Optimization, equilibria, energy and risk

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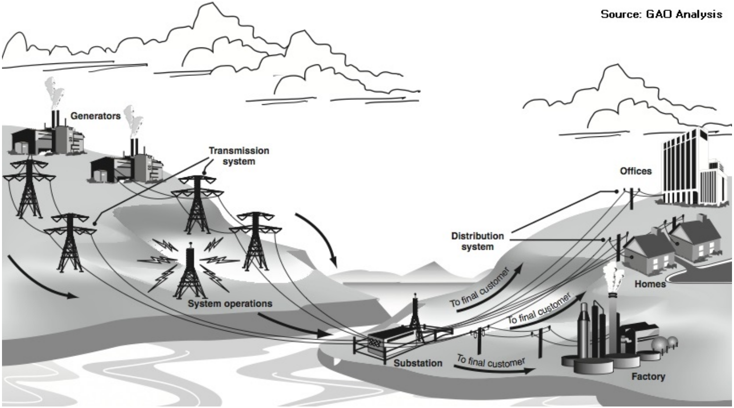
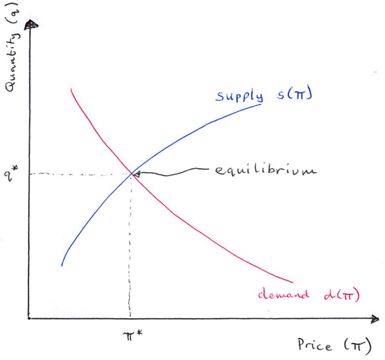
Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $\geq \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

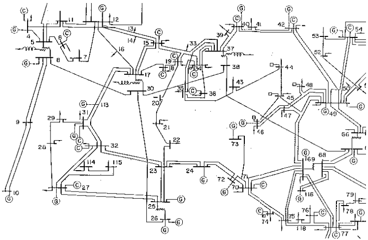
Single market, single good: equilibrium



Walras: $0 \leq s(\pi) - d(\pi) \perp \pi \geq 0$

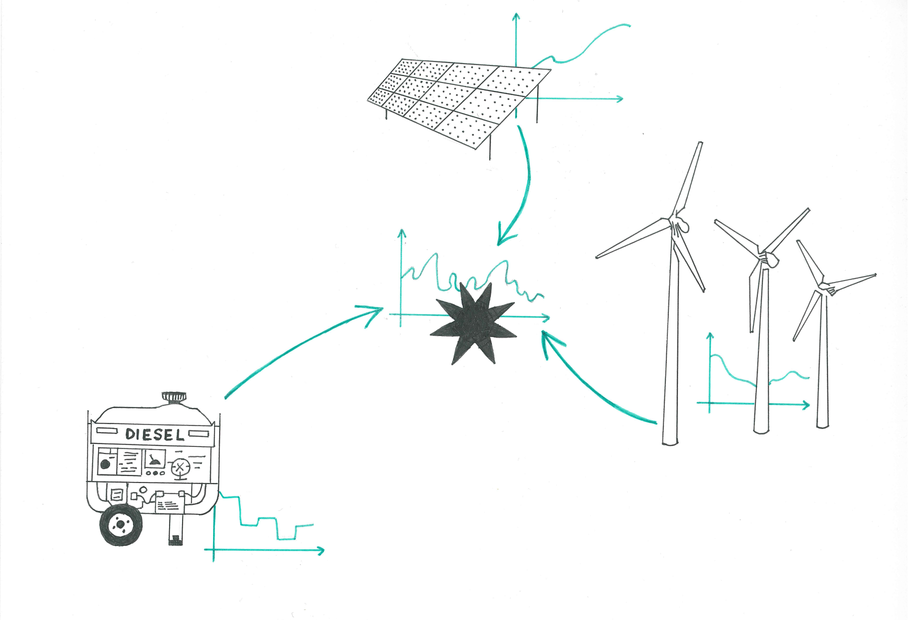
Market design and rules to foster competitive behavior/efficiency

- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network



- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution

The setup: agents a =(solar, wind, diesel, consumer)



A (competitive) equilibrium

$$u_a \text{ solves AO}(a,\pi): \quad \min_{u_a \in \mathcal{U}_{a,\pi}} C_{a,\pi}(u_a)$$

and

$$0 \le \sum_{a \in \mathcal{A}} g_a(u_a(n)) \perp \pi(n) \ge 0$$

- Actions u_a (dispatch, curtail, generate, shed), with costs C_a
- One optimization per agent, coupled with solution of complementarity (equilibrium) constraint: g_a converts actions into energy
- Overall, a Nash Equilibrium problem (or a MOPEC), solvable as a large scale complementarity problem (replacing all the optimization problems by their KKT conditions) using the PATH solver
- Model to understand behaviour of (rational) agents assuming price taking (π) behavior
- What is the gold standard?

System Optimization

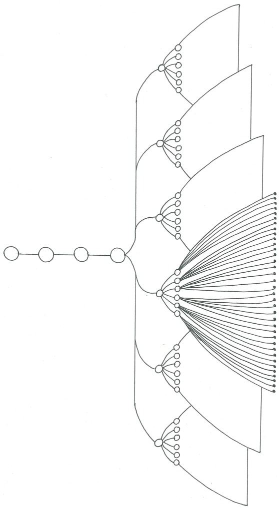
$$\begin{aligned} \text{SO:} \quad \min_u \quad & \sum_{a \in \mathcal{A}} C_a(u_a) \\ \text{s.t.} \quad & \sum_{a \in \mathcal{A}} g_a(u_a(n)) \geq 0 \\ & u_a \in \mathcal{U}_a \end{aligned}$$

- Lagrangian theory shows MOPEC is equivalent to SO under behavioral assumptions (perfectly competitive) and some standard technical assumptions
- Could use as a counter-factual to determine if agents are in practice acting perfectly competitively
- So what's the issue?

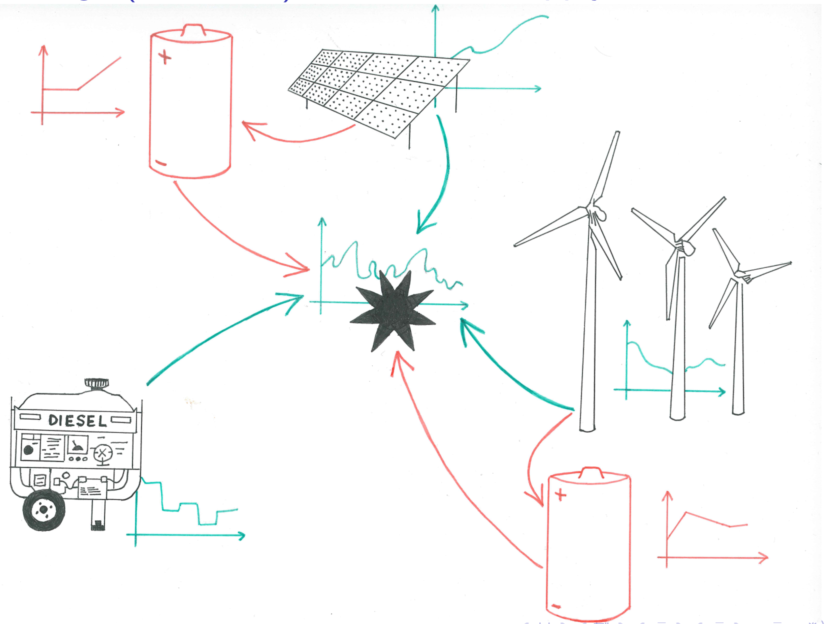
There's more: dynamics and uncertainties

- Lousy solution - no transfer of energy across time: need dynamics
- Storage allows energy to be moved across stages (batteries, pump, compressed air, etc)
- Power distribution not modeled (single consumer location)
- Uncertainties (wind flow, cloud cover, rainfall, demand) $\omega_a(n)$
- Scenario tree is data
- Nodes $n \in \mathcal{N}$, n_+ successors
- State and shared variables (storage, prices)

T stages (e.g. $t \in 0, 1, 2, 3, 4, 5, 6$)



Add storage (smoother) to uncertain supply



The Philpott bach problem

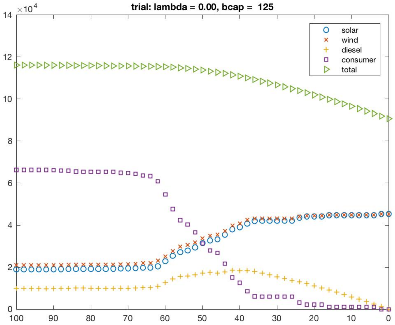
Solar panels:

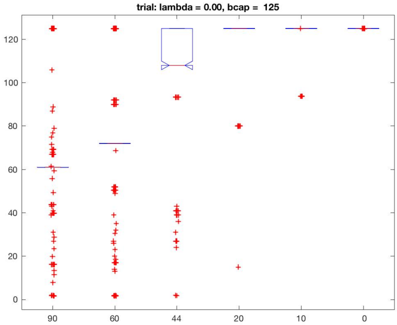
Battery:

Petrol generator:

Pump storage:

Example: 100% renewables

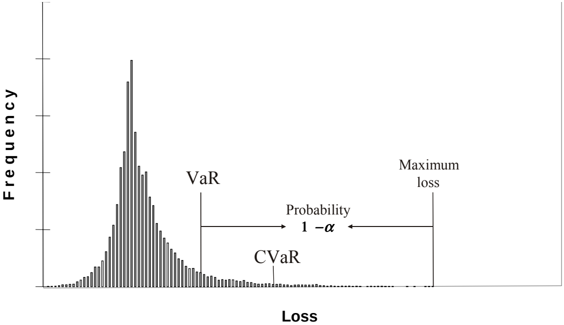




- Motivated by european/US model about batteries
- Allows direct delivery of renewables, models efficiency of charging, and uses installed capacities
- Using risk neutral (expectation) optimization

Risk modeling

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- Dual representation in terms of risk sets: $\mathcal{D}$
- Different agents have different risk profiles
- Recursive (nested) definition of expected cost-to-go

Risk averse equilibrium

Replace each agents problem by:

$$\mathrm{AO}(a, \pi, \mathcal{D}_a): \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad Z_a(0) + \theta_a(0)$$

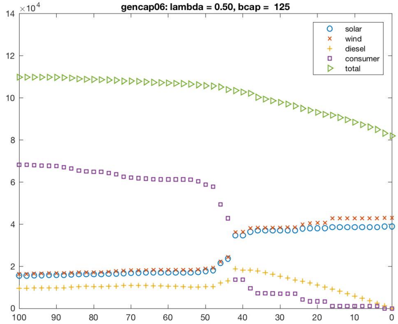
$$\text{s.t.} \quad x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n)$$

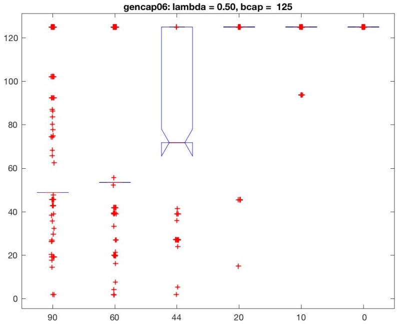
$$\theta_a(n) \geq \sum_{m\in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n)$$

$$Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n))$$

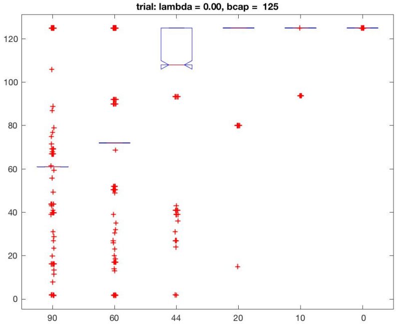
- $p_a^k(m)$ are extreme points of the agents risk set at m
- No longer system optimization
- Must solve using complementarity solver
- Need new techniques to treat stochastic optimization problems within equilibrium

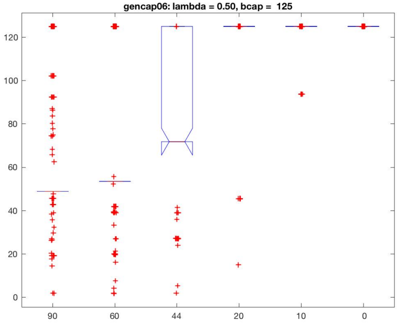
Risk averse ($\lambda = 0.5$) case: 100% renewables





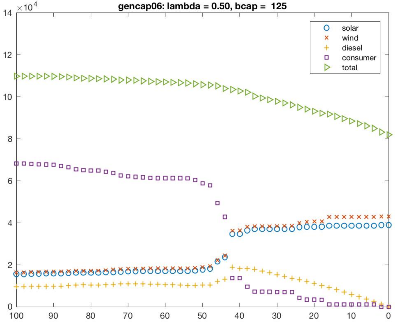
Prices: different values of risk

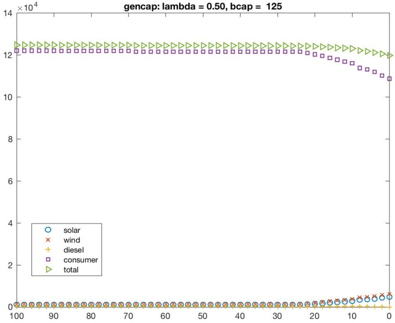




- Right hand figure increases risk aversion to $\lambda = 0.5$
- Equilibrium prices generally lower (different equilibria!)

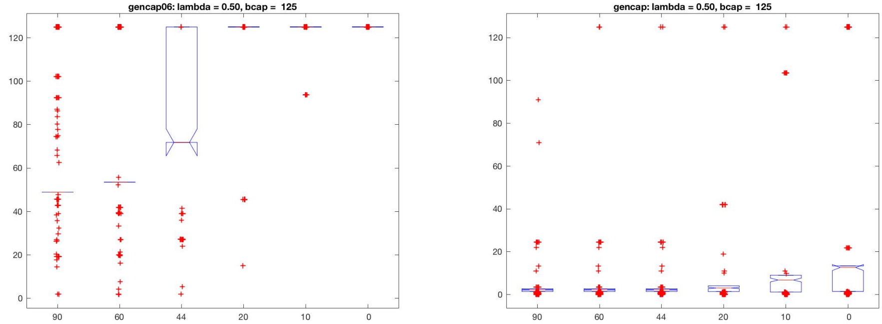
Increase (capture) capability of renewables





- Right hand figure increases capture capability by 50%
- Welfare increasing but not counting infrastructure cost
- May need smaller increase so can pay for investment (see later)

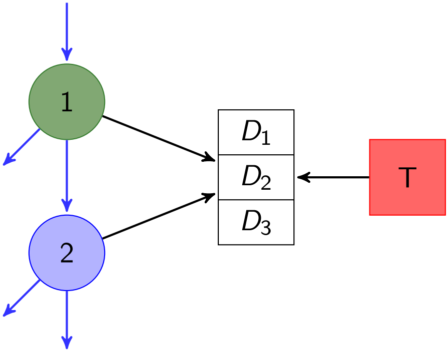
Prices: increased capture capability of renewables



- Right hand figure increases capture capability by 50%
- Distribution of prices at selected decreasing levels of thermal generation capacity

Cascading hydro-thermal system: XMGD

- Two hydros on same river: '1' is above '2': spill or release with generation
- Thermal generator 'T' and consumer (risk neutral)



- Competing firms (collections of consumers, or generators in energy market)
- Each firm minimizes objective independently
- Look at joint ownership issues (firms represented colors: X, M, G)
- Label consumer as 'D' (but can be partitioned into 'D_1','D_2','D_3')

Equilibria with cascades: water prices

T_{ab} encodes the water network, water prices are multipliers on:

$$x_a(n_-) + \sum_b T_{ab} u_b(n) + \omega_a(n) \geq x_a(n)$$

Allows interaction with other water uses (irrigation, tourism, conservation)

$$\text{AO}(a, \pi, \mathcal{D}_a): \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad Z_a(0) + \theta_a(0)$$

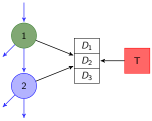
$$\text{s.t. } \theta_a(n) \geq \sum_{m\in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n)$$

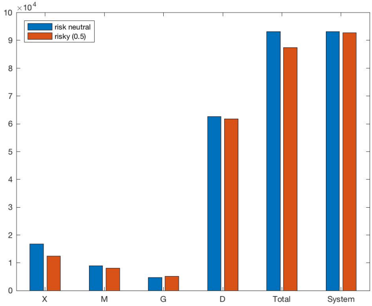
where $Z_a(n)$ is updated to incorporate prices of interactions

$$Z_a(n; u, x) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) + \alpha_a(n)\,(x_a(n) - x_a(n_-) - \omega_a(n)) - \sum_{b\in\mathcal{A}} \alpha_b(n)\, T_{ba} u_a(n),$$

Average inflow 0.6

XMGD
$\text{TotRA} = 87351$
$\text{SysRA} = 92763$
$\text{SysRN} = 93109$





MMGD
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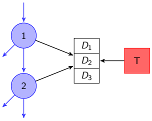
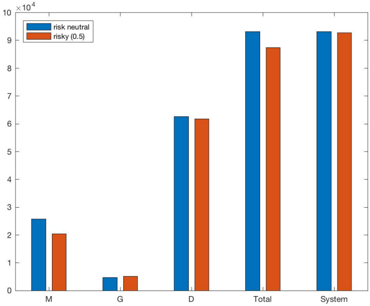




- Ownership of both hydros is not beneficial with competitive pricing of water

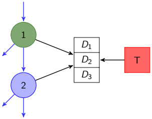
Low inflow 0.1

XMGD

TotRA = 62382

SysRA = 65269

SysRN = 65375

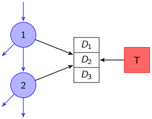


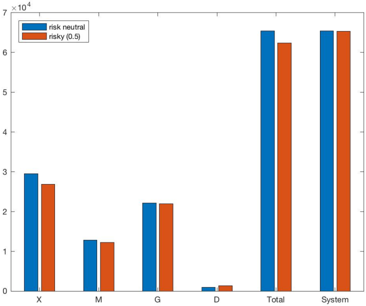
MMGD

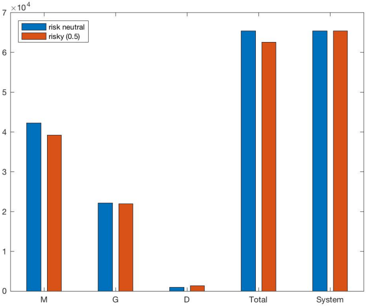
TotRA = 62552

SysRA = 65371

SysRN = 65375



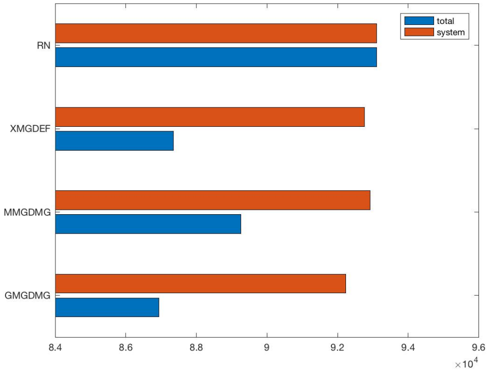




- Not true: risk averse and low inflows shows advantage to co-ownership of hydros

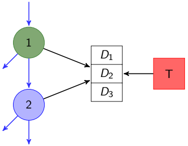
Vertical integration/asset swaps

- SysRN and TotRN in risk neutral case, followed by SysRA and TotRA for three cases depicted on left

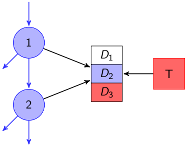


- Vertical integration and risk matters!

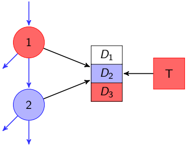
Base: XMGDEF



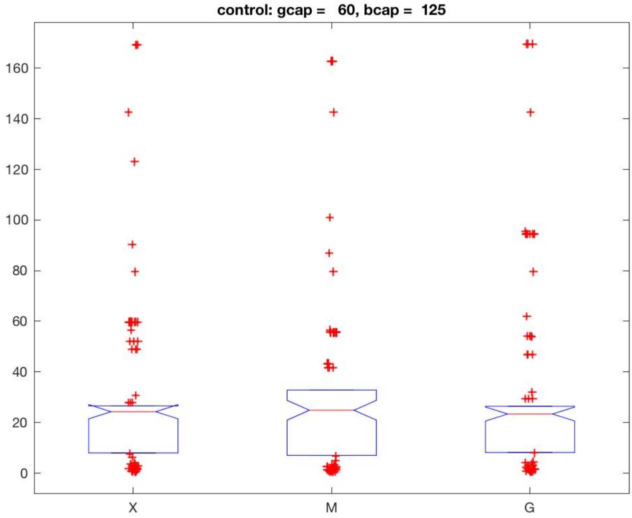
Vertical integration: MMGDMG



VI & Asset Swap: GMGDMG



XMGDEF/MMGDMG/GMGDMG (water price differences)



Equilibrium or optimization?

Theorem

If (u, θ) solves $SO(\mathcal{D}_s)$, then there is a probability distribution $(\bar{\mu}(n), n \in \mathcal{N})$ and prices $(\pi(n), n \in \mathcal{N})$ so that defining $\mathcal{D}_a = \{\bar{\mu}\}$ for all $a \in \mathcal{A}$, (u, π) solves $SE(\mathcal{D}_\mathcal{A})$. That is, the social plan is decomposable into a risk-neutral multi-stage stochastic optimization problem for each agent, with coupling via complementarity constraints.

(Observe that each agent must maximize their own expected profit using probabilities $\bar{\mu}$ that are derived from identifying the worst outcomes as measured by SO. These will correspond to the worst outcomes for each agent only under very special circumstances)

- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Given any node n, an *Arrow-Debreu security* for node $m \in n_+$ is a contract that charges a price $\mu(m)$ in node $n \in \mathcal{N}$, to receive a payment of 1 in node $m \in n_+$.
- Conceptually allows to transfer money from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Such contracts complete the market (RTE)

$$\text{AO}_a(\pi, \mu, \mathcal{D}_a): \min_{(\theta, Z, x, u, W) \in \mathcal{F}(\omega)} Z_a(0) + \theta_a(0)$$

$$\text{s.t. } \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m) - W_a(m)), k \in K(n)$$

$$Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) + \sum_{m \in n_+} \mu(m) W_a(m)$$

coupled to clearing of energy, (water) and contracts

$$0 \leq -\sum_{a \in \mathcal{A}} W_a(n) \perp \mu(n) \geq 0$$

Theorem

Consider agents $a \in \mathcal{A}$, with risk sets $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$. Let (u, θ) solve $SO(\mathcal{D}_s)$ with risk sets $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$. There exist prices $(\bar{\pi}(n), n \in \mathcal{N})$ and $(\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\})$ and actions $\bar{u}_a(n), n \in \mathcal{N}$, $\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}$ that form a multistage risk-trading equilibrium $RTE(\mathcal{D}_{\mathcal{A}})$.

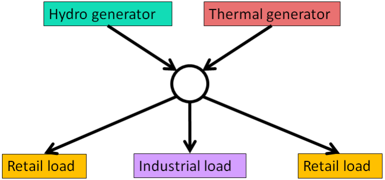
Conversely...

Theorem

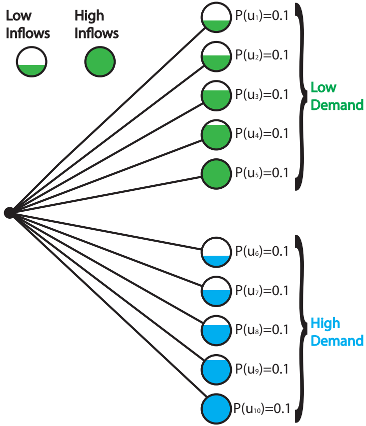
Consider a set of agents $a \in \mathcal{A}$*, each endowed with a polyhedral node-dependent risk set* $\mathcal{D}_a(n)$*,* $n \in \mathcal{N} \setminus \mathcal{L}$*. Suppose* $(\bar{\pi}(n), n \in \mathcal{N})$ *and* $(\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\})$ *form a multistage risk-trading equilibrium* $RTE(\mathcal{D}_{\mathcal{A}})$ *in which agent a solves* $AO_a(\bar{\pi}, \bar{\mu}, \mathcal{D}_a)$ *with a policy defined by* $\bar{u}_a(\cdot)$ *together with a policy of trading Arrow-Debreu securities defined by* $\{\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}\}$*. Then* $(\bar{u}, \bar{\theta})$ *is a solution to* $SO(\mathcal{D}_s)$ *with risk sets* $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$*, where* $\bar{\theta}$ *is defined recursively (above) with* $\mu_\sigma = \bar{\mu}$ *and* $u_a(n) = \bar{u}_a(n)$*.*

- In battery problem can recover by trading the system optimal solution (and its properties) since the retailer/generator agent is risk neutral
- Both theorems are generalizable to the water pricing setting too!

Risked capacity expansion (Corey Kok PhD thesis)

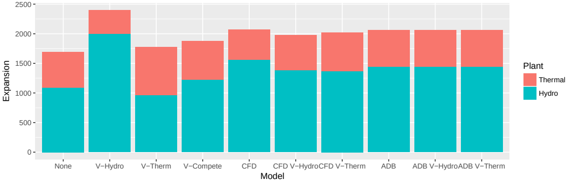


Agents in the model

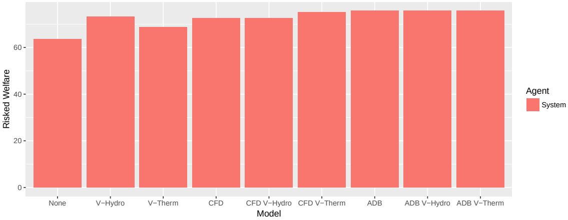


Scenarios for capacity expansion model

Expansion decisions and their social welfare

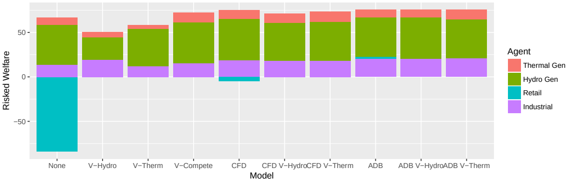


Expansion decisions

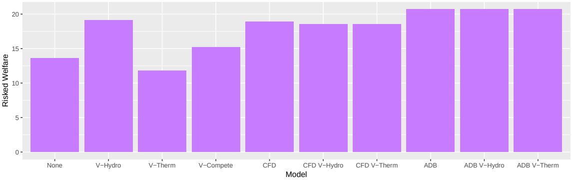


Total risk adjusted welfare

Risk adjusted welfare of agents



Risk adjusted total social welfare



Risk adjusted welfare of industrial agent

Results generated using GAMS/EMP

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- dualvar (use multipliers from one agent as variables for another)
- implicit functions and shared constraints

- Currently available within GAMS
- Some solution algorithms implemented in modeling system - limitations on size, decomposition and advanced algorithms

Conclusions/Questions/Comments

- Risk matters!
- Optimization guides the development of complex interaction processes within application domains
- Combination of models (including transmission) provides effective decision tool at multiple scales
- Problems solved by combination of domain expertise, modeling prowess, good theory/algorithms and efficient implementations: all facets needed
- Policy implications addressable using optimization and complementarity
- Can evaluate effects of regulations and their implementation in a competitive environment
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements