A},$$

$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \qquad \forall j \in \mathcal{T}.$$

Scenario trees linked across agents



- Dynamics link over time
- Complementarity links nodes of scenario tree across agents

Three sources of difficulty:

1. Size: number of scenarios, agents, details
2. Non-convexity: Nash behavior
3. Risk aversion: Nonsmooth or Nonlinear (product of probabilities)

Risk Measures

Problem type

Objective function or Constraint

$$\min_{x \in X} \theta(x) + \rho(F(x)) \qquad \min_{x \in X} \theta(x) \text{ s.t. } \rho(F(x)) \leq \alpha$$

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{y \in \mathcal{D}} \mathbb{E}_y[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{y \in [0, p/(1-\alpha)] : \langle \mathbf{1}, y \rangle = 1\}$, then $\rho(Z) = \overline{CVaR}_\alpha(Z)$
- Combinations - increasing risk aversion as λ increases

$$\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda \overline{CVaR}_\alpha(Z)$$

The transformation to complementarity

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\text{where } \rho(u) = \sup_{y \in \mathcal{D}} \left\{ \langle y, u \rangle - \frac{1}{2} \langle y, My \rangle \right\}$$

optimality condition:

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

calculus:

$$0 \in \partial\theta(x) + \nabla F(x)^T y + N_X(x)$$

$$0 \in -y + \partial\rho(F(x)) \iff 0 \in -F(x) + My + N_{\mathcal{D}}(y)$$

- This is a complementarity problem: opt conds in x coupled with opt conds in y - separated

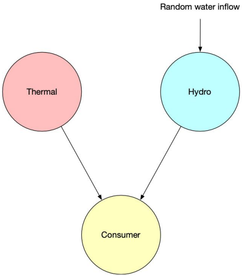
Stochastic Equilibrium as (extended) MOPEC

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \sum_{j \in 1_+} y_{aj} \left(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \sum_{\ell \in j_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \right), \quad \forall a \in \mathcal{A} \tag{1}$$

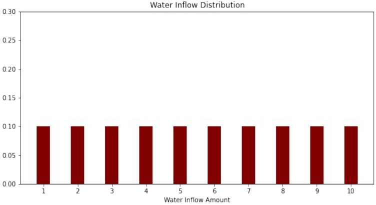
$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T} \tag{2}$$

$$\begin{aligned} r_{a1}(x,\pi) &= \max_{y_{a1_+} \in \mathcal{D}_{a1}} \sum_{j \in 1_+} y_{aj}(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + r_{aj}(x,\pi)) \\ r_{a2}(x,\pi) &= \max_{y_{a2_+} \in \mathcal{D}_{a2}} \sum_{\ell \in 2_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \\ r_{a3}(x,\pi) &= \max_{y_{a3_+} \in \mathcal{D}_{a3}} \sum_{\ell \in 3_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \\ r_{a4}(x,\pi) &= \max_{y_{a4_+} \in \mathcal{D}_{a4}} \sum_{\ell \in 4_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \end{aligned} \tag{3}$$

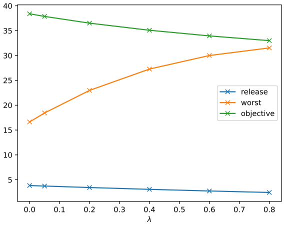
Simple example (3 agents, 2 stages, 10 scenarios)



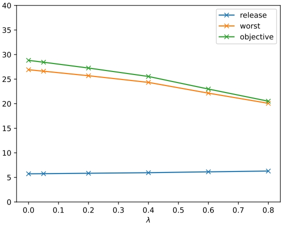
Second stage probabilities:



Low stage 1 inflow:



Higher stage 1 inflow:



Algorithms and problems

- PATH: nonsmooth Newton method (defaults) (blue+red+black)
- PD (Primal-dual): iteratively blue+red then black
- PD-PTH (Primal-dual + PATH)
- PD-CC-PTH (Primal-dual + convex-comb(black) + PATH)
- Homot(λ) + Primal-dual + convex-comb(black) + PATH

- Multistage economic dispatch, capacity expansion, hydroelectric system
- 3 types of demand formulation (I,II and III)
- Two scenario trees (4 stages, 40 nodes) and (4 stages, 156 nodes)
- 32 data instances for each formulation
- Several modulus of convexity and risk aversion parameters

$$\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda\overline{CVaR}_\alpha(Z)$$

Dispatch example, large tree, type I

quad	λ	PATH	PATH-RN	PD	PD-PTH	PD-CC-PTH
0	0.1	0.0	37.5	0.0	59.4	100.0
0	0.3	0.0	0.0	0.0	12.5	96.9
0	0.5	0.0	0.0	0.0	9.4	71.9
0	0.7	0.0	0.0	0.0	3.1	18.8
0	0.9	0.0	0.0	0.0	0.0	9.4
1e-2	0.1	28.1	90.6	15.6	100.0	100.0
1e-2	0.3	0.0	0.0	0.0	90.6	100.0
1e-2	0.5	0.0	0.0	0.0	40.6	100.0
1e-2	0.7	0.0	0.0	0.0	21.9	84.4
1e-2	0.9	0.0	0.0	0.0	6.2	53.1
1e-1	0.1	0.0	100.0	59.4	100.0	100.0
1e-1	0.3	0.0	68.8	43.8	100.0	100.0
1e-1	0.5	0.0	3.1	18.8	96.9	100.0
1e-1	0.7	0.0	0.0	12.5	100.0	100.0
1e-1	0.9	0.0	0.0	15.6	93.8	100.0

Market Type	PATH SR(%)	PATH-RN SR(%)	PD SR(%)	PD-PATH SR(%)	No mixed solution percentage(%)
Type I	56.4	78.0	98.2	100.0	98 .4
Type II	64.8	82.4	97.9	100.0	97 .9
Type III	89.9	93.2	77.4	100.0	78. 0

Summary table of performance of PATH, PATH-RN, PD and PD-PATH over capacity expansion example on smaller scenario tree

Market Type	PATH SR(%)	PATH-RN SR(%)	PD SR(%)	PD-PATH SR(%)	No mixed solution percentage(%)
Type I	50.9	42.1	95.5	99.5	96.8
Type II	77.9	43.5	95.1	99.8	99.9
Type III	58.9	26.6	97.5	100.0	97.9

Summary table of performance of PATH, PATH-RN, PD and PD-PATH over hydroelectricity example on smaller scenario tree

Performance on economic dispatch (*Type I and II*) on smaller tree

quad	λ	*Type I* market				*Type II* market			
		PD-PATH		PD-CC-PATH		PD-PATH		PD-CC-PATH	
		SR(%)	Time(s)	SR(%)	Time(s)	SR(%)	Time (s)	SR(%)	Time(s)
0	0.1	96.9	7.8	100.0	8.9	96.9	9.2	100.0	10.2
0	0.3	78.1	8.6	100.0	12.8	84.4	9.3	100.0	10.7
0	0.5	59.4	7.6	96.9	15.2	62.5	8.0	100.0	12.8
0	0.7	18.8	6.4	96.9	30.9	25.0	7.5	100.0	35.6
0	0.9	3.1	9.0	65.6	32.6	3.1	6.1	68.8	33.6
1e-2	0.1	100.0	6.8	100.0	7.6	100.0	7.0	100.0	7.4
1e-2	0.3	100.0	7.5	100.0	8.6	100.0	7.5	100.0	8.5
1e-2	0.5	90.6	7.6	100.0	9.0	96.9	7.7	100.0	8.7
1e-2	0.7	71.9	6.7	100.0	9.8	84.4	6.7	100.0	10.0
1e-2	0.9	37.5	6.8	100.0	18.7	43.8	8.6	100.0	10.1

- PATH times vary from 0.3 to 2.7 (s)
- All *Type III* problems on small and larger scenario tree solved by PD-CC-PTH

Performance on economic dispatch (*Type I and II*) on larger tree

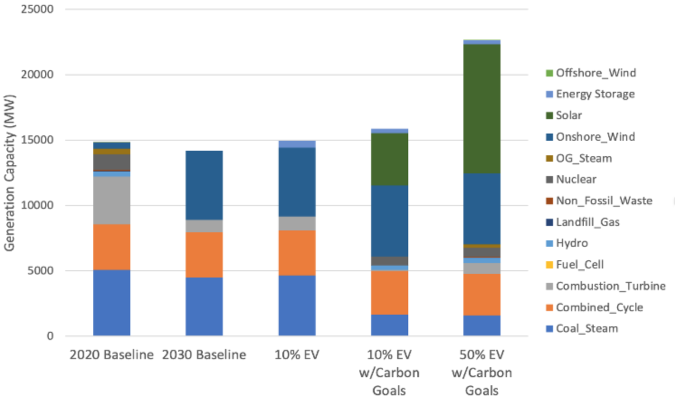
		Type I market		*Type II* market	
quad	λ	PD-CC-PATH	Homot	PD-CC-PATH	Homot
0	0.1	100.0	100.0	100.0	100.0
0	0.3	96.9	100.0	100.0	100.0
0	0.5	71.9	90.6	71.9	87.5
0	0.7	18.8	53.1	31.2	50.0
0	0.9	9.4	21.9	9.4	12.5
1e-2	0.1	100.0	100.0	100.0	100.0
1e-2	0.3	100.0	100.0	100.0	100.0
1e-2	0.5	100.0	100.0	100.0	100.0
1e-2	0.7	84.4	93.8	96.9	100.0
1e-2	0.9	53.1	68.8	65.6	81.2

Large pumped storage investment: Lake Onslow

Technology	Without			With		
	SI	HAY	NI	SI	HAY	NI
ONSLOW	0.0	0.0	0.0	1000.0	0.0	0.0
SLOWBATT	500.0	500.0	500.0	0.0	500.0	500.0
WIND	0.0	2049.9	5000.0	0.0	1407.4	5000.0

- Worried about the effects of dry winters and excess wind capacity
- Pumped storage costs amortized over long period
- Economical if emissions constraint is strict enough (i.e. no more than 5% of 2017 levels)
- Remove large battery in SI, reduce wind capacity at HAY

Impact of Electric Vehicles on Generator Investments



- Carbon Goals: 60% reduction on in-state carbon emissions
- Nuclear (low-carbon) used
- Coal steam generators shut down, supplanted by renewables
- Additional 180,000 MWh demand for EVs
- Storage investment needed
- Additional demand or carbon goals give more dramatic effects

A mathematical modelling approach to planning

- Build and solve a social plannning model that optimizes electricity capacity investment with constraints on CO2 emissions.
- Social planning solution should be stochastic: i.e. account for future uncertainty
- Social planning solution should be risk-averse: because the industry is.
- Approximate the outcomes of the social plan by a competitive equilibrium with risk-averse investors.
- Compensate for market failures from imperfect competition or incomplete markets.