CS 536

Introduction to Programming Languages and Compilers

Charles N. Fischer

Fall 2002

http://www.cs.wisc.edu/~fischer/cs536.html
Class Meets

Lecture 1:
Mondays, Wednesdays & Fridays,
9:55 — 10:45
1221 Computer Sciences

Lecture 2:
Mondays, Wednesdays & Fridays,
1:20 — 2:10
1325 Computer Sciences

Recitation (both sections):
Tuesdays,
2:25 — 3:15
113 Psychology
Instructor

Charles N. Fischer
5397 Computer Sciences
Telephone: 262-6635
E-mail: fischer@cs.wisc.edu
Office Hours:

10:30 - Noon, Tuesdays & Thursdays, or by appointment
Teaching Assistants

- David Mulvihill
  3310 Computer Sciences
  Telephone: 262-1721
  E-mail: mulvihil@cs.wisc.edu
  Office Hours:
    Tuesday: 1:00 - 3:00pm
    Thursday: 1:00 - 3:00pm

- Supanida Arayametee
  1302 Computer Sciences
  Telephone: 262-6600
  E-mail: supanida@cs.wisc.edu
  Office Hours:
    Monday: 2:00 - 4:00pm
    Wednesday: 2:00 - 4:00pm
Key Dates

- September 17: Assignment #1 (Symbol Table Routines)
- October 7: Assignment #2 (CSX Scanner)
- October 28: Assignment #3 (CSX Parser)
- October 30: Midterm Exam (7-9 pm)
- November 20: Assignment #4 (CSX Type Checker)
- December 13: Assignment #5 (CSX Code Generator)
- December 18: Final Exam 12:25 pm-2:25 pm
Class Text

- Draft Chapters from:
  Crafting a Compiler featuring Java.
  (Available from Dolt Tech Store)
- Handouts and Web-based reading will also be used.

Reading Assignment

- Chapters 1-2 of CaC (as background)

Class Notes

- The transparencies used for each lecture will be made available prior to, and after, that lecture on the class Web page (under the “Lecture Nodes” link).
Instructional Computers

Departmental Unix Machines (nova1-nova60) have been assigned to CS 536. All necessary compiler, tools are loaded onto these machines.

You may also use your own PC or workstation. It will be your responsibility to load needed software (instructions on where to find needed software are included on the class web page).

The Systems Lab teaches brief tutorials on Unix if you are unfamiliar with that OS.
Academic Misconduct Policy

• You must do your assignments—NO copying or sharing of solutions.
• You may discuss general concepts and Ideas.
• All cases of Misconduct must be reported to the Dean’s office.
• Penalties may be severe.
Program & Homework Late Policy

• An assignment may be handed in up to one week late.
• Each late day will be debited 3%, up to a maximum of 21%.

Approximate Grade Weights

Program 1 - Symbol Tables 6%
Program 2 - Scanner 11%
Program 3 - Parser 11%
Program 4 - Type Checker 13%
Program 5 - Code Generator 13%
Homework 1 9%
Midterm Exam 19%
Final Exam (non-cumulative) 18%
Partnership Policy

- Program #1 and the written homework must be done individually.

- For undergraduates, programs 2 to 5 may be done individually or by two person teams (your choice). Graduate students must do all assignments individually.
Compilers

Compilers are fundamental to modern computing. They act as translators, transforming human-oriented programming languages into computer-oriented machine languages.

To most users, a compiler can be viewed as a “black box” that performs the transformation shown below.
A compiler allows programmers to ignore the machine-dependent details of programming.

Compilers allow programs and programming skills to be machine-independent.

Compilers also aid in detecting programming errors (which are very common).

Compiler techniques can also help to improve computer security. For example, the Java Bytecode Verifier helps to guarantee that Java security rules are satisfied.
History of Compilers

The term compiler was coined in the early 1950s by Grace Murray Hopper. Translation was viewed as the “compilation” of a sequence of machine-language subprograms selected from a library.

Among the first real compilers were the FORTRAN compilers of the late 1950s. They allowed the programmer to use a problem-oriented, largely machine-independent source language.

Rather ambitious “optimizations” were used to produce efficient machine code, which was vital for early computers with quite limited capabilities.
Efficient use of machine resources is still an essential requirement for modern compilers.
Compilers Enable Programming Languages

Programming languages are used for much more than “ordinary” computation.

- TeX and LaTeX use compilers to translate text and formatting commands into intricate typesetting commands.

- Postscript, generated by text-formatters like LaTeX, Word, and FrameMaker, is really a programming language. It is translated and executed by laser printers and document previewers to produce a readable form of a document. A standardized document representation language allows
documents to be freely interchanged, independent of how they were created and how they will be viewed.

- Mathematica is an interactive system that intermixes programming with mathematics; it is possible to solve intricate problems in both symbolic and numeric form. This system relies heavily on compiler techniques to handle the specification, internal representation, and solution of problems.

- Verilog and VHDL support the creation of VLSI circuits. A silicon compiler specifies the layout and composition of a VLSI circuit mask, using standard cell designs. Just as an ordinary compiler understands and enforces programming language
rules, a silicon compiler understands and enforces the design rules that dictate the feasibility of a given circuit.

- Interactive tools often need a programming language to support automatic analysis and modification of an artifact.

How do you automatically filter or change a MS Word document? You need a text-based specification that can be processed, like a program, to check validity or produce an updated version.
When do We Run a Compiler?

- Prior to execution
  This is standard. We compile a program once, then use it repeatedly.

- At the start of each execution
  We can incorporate values set at the start of the run to improve performance. A program may be partially complied, then completed with values set at execution-time.

- During execution
  Unused code need not be compiled. Active or “hot” portions of a program may be specially optimized.

- After execution
  We can profile a program, looking for heavily used routines, which can be specially optimized for later runs.
What do Compilers Produce?

Pure Machine Code

Compilers may generate code for a particular machine, not assuming any operating system or library routines. This is “pure code” because it includes nothing beyond the instruction set. This form is rare; it is sometimes used with system implementation languages, that define operating systems or embedded applications (like a programmable controller). Pure code can execute on bare hardware without dependence on any other software.
Augmented Machine Code

Commonly, compilers generate code for a machine architecture augmented with operating system routines and run-time language support routines. To use such a program, a particular operating system must be used and a collection of run-time support routines (I/O, storage allocation, mathematical functions, etc.) must be available. The combination of machine instruction and OS and run-time routines define a virtual machine—a computer that exists only as a hardware/software combination.
Virtual Machine Code

Generated code can consist entirely of virtual instructions (no native code at all). This supports transportable code, that can run on a variety of computers.

Java, with its JVM (Java Virtual Machine) is a great example of this approach.

If the virtual machine is kept simple and clean, the interpreter can be quite easy to write. Machine interpretation slows execution speed by a factor of 3:1 to perhaps 10:1 over compiled code.

A “Just in Time” (JIT) compiler can translate “hot” portions of virtual code into native code to speed execution.
Advantages of Virtual Instructions

Virtual instructions serve a variety of purposes.

• They simplify a compiler by providing suitable primitives (such as procedure calls, string manipulation, and so on).

• They contribute to compiler transportability.

• They may decrease in the size of generated code since instructions are designed to match a particular programming language (for example, JVM code for Java).

Almost all compilers, to a greater or lesser extent, generate code for a virtual machine, some of whose operations must be interpreted.
Formats of Translated Programs

Compilers differ in the format of the target code they generate. Target formats may be categorized as assembly language, relocatable binary, or memory-image.

- Assembly Language (Symbolic) Format

A text file containing assembler source code is produced. A number of code generation decisions (jump targets, long vs. short address forms, and so on) can be left for the assembler. This approach is good for instructional projects. Generating assembler code supports cross-compilation (running a compiler on one computer, while its target is a second computer). Generating
assembly language also simplifies debugging and understanding a compiler (since you can see the generated code).

C rather than a specific assembly language can generated, using C as a “universal assembly language.” C is far more machine-independent than any particular assembly language. However, some aspects of a program (such as the run-time representations of program and data) are inaccessible using C code, but readily accessible in assembly language.

- Relocatable Binary Format

Target code may be generated in a binary format with external references and local instruction and data addresses are not yet bound. Instead,
addresses are assigned relative to the beginning of the module or relative to symbolically named locations. A linkage step adds support libraries and other separately compiled routines and produces an absolute binary program format that is executable.

- Memory-Image (Absolute Binary) Form

Compiled code may be loaded into memory and immediately executed. This is faster than going through the intermediate step of link/editing. The ability to access library and precompiled routines may be limited. The program must be recompiled for each execution. Memory-image compilers are useful for student and debugging use, where frequent
changes are the rule and compilation costs far exceed execution costs. Java is designed to use and share classes defined and implemented at a variety of organizations. Rather than use a fixed copy of a class (which may be outdated), the JVM supports dynamic linking of externally defined classes. When first referenced, a class definition may be remotely fetched, checked, and loaded during program execution. In this way “foreign code” can be guaranteed to be up-to-date and secure.
The Structure of a Compiler

A compiler performs two major tasks:

- Analysis of the source program being compiled
- Synthesis of a target program

Almost all modern compilers are syntax-directed: The compilation process is driven by the syntactic structure of the source program.

A parser builds semantic structure out of tokens, the elementary symbols of programming language syntax. Recognition of syntactic structure is a major part of the analysis task.

Semantic analysis examines the meaning (semantics) of the program. Semantic analysis plays a dual role.
It finishes the analysis task by performing a variety of correctness checks (for example, enforcing type and scope rules). Semantic analysis also begins the synthesis phase.

The synthesis phase may translate source programs into some intermediate representation (IR) or it may directly generate target code.

If an IR is generated, it then serves as input to a code generator component that produces the desired machine-language program. The IR may optionally be transformed by an optimizer so that a more efficient program may be generated.
The Structure of a Syntax-Directed Compiler
Scanner

The scanner reads the source program, character by character. It groups individual characters into tokens (identifiers, integers, reserved words, delimiters, and so on). When necessary, the actual character string comprising the token is also passed along for use by the semantic phases.

The scanner does the following:

• It puts the program into a compact and uniform format (a stream of tokens).

• It eliminates unneeded information (such as comments).

• It sometimes enters preliminary information into symbol tables (for
example, to register the presence of a particular label or identifier).

- It optionally formats and lists the source program

Building tokens is driven by token descriptions defined using regular expression notation.

Regular expressions are a formal notation able to describe the tokens used in modern programming languages. Moreover, they can drive the automatic generation of working scanners given only a specification of the tokens. Scanner generators (like Lex, Flex and JLex) are valuable compiler-building tools.
Parser

Given a syntax specification (as a context-free grammar, CFG), the parser reads tokens and groups them into language structures.

Parsers are typically created from a CFG using a parser generator (like Yacc, Bison or Java CUP).

The parser verifies correct syntax and may issue a syntax error message.

As syntactic structure is recognized, the parser usually builds an abstract syntax tree (AST), a concise representation of program structure, which guides semantic processing.
Type Checker (Semantic Analysis)

The type checker checks the static semantics of each AST node. It verifies that the construct is legal and meaningful (that all identifiers involved are declared, that types are correct, and so on).

If the construct is semantically correct, the type checker “decorates” the AST node, adding type or symbol table information to it. If a semantic error is discovered, a suitable error message is issued.

Type checking is purely dependent on the semantic rules of the source language. It is independent of the compiler’s target machine.
If an AST node is semantically correct, it can be translated. Translation involves capturing the run-time “meaning” of a construct.

For example, an AST for a while loop contains two subtrees, one for the loop’s control expression, and the other for the loop’s body. Nothing in the AST shows that a while loop loops! This “meaning” is captured when a while loop’s AST is translated. In the IR, the notion of testing the value of the loop control expression, and conditionally executing the loop body becomes explicit.

The translator is dictated by the semantics of the source language.
Little of the nature of the target machine need be made evident. Detailed information on the nature of the target machine (operations available, addressing, register characteristics, etc.) is reserved for the code generation phase.

In simple non-optimizing compilers (like our class project), the translator generates target code directly, without using an IR.

More elaborate compilers may first generate a high-level IR (that is source language oriented) and then subsequently translate it into a low-level IR (that is target machine oriented). This approach allows a cleaner separation of source and target dependencies.
Optimizer

The IR code generated by the translator is analyzed and transformed into functionally equivalent but improved IR code by the optimizer.

The term optimization is misleading: we don’t always produce the best possible translation of a program, even after optimization by the best of compilers.

Why?

Some optimizations are impossible to do in all circumstances because they involve an undecidable problem. Eliminating unreachable (“dead”) code is, in general, impossible.
Other optimizations are too expensive to do in all cases. These involve NP-complete problems, believed to be inherently exponential. Assigning registers to variables is an example of an NP-complete problem.

Optimization can be complex; it may involve numerous subphases, which may need to be applied more than once.

Optimizations may be turned off to speed translation. Nonetheless, a well designed optimizer can significantly speed program execution by simplifying, moving or eliminating unneeded computations.
Code Generator

IR code produced by the translator is mapped into target machine code by the code generator. This phase uses detailed information about the target machine and includes machine-specific optimizations like register allocation and code scheduling.

Code generators can be quite complex since good target code requires consideration of many special cases.

Automatic generation of code generators is possible. The basic approach is to match a low-level IR to target instruction templates, choosing instructions which best match each IR instruction.

A well-known compiler using automatic code generation
techniques is the GNU C compiler. GCC is a heavily optimizing compiler with machine description files for over ten popular computer architectures, and at least two language front ends (C and C++).
Symbol Tables

A symbol table allows information to be associated with identifiers and shared among compiler phases. Each time an identifier is used, a symbol table provides access to the information collected about the identifier when its declaration was processed.
Example

Our source language will be CSX, a blend of C, C++ and Java.

Our target language will be the Java JVM, using the Jasmin assembler.

• Our source line is
  \[ a = bb + \text{abs}(c-7); \]
  this is a sequence of ASCII characters in a text file.

• The scanner groups characters into tokens, the basic units of a program.
  \[ a = bb + \text{abs}(c-7); \]
  After scanning, we have the following token sequence:
  \[ \text{Id}_a \text{ Asg } \text{Id}_{bb} \text{ Plus } \text{Id}_{\text{abs}} \text{ Lparen } \text{Id}_c \text{ Minus } \text{IntLiteral}_7 \text{ Rparen } \text{Semi} \]
The parser groups these tokens into language constructs (expressions, statements, declarations, etc.) represented in tree form:

(What happened to the parentheses and the semicolon?)
The type checker resolves types and binds declarations within scopes:
Finally, JVM code is generated for each node in the tree (leaves first, then roots):

```
iload 3 ; push local 3 (bb)
iload 2 ; push local 2 (c)
ldc 7  ; Push literal 7
isub   ; compute c-7
invokestatic java/lang/Math/abs(I)I
iadd   ; compute bb+abs(c-7)
istore 1 ; store result into local 1(a)
```
Interpreters

There are two different kinds of interpreters that support execution of programs, machine interpreters and language interpreters.

Machine Interpreters

A machine interpreter simulates the execution of a program compiled for a particular machine architecture. Java uses a bytecode interpreter to simulate the effects of programs compiled for the JVM. Programs like SPIM simulate the execution of a MIPS program on a non-MIPS computer.
Language Interpreters

A language interpreter simulates the effect of executing a program without compiling it to any particular instruction set (real or virtual). Instead some IR form (perhaps an AST) is used to drive execution.

Interpreters provide a number of capabilities not found in compilers:

- Programs may be modified as execution proceeds. This provides a straightforward interactive debugging capability. Depending on program structure, program modifications may require reparsing or repeated semantic analysis. In Python, for example, any string variable may be interpreted as a Python expression or statement and executed.
- Interpreters readily support languages in which the type of a variable denotes may change dynamically (e.g., Python or Scheme). The user program is continuously reexamined as execution proceeds, so symbols need not have a fixed type. Fluid bindings are much more troublesome for compilers, since dynamic changes in the type of a symbol make direct translation into machine code difficult or impossible.

- Interpreters provide better diagnostics. Source text analysis is intermixed with program execution, so especially good diagnostics are available, along with interactive debugging.

- Interpreters support machine independence. All operations are performed within the interpreter. To
move to a new machine, we just recompile the interpreter.

However, interpretation can involve large overheads:

- As execution proceeds, program text is continuously reexamined, with bindings, types, and operations sometimes recomputed at each use. For very dynamic languages this can represent a 100:1 (or worse) factor in execution speed over compiled code. For more static languages (such as C or Java), the speed degradation is closer to 10:1.

- Startup time for small programs is slowed, since the interpreter must be load and the program partially recompiled before execution begins.
Substantial space overhead may be involved. The interpreter and all support routines must usually be kept available. Source text is often not as compact as if it were compiled. This size penalty may lead to restrictions in the size of programs. Programs beyond these built-in limits cannot be handled by the interpreter.

Of course, many languages (including, C, C++, and Java) have both interpreters (for debugging and program development) and compilers (for production work).
Symbol Tables & Scoping

Programming languages use scopes to limit the range an identifier is active (and visible).

Within a scope a name may be defined only once (though overloading may be allowed).

A symbol table (or dictionary) is commonly used to collect all the definitions that appear within a scope.

At the start of a scope, the symbol table is empty. At the end of a scope, all declarations within that scope are available within the symbol table.

A language definition may or may not allow forward references to an identifier.
If forward references are allowed, you may use a name that is defined later in the scope (Java does this for field and method declarations within a class).

If forward references are not allowed, an identifier is visible only after its declaration. C, C++ and Java do this for variable declarations.

In CSX no forward references are allowed.

In terms of symbol tables, forward references require two passes over a scope. First all declarations are gathered. Next, all references are resolved using the complete set of declarations stored in the symbol table.
If forward references are disallowed, one pass through a scope suffices, processing declarations and uses of identifiers together.
Block Structured Languages

- Introduced by Algol 60, includes C, C++, CSX and Java.
- Identifiers may have a non-global scope. Declarations may be local to a class, subprogram or block.
- Scopes may nest, with declarations propagating to inner (contained) scopes.
- The lexically nearest declaration of an identifier is bound to uses of that identifier.
Example (drawn from C):

```c
int x, z;
void A() {
    float x, y;
    print(x, y, z);
}
void B() {
    print(x, y, z)
}
```
Block Structure Concepts

- Nested Visibility
  No access to identifiers outside their scope.
- Nearest Declaration Applies
  Using static nesting of scopes.
- Automatic Allocation and Deallocation of Locals
  Lifetime of data objects is bound to the scope of the Identifiers that denote them.
Block-Structured Symbol Tables

Block structured symbol tables are designed to support the scoping rules of block structured languages. For our CSX project we’ll use class Symb to represent symbols and SymbolTable to implemented operations needed for a block-structured symbol table.

Class Symb will contain a method

   public String name()

that returns the name associated with a symbol.
Class `SymbolTable` contains the following methods:

- `public void openScope()` {
  A new and empty scope is opened.
}

- `public void closeScope() throws EmptySTException`
  The innermost scope is closed. An exception is thrown if there is no scope to close.

- `public void insert(Symb s) throws DuplicateException, EmptySTException`
  A `Symb` is inserted in the innermost scope. An exception is thrown if a `Symb` with the same name is already in the innermost scope or if there is no symbol table to insert into.
• public Symb localLookup(String s)
The innermost scope is searched for a Symb whose name is equal to s. Null is returned if no Symb named s is found.

• public Symb globalLookup(String s)
All scopes, from innermost to outermost, are searched for a Symb whose name is equal to s. The first Symb that matches s is found; otherwise null is returned if no matching Symb is found.
Is Case Significant?

In some languages (C, C++, Java and many others) case is significant in identifiers. This means `aa` and `AA` are different symbols that may have entirely different definitions.

In other languages (Pascal, Ada, Scheme, CSX) case is not significant. In such languages `aa` and `AA` are two alternative spellings of the same identifier.

Data structures commonly used to implement symbol tables usually treat different cases as different symbols. This is fine when case is significant in a language. When case is insignificant, you probably will need to strip case before entering or looking up identifiers.
This just means that identifiers are converted to a uniform case before
they are entered or looked up. Thus if we choose to use lower case
uniformly, the identifiers aaaa, AAA, and AaA are all converted to aaaa for
purposes of insertion or lookup.
BUT, inside the symbol table the identifier is stored in the form it was
declared so that programmers see the form of identifier they expect in
listings, error messages, etc.
How are Symbol Tables Implemented?

There are a number of data structures that can reasonably be used to implement a symbol table:

- **An Ordered List**
  Symbols are stored in a linked list, sorted by the symbol’s name. This is simple, but may be a bit too slow if many identifiers appear in a scope.

- **A Binary Search Tree**
  Lookup is much faster than in a linked list, but rebalancing may be needed. (Entering identifiers in sorted order can turn a search tree into a linked list.)

- **Hash Tables**
  The most popular choice.
Implementing Block-Structured Symbol Tables

To implement a block structured symbol table we need to be able to efficiently open and close individual scopes, and limit insertion to the innermost current scope.

This can be done using one symbol table structure if we tag individual entries with a “scope number.”

It is far easier (but more wasteful of space) to allocate one symbol table for each scope. Open scopes are stacked, pushing and popping tables as scopes are opened and closed.

Be careful though—many preprogrammed stack implementations don’t allow you to
“peek” at entries below the stack top. This is necessary to lookup an identifier in all open scopes.

If a suitable stack implementation (with a peek operation) isn’t available, a linked list of symbol tables will suffice.
More on Hashtables

Hashtables are a very useful data structure. Java provides a predefined Hashtable class. Python includes a built-in dictionary type.

Every Java class has a hashCode method, which allows any object to be entered into a Java Hashtable.

For most objects, hash codes are pretty simple (the address of the corresponding object is often used).

But for strings Java uses a much more elaborate hash function:

$$\sum_{i=0}^{n-1} c_i \times 37^i$$

where $n$ is the length of the string, $c_i$ is the $i$-th character and all arithmetic is done without overflow checking.
Why such an elaborate hash function?

Simpler hash functions can have major problems.

\[ \sum_{i=0}^{n-1} c_i \] (add the characters).

For short identifiers the sum grows slowly, so large indices won’t often be used (leading to non-uniform use of the hash table).

\[ \prod_{i=0}^{n-1} c_i \] (product of characters), but now (surprisingly) the size of the hash table becomes an issue. The problem is that if even one character is encoded as an even number, the product must be even.
If the hash table is even in size (a natural thing to do), most hash table entries will be at even positions. Similarly, if even one character is encoded as a multiple of 3, the whole product will be a multiple of 3, so hash tables that are a multiple of three in size won’t be uniformly used.

To see how bad things can get, consider a hash table with size 210 (which is equal to $2 \times 3 \times 5 \times 7$). This should be a particularly bad table size if a product hash is used. (Why?)

Is it? As an experiment, all the words in the Unix spell checker’s dictionary (26000 words) where entered. Over 50% (56.7% actually) hit position 0 in the table!
Why such non-uniformity?
If an identifier contains characters that are multiples of 2, 3, 5 and 7, then their hash will be a multiple of 210 and will map to position 0.

For example, in Wisconsin, n has an ASCII code of 110 (2×55) and i has a code of 105 (7×5×3).

If we change the table size ever so slightly, to 211, no table entry gets more than 1% of the 26000 words hashed, which is very good.

Why such a big difference? Well 211 is prime and there is a bit a folk-wisdom that states that prime numbers are good choices for hash table sizes. Now our product hash will cover table entries far more uniformly
(small factors in the hash don’t divide the table size evenly).

Now the reason for Java’s more complex string hash function becomes evident—it can uniformly fill a hash table whose size isn’t prime.
How are Collisions Handled?

Since identifiers are often unlimited in length, the set of possible identifiers is infinite. Even if we limit ourselves is short identifiers (say 10 of fewer characters), the range of valid identifiers is greater than $26^{10}$. This means that all hash tables need to contend with collisions, when two different identifiers map to the same place in the table.

How are collisions handled?

The simplest approach is linear resolution. If identifier $id$ hashes to position $p$ in a hash table of size $s$ and position $p$ is already filled, we try $(p+1) \mod s$, then $(p+2) \mod s$, until a free position is found.
As long as the table is not too filled, this approach works well. When we approach an almost-filled situation, long search chains form, and we degenerate to an unordered list. If the table is 100% filled, linear resolution fails.

Some hash table implementations, including Java’s, set a load factor between 0 and 1.0. When the fraction of filled entries in the table exceeds the load factor, table size is increased and all entries are rehashed.

Note that bundling of a `hashCode` method within all Java objects makes rehashing easy to do automatically. If the hash function is external to the symbol table entries, rehashing may need to be done manually by the user.
An alternative to linear resolution is chained resolution, in which symbol table entries contain pointers to chains of symbols rather than a single symbol. All identifiers that hash to the same position appear on the same chain. Now overflowing table size is not catastrophic—as the table fills, chains from each table position get longer. As long as the table is not too overfilled, average chain length will be small.
Reading Assignment

Get and read Chapter 3 of Crafting a Compiler featuring Java. (Available from Dolt Tech Store)
Scanning

A scanner transforms a character stream into a token stream.

A scanner is sometimes called a lexical analyzer or lexer.

Scanners use a formal notation (regular expressions) to specify the precise structure of tokens.

But why bother? Aren’t tokens very simple in structure?

Token structure can be more detailed and subtle than one might expect. Consider simple quoted strings in C, C++ or Java. The body of a string can be any sequence of characters except a quote character (which must be escaped). But is this simple definition really correct?
Can a newline character appear in a string? In C it cannot, unless it is escaped with a backslash.

C, C++ and Java allow escaped newlines in strings, Pascal forbids them entirely. Ada forbids all unprintable characters.

Are null strings (zero-length) allowed? In C, C++, Java and Ada they are, but Pascal forbids them.
(In Pascal a string is a packed array of characters, and zero length arrays are disallowed.)

A precise definition of tokens can ensure that lexical rules are clearly stated and properly enforced.
Regular Expressions

Regular expressions specify simple (possibly infinite) sets of strings. Regular expressions routinely specify the tokens used in programming languages.

Regular expressions can drive a scanner generator.

Regular expressions are widely used in computer utilities:

• The Unix utility `grep` uses regular expressions to define search patterns in files.

• Unix shells allow regular expressions in file lists for a command.
• Most editors provide a “context search” command that specifies desired matches using regular expressions.

• The Windows Find utility allows some regular expressions.
Regular Sets

The sets of strings defined by regular expressions are called regular sets.

When scanning, a token class will be a regular set, whose structure is defined by a regular expression.

Particular instances of a token class are sometimes called lexemes, though we will simply call a string in a token class an instance of that token. Thus we call the string abc an identifier if it matches the regular expression that defines valid identifier tokens.

Regular expressions use a finite character set, or vocabulary (denoted $\Sigma$).

This vocabulary is normally the character set used by a computer.
Today, the ASCII character set, which contains a total of 128 characters, is very widely used.

Java uses the Unicode character set which includes all the ASCII characters as well as a wide variety of other characters.

An empty or null string is allowed (denoted $\lambda$, “lambda”). Lambda represents an empty buffer in which no characters have yet been matched. It also represents optional parts of tokens. An integer literal may begin with a plus or minus, or it may begin with $\lambda$ if it is unsigned.
Catenation

Strings are built from characters in the character set $\Sigma$ via catenation. As characters are catenated to a string, it grows in length. The string $\texttt{do}$ is built by first catenating $\texttt{d}$ to $\lambda$, and then catenating $\texttt{o}$ to the string $\texttt{d}$. The null string, when catenated with any string $s$, yields $s$. That is, $s \lambda \equiv \lambda s \equiv s$. Catenating $\lambda$ to a string is like adding 0 to an integer—nothing changes.

Catenation is extended to sets of strings:

Let $P$ and $Q$ be sets of strings. (The symbol $\in$ represents set membership.) If $s_1 \in P$ and $s_2 \in Q$ then string $s_1s_2 \in (P \cup Q)$. 
Alternation

Small finite sets are conveniently represented by listing their elements. Parentheses delimit expressions, and |, the alternation operator, separates alternatives.

For example, D, the set of the ten single digits, is defined as

\[ D = (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9). \]

The characters (, ), ′, ∗, +, and | are meta-characters (punctuation and regular expression operators).

Meta-characters must be quoted when used as ordinary characters to avoid ambiguity.
For example the expression
(((' | ')') | ; | , )
defines four single character tokens
(left parenthesis, right parenthesis, semicolon and comma). The
parentheses are quoted when they represent individual tokens and are
not used as delimiters in a larger regular expression.

Alternation is extended to sets of strings:
Let P and Q be sets of strings.

Then string \( s \in (P \mid Q) \) if and only if
\( s \in P \) or \( s \in Q \).

For example, if LC is the set of lower-case letters and UC is the set
of upper-case letters, then \( (LC \mid UC) \) is the set of all letters (in either case).
Kleene Closure

A useful operation is Kleene closure represented by a postfix \( * \) operator.

Let \( P \) be a set of strings. Then \( P^* \) represents all strings formed by the catenation of zero or more selections (possibly repeated) from \( P \).

Zero selections are represented by \( \lambda \).

For example, \( LC^* \) is the set of all words composed only of lower-case letters, of any length (including the zero length word, \( \lambda \)).

Precisely stated, a string \( s \in P^* \) if and only if \( s \) can be broken into zero or more pieces: \( s = s_1 s_2 \ldots s_n \) so that each \( s_i \in P \) (\( n \geq 0, 1 \leq i \leq n \)).

We allow \( n = 0 \), so \( \lambda \) is always in \( P^* \).
Definition of Regular Expressions

Using catenations, alternation and Kleene closure, we can define regular expressions as follows:

- $\emptyset$ is a regular expression denoting the empty set (the set containing no strings). $\emptyset$ is rarely used, but is included for completeness.

- $\lambda$ is a regular expression denoting the set that contains only the empty string. This set is not the same as the empty set, because it contains one element.

- A string $s$ is a regular expression denoting a set containing the single string $s$. 
If A and B are regular expressions, then A | B, A B, and A* are also regular expressions, denoting the alternation, catenation, and Kleene closure of the corresponding regular sets.

Each regular expression denotes a set of strings (a regular set). Any finite set of strings can be represented by a regular expression of the form \((s_1 \mid s_2 \mid ... \mid s_k)\). Thus the reserved words of ANSI C can be defined as (auto | break | case | ...).
The following additional operations useful. They are not strictly necessary, because their effect can be obtained using alternation, catenation, Kleene closure:

- \( P^+ \) denotes all strings consisting of one or more strings in \( P \) catenated together:
  \[
P^* = (P^+ | \lambda) \quad \text{and} \quad P^+ = P \ P^*.
  \]
  For example, \((0 | 1)^+\) is the set of all strings containing one or more bits.

- If \( A \) is a set of characters, \( \text{Not}(A) \) denotes \((\Sigma - A)\); that is, all characters in \( \Sigma \) not included in \( A \). Since \( \text{Not}(A) \) can never be larger than \( \Sigma \) and \( \Sigma \) is finite, \( \text{Not}(A) \) must also be finite, and is therefore regular. \( \text{Not}(A) \) does not contain \( \lambda \) since \( \lambda \) is not a
character (it is a zero-length string). For example, $\text{Not}(\text{Eol})$ is the set of all characters excluding Eol (the end of line character, '\n' in Java or C).

It is possible to extend Not to strings, rather than just $\Sigma$. That is, if $S$ is a set of strings, we define $\overline{S}$ to be $(\Sigma^* - S)$; the set of all strings except those in $S$. Though $\overline{S}$ is usually infinite, it is also regular if $S$ is.

- If $k$ is a constant, the set $A^k$ represents all strings formed by catenating $k$ (possibly different) strings from $A$. That is, $A^k = (A \ A \ A \ ...)$ ($k$ copies). Thus $(0 \ | \ 1)^{32}$ is the set of all bit strings exactly 32 bits long.
Examples

Let $D$ be the ten single digits and let $L$ be the set of all 52 letters. Then

- A Java or C++ single-line comment that begins with `//` and ends with `Eol` can be defined as:

  $$\text{Comment} = // \text{Not(Eol)}^* \text{Eol}$$

- A fixed decimal literal (e.g., `12.345`) can be defined as:

  $$\text{Lit} = D^+. D^+$$

- An optionally signed integer literal can be defined as:

  $$\text{IntLiteral} = ( '+' | - | \lambda ) \text{D}^+$$

(Why the quotes on the plus?)
A comment delimited by ## markers, which allows single #'s within the comment body:

Comment2 =

## ((# | λ) Not(#))^* ##

All finite sets and many infinite sets are regular. But not all infinite sets are regular. Consider the set of balanced brackets of the form


This set is defined formally as

{ [^m]^m | m ≥ 1 }.

This set is known not to be regular. Any regular expression that tries to define it either does not get all balanced nestings or it includes extra, unwanted strings.
Finite Automata and Scanners

A finite automaton (FA) can be used to recognize the tokens specified by a regular expression. FAs are simple, idealized computers that recognize strings belonging to regular sets. An FA consists of:

- A finite set of states
- A set of transitions (or moves) from one state to another, labeled with characters in \( \Sigma \)
- A special state called the start state
- A subset of the states called the accepting, or final, states
These four components of a finite automaton are often represented graphically:

- is a state
- is a transition
- is the start state
- is an accepting state

Finite automata (the plural of automaton is automata) are represented graphically using transition diagrams. We start at the start state. If the next input character matches the label on a transition
from the current state, we go to the state it points to. If no move is possible, we stop. If we finish in an accepting state, the sequence of characters read forms a valid token; otherwise, we have not seen a valid token.

In this diagram, the valid tokens are the strings described by the regular expression \((a \ b \ (c)^+ \ )^+\).
Deterministic Finite Automata

As an abbreviation, a transition may be labeled with more than one character (for example, Not(c)). The transition may be taken if the current input character matches any of the characters labeling the transition.

If an FA always has a unique transition (for a given state and character), the FA is deterministic (that is, a deterministic FA, or DFA). Deterministic finite automata are easy to program and often drive a scanner.

If there are transitions to more than one state for some character, then the FA is nondeterministic (that is, an NFA).
A DFA is conveniently represented in a computer by a transition table. A transition table, \( T \), is a two dimensional array indexed by a DFA state and a vocabulary symbol.

Table entries are either a DFA state or an error flag (often represented as a blank table entry). If we are in state \( s \), and read character \( c \), then \( T[s,c] \) will be the next state we visit, or \( T[s,c] \) will contain an error marker indicating that \( c \) cannot extend the current token. For example, the regular expression

\[
// \text{Not(Eol)}^* \text{Eol}
\]

which defines a Java or C++ single-line comment, might be translated into
The corresponding transition table is:

<table>
<thead>
<tr>
<th>State</th>
<th>Character</th>
</tr>
</thead>
<tbody>
<tr>
<td>/</td>
<td>Eol</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3 4 3 3 3</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

A complete transition table contains one column for each character. To save space, table compression may be used. Only non-error entries are explicitly represented in the table, using hashing, indirection or linked structures.
All regular expressions can be translated into DFAs that accept (as valid tokens) the strings defined by the regular expressions. This translation can be done manually by a programmer or automatically using a scanner generator.

A DFA can be coded in:
- Table-driven form
- Explicit control form

In the table-driven form, the transition table that defines a DFA’s actions is explicitly represented in a run-time table that is “interpreted” by a driver program.

In the direct control form, the transition table that defines a DFA’s actions appears implicitly as the control logic of the program.
For example, suppose `CurrentChar` is the current input character. End of file is represented by a special character value, `eof`. Using the DFA for the Java comments shown earlier, a table-driven scanner is:

```java
State = StartState
while (true) {
    if (CurrentChar == eof)
        break
    NextState =
        T[State][CurrentChar]
    if (NextState == error)
        break
    State = NextState
    read(CurrentChar)
}
if (State in AcceptingStates)
    // Process valid token
else // Signal a lexical error
```
This form of scanner is produced by a scanner generator; it is definition-independent. The scanner is a driver that can scan any token if $T$ contains the appropriate transition table.

Here is an explicit-control scanner for the same comment definition:

```plaintext
if (CurrentChar == '/'){
    read(CurrentChar)
    if (CurrentChar == '/')
        repeat
            read(CurrentChar)
        until (CurrentChar in {eol, eof})
    else // Signal lexical error
else // Signal lexical error
if (CurrentChar == eol)
    // Process valid token
else // Signal lexical error
```
The token being scanned is "hardwired" into the logic of the code. The scanner is usually easy to read and often is more efficient, but is specific to a single token definition.
More Examples

- A FORTRAN-like real literal (which requires digits on either or both sides of a decimal point, or just a string of digits) can be defined as

\[ \text{RealLit} = (D^+ (\lambda \mid . \ ))) \mid (D^* \cdot D^+) \]

This corresponds to the DFA
An identifier consisting of letters, digits, and underscores, which begins with a letter and allows no adjacent or trailing underscores, may be defined as

\[
\text{ID} = \text{L} \ (\text{L} \mid \text{D})^* \ (\ _ \ (\text{L} \mid \text{D})^+)^*
\]

This definition includes identifiers like `sum` or `unit_cost`, but excludes `_one` and `two_` and `grand___total`. The DFA is:
Transducers

So far our scanners haven’t saved or processed the characters we’ve read—they are matched and then thrown away!

It is useful to add an output facility to an FA; this makes the FA a transducer.

As characters are read, they can be transformed and catenated to an output string. For our purposes, we shall limit the transformation operations to saving or deleting input characters. After a token is recognized, the transformed input can be passed to other compiler phases for further processing.
We use this notation:

- a → means save a in a token buffer
- T(a) → means don’t save a (Toss it away)

For example, for Java and C++ comments, we might write

```
T(/) → T(/) → T(Eol)
```

T(Not(Eol))
A more interesting example is given by Pascal-style quoted strings, according to the regular expression

" ( Not(" | " " )* ")"

A corresponding transducer might be

The input """Hi"""" would produce output "Hi"."
Other Scanner Issues

We will consider other practical issues in building real scanners for real programming languages. Our finite automaton model sometimes needs to be augmented. Moreover, error handling must be incorporated into any practical scanner.
Identifiers vs. Reserved Words

Most programming languages contain reserved words like if, while, switch, etc. These tokens look like ordinary identifiers, but aren’t. It is up to the scanner to decide if what looks like an identifier is really a reserved word. This distinction is vital as reserved words have different token codes than identifiers and are parsed differently.

How can a scanner decide which tokens are identifiers and which are reserved words?

- We can scan identifiers and reserved words using the same pattern, and then look up the token in a special “reserved word” table.
• It is known that any regular expression may be \textit{complemented to obtain} all strings not in the original regular expression. Thus $\overline{A}$, the complement of $A$, is regular if $A$ is. Using complementation we can write a regular expression for nonreserved identifiers: \( (ident|if|while|...) \)

Since scanner generators don’t usually support complementation of regular expressions, this approach is more of theoretical than practical interest.

• We can give distinct regular expression definitions for each reserved word, and for identifiers. Since the definitions overlap ($\texttt{if}$ will match a reserved word and the general identifier pattern), we give
priority to reserved words. Thus a token is scanned as an identifier if it matches the identifier pattern and does not match any reserved word pattern. This approach is commonly used in scanner generators like Lex and JLex.
Converting Token Values

For some tokens, we may need to convert from string form into numeric or binary form.

For example, for integers, we need to transform a string a digits into the internal (binary) form of integers.

We know the format of the token is valid (the scanner checked this), but:

- The string may represent an integer too large to represent in 32 or 64 bit form.
- Languages like CSX and ML use a non-standard representation for negative values (~123 instead of −123)
We can safely convert from string to integer form by first converting the string to double form, checking against max and min int, and then converting to int form if the value is representable.

Thus $d = \text{new Double}(\text{str})$ will create an object $d$ containing the value of $\text{str}$ in double form. If $\text{str}$ is too large or too small to be represented as a double, plus or minus infinity is automatically substituted.

$d.doubleValue()$ will give $d$’s value as a Java double, which can be compared against $\text{Integer.MAX\_VALUE}$ or $\text{Integer.MIN\_VALUE}$.
If `d.doubleValue()` represents a valid integer, 
`(int) d.doubleValue()` 
will create the appropriate integer value.

If a string representation of an integer begins with a “~” we can strip the “~”, convert to a double and then negate the resulting value.
Scanner Termination

A scanner reads input characters and partitions them into tokens.

What happens when the end of the input file is reached? It may be useful to create an Eof pseudo-character when this occurs. In Java, for example, InputStream.read(), which reads a single byte, returns -1 when end of file is reached. A constant, Eof, defined as -1 can be treated as an “extended” ASCII character. This character then allows the definition of an Eof token that can be passed back to the parser.

An Eof token is useful because it allows the parser to verify that the logical end of a program corresponds
to its physical end. Most parsers require an end of file token.

Lex and Jlex automatically create an Eof token when the scanner they build tries to scan an EOF character (or tries to scan when eof() is true).
**Multi Character Lookahead**

We may allow finite automata to look beyond the next input character. This feature is necessary to implement a scanner for FORTRAN.

In FORTRAN, the statement

```
DO 10 J = 1,100
```

specifies a loop, with index \( J \) ranging from 1 to 100.

The statement

```
DO 10 J = 1.100
```

is an assignment to the variable \( \text{DO10J} \). (Blanks are not significant except in strings.)

A FORTRAN scanner decides whether the \( \circ \) is the last character of a \( \text{DO} \) token only after reading as far as the comma (or period).
A milder form of extended lookahead problem occurs in Pascal and Ada. The token 10.50 is a real literal, whereas 10..50 is three different tokens.

We need two-character lookahead after the 10 prefix to decide whether we are to return 10 (an integer literal) or 10.50 (a real literal).
Suppose we use the following FA.

Given 10..100 we scan three characters and stop in a non-accepting state.

Whenever we stop reading in a non-accepting state, we back up along accepted characters until an accepting state is found.

Characters we back up over are rescanned to form later tokens. If no accepting state is reached during backup, we have a lexical error.
Performance Considerations

Because scanners do so much character-level processing, they can be a real performance bottleneck in production compilers.

Speed is not a concern in our project, but let’s see why scanning speed can be a concern in production compilers.

Let’s assume we want to compile at a rate of 1000 lines/sec. (so that most programs compile in just a few seconds).

Assuming 30 characters/line (on average), we need to scan 30,000 char/sec.
On a 30 SPECmark machine (30 million instructions/sec.), we have 1000 instructions per character to spend on all compiling steps.

If we allow 25% of compiling to be scanning (a compiler has a lot more to do than just scan!), that’s just 250 instructions per character.

A key to efficient scanning is to group character-level operations whenever possible. It is better to do one operation on n characters rather than n operations on single characters.

In our examples we’ve read input one character at a time. A subroutine call can cost hundreds or thousands of instructions to execute—far too much to spend on a single character.
We prefer routines that do block reads, putting an entire block of characters directly into a buffer. Specialized scanner generators can produce particularly fast scanners. The GLA scanner generator claims that the scanners it produces run as fast as:

```c
while(c != Eof) {
    c = getchar();
}
```
Lexical Error Recovery

A character sequence that can’t be scanned into any valid token is a lexical error.

Lexical errors are uncommon, but they still must be handled by a scanner. We won’t stop compilation because of so minor an error.

Approaches to lexical error handling include:

• Delete the characters read so far and restart scanning at the next unread character.

• Delete the first character read by the scanner and resume scanning at the character following it.
Both of these approaches are reasonable.

The first is easy to do. We just reset the scanner and begin scanning anew.

The second is a bit harder but also is a bit safer (less is immediately deleted). It can be implemented using scanner backup.

Usually, a lexical error is caused by the appearance of some illegal character, mostly at the beginning of a token.

(Why at the beginning?)

In these case, the two approaches are equivalent.
The effects of lexical error recovery might well create a later syntax error, handled by the parser.

Consider

\[ \ldots \text{for$tnight\ldots} \]

The $ terminates scanning of for. Since no valid token begins with $, it is deleted. Then tnight is scanned as an identifier. In effect we get

\[ \ldots \text{for tnight\ldots} \]

which will cause a syntax error. Such “false errors” are unavoidable, though a syntactic error-repair may help.
**Error Tokens**

Certain lexical errors require special care. In particular, runaway strings and runaway comments ought to receive special error messages.

In Java strings may not cross line boundaries, so a runaway string is detected when an end of a line is read within the string body. Ordinary recovery rules are inappropriate for this error. In particular, deleting the first character (the double quote character) and restarting scanning is a bad decision.

It will almost certainly lead to a cascade of “false” errors as the string text is inappropriately scanned as ordinary input.
One way to handle runaway strings is to define an error token.

An error token is not a valid token; it is never returned to the parser. Rather, it is a pattern for an error condition that needs special handling. We can define an error token that represents a string terminated by an end of line rather than a double quote character.

For a valid string, in which internal double quotes and back slashes are escaped (and no other escaped characters are allowed), we can use
"
( Not( " | Eol | \ ) | \" | \\ )* "
"
For a runaway string we use
"
( Not( " | Eol | \ ) | \" | \\ )* Eol
(Eol is the end of line character.)
When a runaway string token is recognized, a special error message should be issued.

Further, the string may be “repaired” into a correct string by returning an ordinary string token with the closing Eol replaced by a double quote.

This repair may or may not be “correct.” If the closing double quote is truly missing, the repair will be good; if it is present on a succeeding line, a cascade of inappropriate lexical and syntactic errors will follow.

Still, we have told the programmer exactly what is wrong, and that is our primary goal.
In languages like C, C++, Java and CSX, which allow multiline comments, improperly terminated (runaway) comments present a similar problem. A runaway comment is not detected until the scanner finds a close comment symbol (possibly belonging to some other comment) or until the end of file is reached. Clearly a special, detailed error message is required.

Let’s look at Pascal-style comments that begin with a { and end with a }. Comments that begin and end with a pair of characters, like /* and */ in Java, C and C++, are a bit trickier.
Correct Pascal comments are defined quite simply:

```pascal
{ Not( } )* }
```

To handle comments terminated by Eof, this error token can be used:

```pascal
{ Not( } )* Eof
```

We want to handle comments unexpectedly closed by a close comment belonging to another comment:

```pascal
{ ... missing close comment 
  ... { normal comment } ... 
```

We will issue a warning (this form of comment is lexically legal).

Any comment containing an open comment symbol in its body is most probably a missing } error.
We split our legal comment definition into two token definitions.

The definition that accepts an open comment in its body causes a warning message ("Possible unclosed comment") to be printed.

We now use:

\[
\{ \text{Not( } \{ \} \text{ )} \text{)* }\} \text{ and } \\
\{ \text{(Not( } \{ | \} \text{ )* } \{ \text{Not( } \{ | \} \text{ )* }\text{)+ } \}
\]

The first definition matches correct comments that do not contain an open comment in their body.

The second definition matches correct, but suspect, comments that contain at least one open comment in their body.
Single line comments, found in Java, CSX and C++, are terminated by Eol. They can fall prey to a more subtle error—what if the last line has no Eol at its end?

The solution?

Another error token for single line comments:

    // Not(Eol)*

This rule will only be used for comments that don’t end with an Eol, since scanners always match the longest rule possible.
Regular Expressions and Finite Automata

Regular expressions are fully equivalent to finite automata.

The main job of a scanner generator like JLex is to transform a regular expression definition into an equivalent finite automaton.

First it transforms a regular expression into a nondeterministic finite automaton (NFA).

Unlike an ordinary deterministic finite automaton, an NFA need not make a unique (deterministic) choice of a successor state to visit. For example, as shown below, an NFA is allowed to have a state that has two transitions (arrows) coming out of it, labeled by
the same symbol. An NFA may also have transitions labeled with $\lambda$.

Transitions are normally labeled with individual characters in $\Sigma$, and although $\lambda$ is a string (the string with no characters in it), it is definitely not a character. In the above example, when the automaton is in the state at the left and the next input character is $a$, it may choose to use the
transition labeled a or first follow the \( \lambda \) transition (you can always find \( \lambda \) wherever you look for it) and then follow an a transition. FAs that contain no \( \lambda \) transitions and that always have unique successor states for any symbol are deterministic.
Building Finite Automata From Regular Expressions

We make an FA from a regular expression in two steps:

- Transform the regular expression into an NFA.
- Transform the NFA into a deterministic FA.

The first step is easy.

Regular expressions are all built out of the atomic regular expressions $a$ (where $a$ is a character in $\Sigma$) and $\lambda$ by using the three operations $A \cdot B$ and $A \mid B$ and $A^*$. 
Other operations (like $A^+$) are just abbreviations for combinations of these.

NFAs for $a$ and $\lambda$ are trivial:
Suppose we have NFAs for A and B and want one for A | B. We construct the NFA shown below:

The states labeled A and B were the accepting states of the automata for A and B; we create a new accepting state for the combined automaton.

A path through the top automaton accepts strings in A, and a path through the bottom automation accepts strings in B, so the whole automaton matches A | B.
As shown below, the construction for $A \, B$ is even easier. The accepting state of the combined automaton is the same state that was the accepting state of $B$. We must follow a path through $A$’s automaton, then through $B$’s automaton, so overall $A \, B$ is matched.

We could also just merge the accepting state of $A$ with the initial state of $B$. We chose not to only because the picture would be more difficult to draw.
Finally, let’s look at the NFA for $A^*$. The start state reaches an accepting state via $\lambda$, so $\lambda$ is accepted. Alternatively, we can follow a path through the FA for $A$ one or more times, so zero or more strings that belong to $A$ are matched.
Creating Deterministic Automata

The transformation from an NFA $N$ to an equivalent DFA $D$ works by what is sometimes called the subset construction.

Each state of $D$ corresponds to a set of states of $N$.

The idea is that $D$ will be in state $\{x, y, z\}$ after reading a given input string if and only if $N$ could be in any one of the states $x$, $y$, or $z$, depending on the transitions it chooses. Thus $D$ keeps track of all the possible routes $N$ might take and runs them simultaneously.

Because $N$ is a finite automaton, it has only a finite number of states. The
number of subsets of N’s states is also finite, which makes tracking various sets of states feasible.

An accepting state of D will be any set containing an accepting state of N, reflecting the convention that N accepts if there is any way it could get to its accepting state by choosing the “right” transitions.

The start state of D is the set of all states that N could be in without reading any input characters—that is, the set of states reachable from the start state of N following only \( \lambda \) transitions. Algorithm close computes those states that can be reached following only \( \lambda \) transitions.

Once the start state of D is built, we begin to create successor states:
We take each state $S$ of $D$, and each character $c$, and compute $S$’s successor under $c$.

$S$ is identified with some set of $N$’s states, $\{n_1, n_2, \ldots\}$.

We find all the possible successor states to $\{n_1, n_2, \ldots\}$ under $c$, obtaining a set $\{m_1, m_2, \ldots\}$.

Finally, we compute

$$T = \text{CLOSE}(\{m_1, m_2, \ldots\}).$$

$T$ becomes a state in $D$, and a transition from $S$ to $T$ labeled with $c$ is added to $D$.

We continue adding states and transitions to $D$ until all possible successors to existing states are added.
Because each state corresponds to a finite subset of $N$’s states, the process of adding new states to $D$ must eventually terminate.

Here is the algorithm for $\lambda$-closure, called close. It starts with a set of NFA states, $S$, and adds to $S$ all states reachable from $S$ using only $\lambda$ transitions.

```c
void close(NFASet S) {
    while (x in S and x $\xrightarrow{\lambda}$ y and y notin S) {
        S = S U {y}
    }
}
```

Using `close`, we can define the construction of a DFA, $D$, from an NFA, $N$: 
DFA MakeDeterministic(NFA N) {
    DFA D ; NFASet T
    D.StartState = { N.StartState }
    close(D.StartState)
    D.States = { D.StartState }
    while (states or transitions can be added to D) {
        Choose any state S in D.States and any character c in Alphabet
        T = {y in N.States such that x \xrightarrow{c} y for some x in S}
        close(T);
        if (T notin D. States) {
            D.States = D.States U {T}
            D.Transitions =
            D.Transitions U
            {the transition S \xrightarrow{c} T}
        }
    }
    D.AcceptingStates =
    { S in D.States such that an accepting state of N in S}
}
Example

To see how the subset construction operates, consider the following NFA:

We start with state 1, the start state of N, and add state 2 its $\lambda$-successor. D’s start state is $\{1,2\}$.

Under a, $\{1,2\}$’s successor is $\{3,4,5\}$.

State 1 has itself as a successor under b. When state 1’s $\lambda$-successor, 2, is included, $\{1,2\}$’s successor is $\{1,2\}$.
{3,4,5}'s successors under a and b are {5} and {4,5}.

{4,5}'s successor under b is {5}.

Accepting states of D are those state sets that contain N's accepting state which is 5.

The resulting DFA is:
It is not too difficult to establish that the DFA constructed by \texttt{MakeDeterministic} is equivalent to the original NFA.

The idea is that each path to an accepting state in the original NFA has a corresponding path in the DFA. Similarly, all paths through the constructed DFA correspond to paths in the original NFA.

What is less obvious is the fact that the DFA that is built can sometimes be much larger than the original NFA. States of the DFA are identified with sets of NFA states.

If the NFA has $n$ states, there are $2^n$ distinct sets of NFA states, and hence the DFA may have as many as $2^n$ states. Certain NFAs actually exhibit
this exponential blowup in size when made deterministic.
Fortunately, the NFAs built from the kind of regular expressions used to specify programming language tokens do not exhibit this problem when they are made deterministic.
As a rule, DFAs used for scanning are simple and compact.
If creating a DFA is impractical (because of size or speed-of-generation concerns), we can scan using an NFA. Each possible path through an NFA is tracked, and reachable accepting states are identified. Scanning is slower using this approach, so it is used only when construction of a DFA is not practical.
Optimizing Finite Automata

We can improve the DFA created by MakeDeterministic.

Sometimes a DFA will have more states than necessary. For every DFA there is a unique smallest equivalent DFA (fewest states possible).

Some DFA’s contain unreachable states that cannot be reached from the start state.

Other DFA’s may contain dead states that cannot reach any accepting state.

It is clear that neither unreachable states nor dead states can participate in scanning any valid token. We therefore eliminate all such states as part of our optimization process.
We optimize a DFA by merging together states we know to be equivalent.

For example, two accepting states that have no transitions at all out of them are equivalent.

Why? Because they behave exactly the same way—they accept the string read so far, but will accept no additional characters.

If two states, \( s_1 \) and \( s_2 \), are equivalent, then all transitions to \( s_2 \) can be replaced with transitions to \( s_1 \). In effect, the two states are merged together into one common state.
How do we decide what states to merge together?

We take a greedy approach and try the most optimistic merger of states. By definition, accepting and non-accepting states are distinct, so we initially try to create only two states: one representing the merger of all accepting states and the other representing the merger of all non-accepting states.

This merger into only two states is almost certainly too optimistic. In particular, all the constituents of a merged state must agree on the same transition for each possible character. That is, for character $c$ all the merged states must have no successor under $c$ or they must all go to a single (possibly merged) state.
If all constituents of a merged state do not agree on the transition to follow for some character, the merged state is split into two or more smaller states that do agree.

As an example, assume we start with the following automaton:

Initially we have a merged non-accepting state \( \{1,2,3,5,6\} \) and a merged accepting state \( \{4,7\} \).

A merger is legal if and only if all constituent states agree on the same successor state for all characters. For example, states 3 and 6 would go to
an accepting state given character \( c \); states 1, 2, 5 would not, so a split must occur.

We will add an error state \( s_E \) to the original DFA that is the successor state under any illegal character. (Thus reaching \( s_E \) becomes equivalent to detecting an illegal token.) \( s_E \) is not a real state; rather it allows us to assume every state has a successor under every character. \( s_E \) is never merged with any real state.

Algorithm \texttt{Split}, shown below, splits merged states whose constituents do not agree on a common successor state for all characters. When \texttt{Split} terminates, we know that the states that remain merged are equivalent in
that they always agree on common successors.

Split(FASet StateSet) {
    repeat
        for(each merged state
            S in StateSet) {
            Let S corresponds to \{s_1, \ldots, s_n\}
            for(each char c in Alphabet){
                Let t_1, \ldots, t_n be the successor states to s_1, \ldots, s_n under c
                if(t_1, \ldots, t_n do not all belong to the same merged state){
                    Split S into two or more new states such that s_i and s_j remain in the same merged state if and only if t_i and t_j are in the same merged state
                }
            }
        }
    until no more splits are possible
}
Returning to our example, we initially have states \{1,2,3,5,6\} and \{4,7\}. Invoking \texttt{Split}, we first observe that states 3 and 6 have a common successor under \(c\), and states 1, 2, and 5 have no successor under \(c\) (or, equivalently, have the error state \(s_E\)).

This forces a split, yielding \{1,2,5\}, \{3,6\} and \{4,7\}.

Now, for character \(b\) states 2 and 5 would go to the merged state \{3,6\}, but state 1 would not, so another split occurs.

We now have: \{1\}, \{2,5\}, \{3,6\} and \{4,7\}.

At this point we are done, as all constituents of merged states agree on the same successor for each input symbol.
Once $\text{Split}$ is executed, we are essentially done.

Transitions between merged states are the same as the transitions between states in the original DFA.

Thus, if there was a transition between state $s_i$ and $s_j$ under character $c$, there is now a transition under $c$ from the merged state containing $s_i$ to the merged state containing $s_j$. The start state is that merged state containing the original start state.

Accepting states are those merged states containing accepting states (recall that accepting and non-accepting states are never merged).
Returning to our example, the minimum state automaton we obtain is

```
1  a | d  2,5  b  3,6  c  4,7
```
Translating Finite Automata to Regular Expressions

We can convert any regular expression into an equivalent finite automaton. JLex does this in building a scanner from token patterns.

It is also possible to derive an equivalent regular expression for any finite automaton.

This is useful when you already have a finite automaton you want to use, but need an regular expression to program JLex.

The idea is simple and elegant:
We start with a finite automaton and simplify it by removing states, one by one.
Simplified automata are equivalent to the original except that transitions are now labeled with regular expressions rather than individual characters.

We continue removing states until we have only a single transition from the start state to a single accepting state. The regular expression labeling that single transition correctly describes the effect of the original automaton.

To start, assume our finite automaton has a start state with no transitions into it and a single accepting state with no transitions out of it.

If the automaton we start with doesn’t meet these requirements, we can easily transform it by adding a new start state and a new accepting
state linked to the original automaton with $\lambda$-transitions. For example:

Original Automaton

New Automaton with Start and Accepting States Added
We define three simple transformations, T1, T2 and T3 that can simplify finite automata.

The first, illustrated below, notes that if we have two different transitions between the same pair of states, with one labeled R and the other labeled S, we can replace the two transitions with a new one, labeled R | S.

\[
\begin{array}{ccc}
\text{Original Transitions} & \text{Combined Transition} \\
\text{The T1 Transformation} & \\
\end{array}
\]

T1 simply reflects that we can choose to use the first transition or the second.
Transformation T2, illustrated below allows us to “by-pass” a state. If state s has a transition to state r labeled X and state r has a transition to state u labeled Y, then we can go directly from state s to state u with a transition labeled XY.

The T2 Transformation
Transformation T3, illustrated below is similar to transformation T2. It again allows us to by-pass a state.

If state s has a transition to state r labeled X, and state r has a transition to itself labeled Z, and state r also has a transition to state u labeled Y, we can go directly from state s to state u with a transition labeled $XZ^*Y$.

The $Z^*$ term reflects that once we reach state r we can cycle back into r zero or more times before finally going to u.
We will use transformations T2 and T3 as follows.

We consider, in turn, each pair of predecessors and successors a state s has, and use T2 or T3 to link a predecessor state directly to a successor state.

Then s is no longer needed—all paths through the finite automaton can bypass it!

Since s isn’t needed, we remove it. The finite automaton is now simpler because it has one fewer states.

If we remove all states other than the start state and the accepting state (using transformation T1 when necessary), we will reach our goal.
We will have an automaton with only one transition, and the label on this transition will be the regular expression we want. $\text{FindRe}$, shown below, implements this algorithm.
RegularExpr FindRe(NFa Fa) {
    if (Fa’s start state has a transition into it) {
        Create a new start state;
        Link it to the original start state with a $\lambda$-transition }
    if (Fa has > 1 accepting state || Fa has an accepting state with out transitions) {
        Create a new and unique accepting state and link it to the original accepting states with $\lambda$-transitions}
    while (Fa has > 1 transition) {
        while (Any pair of states have more than 1 transition between them) {
            Use a T1 transform to obtain a single transition }
        Let S be any state != the start or accepting state;
        for (each predecessor P of S where P != S)
            for (each successor U of S where U != S)
                if (S has no transition to itself)
                    Create a transition from P to U using a T2 transformation
                else
                    Create a transition from P to U using a T3 transformation
        Remove S from Fa
    }
    return the regular expression labeling the last remaining transition
As an example, we will find the regular expression corresponding to:

The original automaton (renumbered), with a new start state and accepting state added, is
State 1 has a single predecessor, state 0 and a single successor, 2. Using a T3 transformation, an arc directly from state 0 to state 2 is added, and state 1 is removed. We now have:

State 2 has a single predecessor, state 0 and three successors, 2, 4 and 5. Using three T2 transformations, arcs directly from state 0 to states 3, 4 and 5 are added. State 2 is removed.
We have:

State 4 has two predecessors, states 0 and 3. It has one successor, state 5. Using two T2 transformations, arcs directly from states 0 and 3 to state 5 are added. State 4 is removed. We obtain:
Two pairs of transitions are merged using T1 transformations, producing:

Finally, state 3 is by-passed with a T2 transformation and a pair of
transitions are merged with a T1 transformation:

\[ b^*ab (a | b | \lambda) | b^*aa | b^*a \]

The regular expression we obtain is
\[ b^*a b (a | b | \lambda) | b^*a a | b^*a \]

By expanding the parenthesized subterm and then factoring a common term, we obtain
\[ b^*a b a | b^*a b b | b^*a b | b^*a a | b^*a \]
\[ = b^*a (ba | bb | b | a | \lambda) \]

We can see that
\[ b^*a (ba | bb | b | a | \lambda) \]
is equivalent to the original automaton:

Properties of Regular
Expressions and Finite Automata

- Some token patterns can’t be defined as regular expressions or finite automata. Consider the set of balanced brackets of the form
  \[
  \[ \[ \ldots \] \] .
  \]
This set is defined formally as
\[
\{ \[^m \]_m | m \geq 1 \} .
\]
This set is not regular. No finite automaton that recognizes exactly this set can exist. Why?
Consider the inputs [, [[, [[[, ...
For two different counts (call them i and j). \[^i\] and \[^j\] must reach the same state of any given FA! (Why)
Once that happens, we know that if \[^i\] is
accepted (as it should be), the $[j^i$ will also be accepted (and that should not happen).

- $\overline{R} = V^* - R$ is regular if $R$ is. Why?
  Build a finite automaton for $R$. Be careful to include transitions to an “error state” $s_E$ for illegal characters.
  Now invert final and non-final states. What was previously accepted is now rejected, and what was rejected is now accepted. That is, $\overline{R}$ is accepted by the modified automaton.

- Not all subsets of a regular set are themselves regular. The regular
expression \([+]^+\) has a subset that isn’t regular. (What is that subset??)

- Let \(R\) be a set of strings. \(R^{rev}\) is defined to be the strings in \(R\), in reversed (backward) character order. Thus if \(R = \{abc, def\}\) then \(R^{rev} = \{cba, fed\}\).

If \(R\) is regular, then \(R^{rev}\) is too. Why?

Build a finite automaton for \(R\). Make sure the automaton has only one final state. Now reverse the direction of all transitions, and interchange the start and final states. What does the modified automation accept?

- If \(R_1\) and \(R_2\) are both regular, then \(R_1 \cap R_2\) is also regular. We can show this two different ways:
1. Build two finite automata, one for $R_1$ and one for $R_2$. Pair together states of the two automata to match $R_1$ and $R_2$ simultaneously. The paired-state automaton accepts only if both $R_1$ and $R_2$ would, so $R_1 \cap R_2$ is matched.

2. We can use the fact that $R_1 \cap R_2$ is equal to $\overline{R_1} \cup \overline{R_2}$. We already know union and complementation are regular.

Reading Assignment

Get and read Chapter 4 of
Context Free Grammars

A context-free grammar (CFG) is defined as:
• A finite terminal set $V_t$; these are the tokens produced by the scanner.

• A set of intermediate symbols, called non-terminals, $V_n$.

• A start symbol, a designated non-terminal, that starts all derivations.

• A set of productions (sometimes called rewriting rules) of the form

$$A \rightarrow X_1 \ldots X_m$$

$X_1$ to $X_m$ may be any combination of terminals and non-terminals. If $m = 0$ we have $A \rightarrow \lambda$ which is a valid production.

Example

$$\text{Prog} \rightarrow \{ \text{Stmts} \}$$
Stmts → Stmts ; Stmt
Stmts → Stmt
Stmt → id = Expr
Expr → id
Expr → Expr + id

Often more than one production shares the same left-hand side.
Rather than repeat the left hand side, an “or notation” is used:

\[
\begin{align*}
\text{Prog} & \rightarrow \{ \text{Stmts} \} \\
\text{Stmts} & \rightarrow \text{Stmts} \ ; \ \text{Stmt} \\
& \quad | \ \text{Stmt} \\
\text{Stmt} & \rightarrow \text{id} = \text{Expr} \\
\text{Expr} & \rightarrow \text{id} \\
& \quad | \ \text{Expr} + \text{id}
\end{align*}
\]
Derivations

Starting with the start symbol, non-terminals are rewritten using productions until only terminals remain.

Any terminal sequence that can be generated in this manner is syntactically valid.

If a terminal sequence can’t be generated using the productions of the grammar it is invalid (has syntax errors).

The set of strings derivable from the start symbol is the language of the grammar (sometimes denoted L(G)).
For example, starting at Prog we generate a terminal sequence, by repeatedly applying productions:

Prog
{ Stmts }
{ Stmts ; Stmt }
{ Stmt ; Stmt }
{ id = Expr ; Stmt }
{ id = id ; Stmt }
{ id = id ; id = Expr }
{ id = id ; id = Expr + id}
{ id = id ; id = id + id}
Parse Trees

To illustrate a derivation, we can draw a derivation tree (also called a parse tree):

```
Prog
  /\      /
{ Stmts }Stmts ; Stmt
   /\      /
 Stmt  id = Expr
  /\     /\      /
 id = Expr Expr + id
    /\     /\      /
   id   id
```
An abstract syntax tree (AST) shows essential structure but eliminates unnecessary delimiters and intermediate symbols:

```
Prog
  ↓
Stmts
  ↓
Stmts =
    ↓
    id =
      ↓
      id +
        ↓
        id id id id
```
If $A \rightarrow \gamma$ is a production then
$\alpha A \beta \Rightarrow \alpha \gamma \beta$
where $\Rightarrow$ denotes a one step derivation (using production $A \rightarrow \gamma$).

We extend $\Rightarrow$ to $\Rightarrow^+ \hspace{1cm}$ (derives in one or more steps), and $\Rightarrow^* \hspace{1cm}$ (derives in zero or more steps).

We can show our earlier derivation as

\begin{align*}
\text{Prog} & \Rightarrow \\
\{ \text{Stmts} \} & \Rightarrow \\
\{ \text{Stmts ; Stmt} \} & \Rightarrow \\
\{ \text{Stmt ; Stmt} \} & \Rightarrow \\
\{ \text{id = Expr ; Stmt} \} & \Rightarrow \\
\{ \text{id = id ; Stmt} \} & \Rightarrow \\
\{ \text{id = id ; id = Expr} \} & \Rightarrow \\
\{ \text{id = id ; id = Expr + id} \} & \Rightarrow \\
\{ \text{id = id ; id = id + id} \} & 
\end{align*}
Prog \implies^+ \{ \text{id = id} ; \text{id = id + id} \}

When deriving a token sequence, if more than one non-terminal is present, we have a choice of which to expand next.

We must specify, at each step, which non-terminal is expanded, and what production is applied.

For simplicity we adopt a convention on what non-terminal is expanded at each step.

We can choose the leftmost possible non-terminal at each step.

A derivation that follows this rule is a leftmost derivation.

If we know a derivation is leftmost, we need only specify what
productions are used; the choice of non-terminal is always fixed.

To denote derivations that are leftmost,

we use \( \Rightarrow_L, \Rightarrow^+_L, \) and \( \Rightarrow^*_L \)

The production sequence discovered by a large class of parsers (the top-down parsers) is a leftmost derivation, hence these parsers produce a leftmost parse.

\[
\text{Prog} \Rightarrow_L \\
\{ \text{Stmts } \} \Rightarrow_L \\
\{ \text{Stmts} ; \text{Stmt} \} \Rightarrow_L \\
\{ \text{Stmt} ; \text{Stmt} \} \Rightarrow_L \\
\{ \text{id} = \text{Expr} ; \text{Stmt} \} \Rightarrow_L \\
\{ \text{id} = \text{id} ; \text{Stmt} \} \Rightarrow_L
\]
\{ \text{id} = \text{id} \, ; \, \text{id} = \text{Expr} \} \Rightarrow_L
\{ \text{id} = \text{id} \, ; \, \text{id} = \text{Expr} + \text{id} \} \Rightarrow_L
\{ \text{id} = \text{id} \, ; \, \text{id} = \text{id} + \text{id} \}
\text{Prog} \Rightarrow^+_L \{ \text{id} = \text{id} \, ; \, \text{id} = \text{id} + \text{id} \}
Rightmost Derivations

An alternative to a leftmost derivation is a rightmost derivation, in which the rightmost possible non-terminal is always expanded.

This derivation sequence may seem less intuitive given our normal left-to-right bias, but it corresponds to an important class of parsers (the bottom-up parsers, including CUP).

As a bottom-up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in reverse order.

The last production applied in a rightmost derivation is the first that is discovered, while the first production used, involving the start symbol, is the last to be discovered.
The sequence of productions recognized by a bottom-up parser is a rightmost parse.
It is the exact reverse of the production sequence that represents a rightmost derivation.
For derivations that are rightmost, we use the notation \( \Rightarrow_R \), \( \Rightarrow_R^+ \), and \( \Rightarrow_R^* \).

\[
\text{Prog} \Rightarrow_R \\
\{ \text{Stmts} \} \Rightarrow_R \\
\{ \text{Stmts} ; \text{Stmt} \} \Rightarrow_R \\
\{ \text{Stmts} ; \text{id} = \text{Expr} \} \Rightarrow_R \\
\{ \text{Stmts} ; \text{id} = \text{Expr} + \text{id} \} \Rightarrow_R \\
\{ \text{Stmts} ; \text{id} = \text{id} + \text{id} \} \Rightarrow_R \\
\{ \text{Stmt} ; \text{id} = \text{id} + \text{id} \} \Rightarrow_R \\
\{ \text{id} = \text{Expr} ; \text{id} = \text{id} + \text{id} \} \Rightarrow_R 
\]
\{ id = id \ ; \ id = id + id \} \\
Prog \Rightarrow^+ \{ id = id \ ; \ id = id + id \}

You can derive the same set of tokens using leftmost and rightmost derivations; the only difference is the order in which productions are used.
Ambiguous Grammars

Some grammars allow more than one parse tree for the same token sequence. Such grammars are ambiguous. Because compilers use syntactic structure to drive translation, ambiguity is undesirable—it may lead to an unexpected translation.

Consider

\[ E \rightarrow E - E \]
\[ \mid \text{id} \]

When parsing the input a - b - c (where a, b and c are scanned as identifiers)
we can build the following two parse trees:

The effect is to parse $a - b - c$ as either $(a - b) - c$ or $a - (b - c)$. These two groupings are certainly not equivalent.

Ambiguous grammars are usually voided in building compilers; the tools we use, like Yacc and CUP, strongly prefer unambiguous grammars.
To correct this ambiguity, we can use

\[ E \rightarrow E - id \]
\[ \mid id \]

Now \( a-b-c \) can only be parsed as:

```
  E
 / \   \
/   \  \
E -  E -
    / \   \
   /   \  \
  id  id id
```
Operator Precedence

Most programming languages have operator precedence rules that state the order in which operators are applied (in the absence of explicit parentheses). Thus in C and Java and CSX, \(a+b*c\) means compute \(b*c\), then add in \(a\).

These operators precedence rules can be incorporated directly into a CFG. Consider

\[
E \rightarrow E + T \\
| \ T \\

T \rightarrow T * P \\
| \ P \\
P \rightarrow id \\
| (E)
\]
Does $a+b*c$ mean $(a+b)*c$ or $a+(b*c)$? The grammar tells us! Look at the derivation tree:

The other grouping can’t be obtained unless explicit parentheses are used. (Why?)
Errors in Context-Free Grammars

Context-free grammars can contain errors, just as programs do. Some errors are easy to detect and fix; others are more subtle.

In context-free grammars we start with the start symbol, and apply productions until a terminal string is produced.

Some context-free grammars may contain useless non-terminals.

Non-terminals that are unreachable (from the start symbol) or that derive no terminal string are considered useless.

Useless non-terminals (and productions that involve them) can be
safely removed from a grammar without changing the language defined by the grammar.

A grammar containing useless non-terminals is said to be non-reduced.

After useless non-terminals are removed, the grammar is reduced.

Consider

\[
\begin{align*}
S & \rightarrow A \ B \\
& \mid \ x \\
B & \rightarrow b \\
A & \rightarrow a \ A \\
C & \rightarrow d
\end{align*}
\]

Which non-terminals are unreachable? Which derive no terminal string?
Finding Useless Non-terminals

To find non-terminals that can derive one or more terminal strings, we’ll use a marking algorithm. We iteratively mark terminals that can derive a string of terminals, until no more non-terminals can be marked. Unmarked non-terminals are useless.

(1) Mark all terminal symbols

(2) Repeat
    If all symbols on the righthand side of a production are marked
    Then mark the lefthand side
    Until no more non-terminals can be marked
We can use a similar marking algorithm to determine which non-terminals can be reached from the start symbol:

(1) Mark the Start Symbol

(2) Repeat
    If the lefthand side of a production is marked
    Then mark all non-terminals in the righthand side
    Until no more non-terminals can be marked
\[ \lambda \] Derivations

When parsing, we’ll sometimes need to know which non-terminals can derive \( \lambda \). (\( \lambda \) is “invisible” and hence tricky to parse).

We can use the following marking algorithm to decide which non-terminals derive \( \lambda \)

(1) For each production \( A \rightarrow \lambda \)
mark \( A \)

(2) Repeat
   If the entire righthand side of a production is marked
   Then mark the lefthand side
   Until no more non-terminals can be marked
As an example consider

\[ S \rightarrow A \ B \ C \]
\[ A \rightarrow a \]
\[ B \rightarrow C \ D \]
\[ D \rightarrow d \]
\[ C \rightarrow c \]

| \lambda |
| \lambda |
Recall that compilers prefer an unambiguous grammar because a unique parse tree structure can be guaranteed for all inputs. Hence a unique translation, guided by the parse tree structure, will be obtained.

We would like an algorithm that checks if a grammar is ambiguous. Unfortunately, it is undecidable whether a given CFG is ambiguous, so such an algorithm is impossible to create.

Fortunately for certain grammar classes, including those for which we can generate parsers, we can prove included grammars are unambiguous.
Potentially, the most serious flaw that a grammar might have is that it generates the “wrong language.”

This is a subtle point as a grammar serves as the definition of a language.

For established languages (like C or Java) there is usually a suite of programs created to test and validate new compilers. An incorrect grammar will almost certainly lead to incorrect compilations of test programs, which can be automatically recognized.

For new languages, initial implementors must thoroughly test the parser to verify that inputs are scanned and parsed as expected.
Parsers and Recognizers

Given a sequence of tokens, we can ask:
"Is this input syntactically valid?"
(Is it generable from the grammar?).
A program that answers this question is a recognizer.

Alternatively, we can ask:
"Is this input valid and, if it is, what is its structure (parse tree)?"

A program that answers this more general question is termed a parser.

We plan to use language structure to drive compilers, so we will be especially interested in parsers.
Two general approaches to parsing exist.

The first approach is top-down. A parser is top-down if it "discovers" the parse tree corresponding to a token sequence by starting at the top of the tree (the start symbol), and then expanding the tree (via predictions) in a depth-first manner.

Top-down parsing techniques are predictive in nature because they always predict the production that is to be matched before matching actually begins.
Consider

\[
E \rightarrow E + T \mid T \\
T \rightarrow T \ast id \mid id
\]

To parse id+id in a top-down manner, a parser builds a parse tree in the following steps:

```
   E  \rightarrow  E   \rightarrow  E          \rightarrow
     E + T       E + T
       \downarrow     \downarrow
         T
   id
```

```
   E  \rightarrow  E   \rightarrow  E          \rightarrow
     E + T       E + T
       \downarrow     \downarrow
         T        id
   id
```
A wide variety of parsing techniques take a different approach. They belong to the class of bottom-up parsers.

As the name suggests, bottom-up parsers discover the structure of a parse tree by beginning at its bottom (at the leaves of the tree which are terminal symbols) and determining the productions used to generate the leaves.

Then the productions used to generate the immediate parents of the leaves are discovered.

The parser continues until it reaches the production used to expand the start symbol.

At this point the entire parse tree has been determined.
A bottom-up parse of $id_1 + id_2$ would follow the following steps:

$$
\begin{align*}
&T \\
&\quad \Rightarrow \\
&\quad \quad id_1 \\
&T \\
&\quad \Rightarrow \\
&\quad \quad id_1 \\
&E \\
&\quad \Rightarrow \\
&\quad \quad T \\
&\quad \quad \quad \Rightarrow \\
&E + T \\
&\quad \quad \quad \Rightarrow \\
&\quad \quad \quad \quad T \\
&\quad \quad \quad \quad \Rightarrow \\
&E \\
&\quad \quad \quad \quad \Rightarrow \\
\end{align*}
$$
A Simple Top-Down Parser

We’ll build a rudimentary top-down parser that simply tries each possible expansion of a non-terminal, in order of production definition. If an expansion leads to a token sequence that doesn’t match the current token being parsed, we backup and try the next possible production choice. We stop when all the input tokens are correctly matched or when all possible production choices have been tried.
Example

Given the productions

\[ S \rightarrow a \]
\[ | ( S ) \]

we try a, then (a), then ((a)), etc.

Let’s next try an additional alternative:

\[ S \rightarrow a \]
\[ | ( S ) \]
\[ | ( S ) \]
\[ | ( S ] \]

Let’s try to parse a, then (a], then ((a]], etc.

We’ll count the number of productions we try for each input.
• For input = a
  We try $S \rightarrow a$ which works.
  Matches needed = 1

• For input = ( a ]
  We try $S \rightarrow a$ which fails.
  We next try $S \rightarrow ( S )$.
  We expand the inner $S$ three different ways; all fail.
  Finally, we try $S \rightarrow ( S ]$.
  The inner $S$ expands to a, which works.
  Total matches tried = $1 + (1+3)+(1+1) = 7$.

• For input = (( a ]]
  We try $S \rightarrow a$ which fails.
  We next try $S \rightarrow ( S )$.
  We match the inner $S$ to (a] using 7 steps, then fail to match the last ].
  Finally, we try $S \rightarrow ( S ]$.
  We match the inner $S$ to (a] using 7
steps, then match the last ].
Total matches tried = 1 + (1+8)+(1+8) = 17.

- For input = ((( a ]])
  We try S → a which fails.
  We next try S → ( S ).
  We match the inner S to ((a]) using 17 steps, then fail to match the last ].
  Finally, we try S → ( S ].
  We match the inner S to ((a]) using 17 steps, then match the last ].
  Total matches tried = 1 + (1+17) + (1+17) = 37.

Adding one extra ( ... ] pair doubles the number of matches we need to do the parse.

In fact to parse (i a) takes 5*2^i - 3 matches. This is exponential growth!
With a more effective dynamic programming approach, in which results of intermediate parsing steps are cached, we can reduce the number of matches needed to $n^3$ for an input with $n$ tokens. Is this acceptable? No!

Typical source programs have at least 1000 tokens, and $1000^3 = 10^9$ is a lot of steps, even for a fast modern computer. The solution? —Smarter selection in the choice of productions we try.
Reading Assignment

Get and read Chapter 5 of Crafting a Compiler featuring Java. (Available from DoIt Tech Store)

Computer Sciences Undergraduate Party

- Wednesday, November 6 at Noon in Union South

- To RSVP, go to:
  http://www-auth.cs.wisc.edu
  Click on "Continue to Web Forms."
  Under the Misc section go to the "RSVP for the undergraduate reception" form and check that you will attend.
**Prediction**

We want to avoid trying productions that can’t possibly work.

For example, if the current token to be parsed is an identifier, it is useless to try a production that begins with an integer literal.

Before we try a production, we’ll consider the set of terminals it might initially produce. If the current token is in this set, we’ll try the production.

If it isn’t, there is no way the production being considered could be part of the parse, so we’ll ignore it.

A predict function will tell us the set of tokens that might be initially generated from any production.
For $A \rightarrow X_1...X_n$, $\text{Predict}(A \rightarrow X_1...X_n)$

$= \text{Set of all initial (first) tokens derivable from } A \rightarrow X_1...X_n =$

$\{a \text{ in } V_t \mid A \rightarrow X_1...X_n \Rightarrow^* a...\}$

For example, given

$\text{Stmt} \rightarrow \text{Label id} = \text{Expr} ;$
$\mid \text{Label if Expr then Stmt} ;$
$\mid \text{Label read( IdList )} ;$
$\mid \text{Label id( Args )} ;$

$\text{Label} \rightarrow \text{intlit} :$
$\mid \lambda$

<table>
<thead>
<tr>
<th>Production</th>
<th>Predict Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Stmt} \rightarrow \text{Label id} = \text{Expr} ;$</td>
<td>${\text{id, intlit}}$</td>
</tr>
<tr>
<td>$\text{Stmt} \rightarrow \text{Label if Expr then Stmt} ;$</td>
<td>${\text{if, intlit}}$</td>
</tr>
<tr>
<td>$\text{Stmt} \rightarrow \text{Label read( IdList )} ;$</td>
<td>${\text{read, intlit}}$</td>
</tr>
<tr>
<td>$\text{Stmt} \rightarrow \text{Label id( Args )} ;$</td>
<td>${\text{id, intlit}}$</td>
</tr>
</tbody>
</table>
We now will match a production $p$ only if the next unmatched token is in $p$’s predict set. We’ll avoid trying productions that clearly won’t work, so parsing will be faster.

But what is the predict set of a $\lambda$-production?

It can’t be what’s generated by $\lambda$ (which is nothing!), so we’ll define it as the tokens that can follow the use of a $\lambda$-production.

That is, $\text{Predict}(A \rightarrow \lambda) = \text{Follow}(A)$

where (by definition)

$\text{Follow}(A) = \{ a \in V_t \mid S \Rightarrow^+ ...Aa... \}$

In our example,
$\text{Follow}(\text{Label} \rightarrow \lambda) = \{ \text{id, if, read} \}$
(since these terminals can immediately follow uses of Label in the given productions).
Now let’s parse

\[ \text{id ( intlit ) ;} \]

Our start symbol is Stmt and the initial token is id.
id can predict \[ \text{Stmt} \rightarrow \text{Label id} = \text{Expr} ; \]
id then predicts \[ \text{Label} \rightarrow \lambda \]
The id is matched, but "(" doesn’t match "=" so be backup and try a different production for Stmt.
id also predicts \[ \text{Stmt} \rightarrow \text{Label id ( Args ) ;} \]
Again, \[ \text{Label} \rightarrow \lambda \] is predicted and used, and the input tokens can match the rest of the remaining production.
We had only one misprediction, which is better than before.
Now we’ll rewrite the productions a bit to make predictions easier.

We remove the Label prefix from all the statement productions (now intlit won’t predict all four productions).

We now have

\[
\begin{align*}
\text{Stmt} & \rightarrow \text{Label} \ \text{BasicStmt} \\
\text{BasicStmt} & \rightarrow \text{id} = \text{Expr} ; \\
& \quad | \quad \text{if Expr then Stmt} ; \\
& \quad | \quad \text{read ( IdList )} ; \\
& \quad | \quad \text{id ( Args )} ; \\
\text{Label} & \rightarrow \text{intlit } : \\
& \quad | \quad \lambda \\
\end{align*}
\]

Now id predicts two different BasicStmt productions. If we rewrite these two productions into

\[
\begin{align*}
\text{BasicStmt} & \rightarrow \text{id} \ \text{StmtSuffix} \\
\text{StmtSuffix} & \rightarrow = \ \text{Expr} ; \\
& \quad | \quad ( \text{Args} ) ;
\end{align*}
\]
we no longer have any doubt over which production id predicts.

We now have

<table>
<thead>
<tr>
<th>Production</th>
<th>Predict Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stmt \rightarrow Label BasicStmt</td>
<td>Not needed!</td>
</tr>
<tr>
<td>BasicStmt \rightarrow id StmtSuffix</td>
<td>{id}</td>
</tr>
<tr>
<td>BasicStmt \rightarrow if Expr then Stmt ;</td>
<td>{if}</td>
</tr>
<tr>
<td>BasicStmt \rightarrow read ( IdList ) ;</td>
<td>{read}</td>
</tr>
<tr>
<td>StmtSuffix \rightarrow ( Args ) ;</td>
<td>{(}}</td>
</tr>
<tr>
<td>StmtSuffix \rightarrow = Expr ;</td>
<td>{=}</td>
</tr>
<tr>
<td>Label \rightarrow intlit :</td>
<td>{intlit}</td>
</tr>
<tr>
<td>Label \rightarrow \lambda</td>
<td>{if, id, read}</td>
</tr>
</tbody>
</table>

This grammar generates the same statements as our original grammar did, but now prediction never fails!
Whenever we must decide what production to use, the predict sets for productions with the same lefthand side are always disjoint.

Any input token will predict a unique production or no production at all (indicating a syntax error).

If we never mispredict a production, we never backup, so parsing will be fast and absolutely accurate!
LL(1) Grammars

A context-free grammar whose Predict sets are always disjoint (for the same non-terminal) is said to be LL(1).

LL(1) grammars are ideally suited for top-down parsing because it is always possible to correctly predict the expansion of any non-terminal. No backup is ever needed.

Formally, let

First(X₁...Xₙ) =
{a in Vₜ | A → X₁...Xₙ ⇒ * a...}

Follow(A) = {a in Vₜ | S ⇒⁺ ...Aa...}
\[ \text{Predict}(A \rightarrow X_1...X_n) = \]
\text{If } X_1...X_n \Rightarrow^* \lambda \text{ Then } \text{First}(X_1...X_n) \cup \text{Follow}(A) \text{ Else } \text{First}(X_1...X_n) \]

If some CFG, G, has the property that for all pairs of distinct productions with the same lefthand side, \( A \rightarrow X_1...X_n \) and \( A \rightarrow Y_1...Y_m \) it is the case that
\[ \text{Predict}(A \rightarrow X_1...X_n) \cap \text{Predict}(A \rightarrow Y_1...Y_m) = \emptyset \]
then G is LL(1).

LL(1) grammars are easy to parse in a top-down manner since predictions are always correct.
Example

<table>
<thead>
<tr>
<th>Production</th>
<th>Predict Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>S → A a</td>
<td>{b,d,a}</td>
</tr>
<tr>
<td>A → B D</td>
<td>{b, d, a}</td>
</tr>
<tr>
<td>B → b</td>
<td>{ b }</td>
</tr>
<tr>
<td>B → λ</td>
<td>{d, a}</td>
</tr>
<tr>
<td>D → d</td>
<td>{ d }</td>
</tr>
<tr>
<td>D → λ</td>
<td>{ a }</td>
</tr>
</tbody>
</table>

Since the predict sets of both B productions and both D productions are disjoint, this grammar is LL(1).
Recursive Descent Parsers

An early implementation of top-down (LL(1)) parsing was recursive descent. A parser was organized as a set of parsing procedures, one for each non-terminal. Each parsing procedure was responsible for parsing a sequence of tokens derivable from its non-terminal.

For example, a parsing procedure, A, when called, would call the scanner and match a token sequence derivable from A.

Starting with the start symbol’s parsing procedure, we would then match the entire input, which must be derivable from the start symbol.
This approach is called recursive descent because the parsing procedures were typically recursive, and they descended down the input’s parse tree (as top-down parsers always do).
Building A Recursive Descent Parser

We start with a procedure `Match`, that matches the current input token against a predicted token:

```c
void Match(Terminal a) {
    if (a == currentToken)
        currentToken = Scanner();
    else SyntaxError();
}
```

To build a parsing procedure for a non-terminal `A`, we look at all productions with `A` on the lefthand side:

\[ A \rightarrow X_1 \ldots X_n \mid A \rightarrow Y_1 \ldots Y_m \mid \ldots \]

We use predict sets to decide which production to match (LL(1) grammars always have disjoint predict sets).
We match a production’s righthand side by calling `Match` to match terminals, and calling parsing procedures to match non-terminals. The general form of a parsing procedure for

\[ A \rightarrow X_1...X_n | A \rightarrow Y_1...Y_m | \ldots \]

is

```c
void A() {
    if (currentToken in Predict(A\rightarrow X_1...X_n))
        for(i=1;i<=n;i++)
            if (X[i] is a terminal)
                Match(X[i]);
            else X[i]();
    else
        if (currentToken in Predict(A\rightarrow Y_1...Y_m))
            for(i=1;i<=m;i++)
                if (Y[i] is a terminal)
                    Match(Y[i]);
                else Y[i]();
        else
            // Handle other A \rightarrow \ldots productions
    else // No production predicted
        SyntaxError();
}
```
Usually this general form isn’t used. Instead, each production is “macro-expanded” into a sequence of \texttt{Match} and parsing procedure calls.
## Example: CSX-Lite

<table>
<thead>
<tr>
<th>Production</th>
<th>Predict Set</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prog</strong> → { <strong>Stmts</strong> } Eof</td>
<td>{</td>
</tr>
<tr>
<td><strong>Stmts</strong> → <strong>Stmt</strong> <strong>Stmts</strong></td>
<td>id if</td>
</tr>
<tr>
<td><strong>Stmts</strong> → λ</td>
<td>)</td>
</tr>
<tr>
<td><strong>Stmt</strong> → id = <strong>Expr</strong> ;</td>
<td>id</td>
</tr>
<tr>
<td><strong>Stmt</strong> → if ( <strong>Expr</strong> ) <strong>Stmt</strong></td>
<td>if</td>
</tr>
<tr>
<td><strong>Expr</strong> → id <strong>Etail</strong></td>
<td>id</td>
</tr>
<tr>
<td><strong>Etail</strong> → + <strong>Expr</strong></td>
<td>+</td>
</tr>
<tr>
<td><strong>Etail</strong> → - <strong>Expr</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Etail</strong> → λ</td>
<td>) ;</td>
</tr>
</tbody>
</table>
CSX-Lite Parsing Procedures

```c
void Prog() {
    Match("{");
    Stmts();
    Match("}");
    Match(Eof);
}

void Stmts() {
    if (currentToken == id ||
        currentToken == if) {
        Stmt();
        Stmts();
    } else {
        /* null */
    }
}
```
void Stmt() {
    if (currentToken == id)
    {
        Match(id);
        Match("=");
        Expr();
        Match(";");
    } else {
        Match(if);
        Match("(");
        Expr();
        Match(")");
        Stmt();
    }
}

void Expr() {
    Match(id);
    Etail();
}

void Etail() {
    if (currentToken == "+")
    {
        Match("+");
        Expr();
    } else if (currentToken == "-")
    {
        Match("-");
        Expr();
    } else {
        /* null */
    }
}
Let's use recursive descent to parse
{ a = b + c; } Eof
We start by calling $Prog()$ since this represents the start symbol.

<table>
<thead>
<tr>
<th>Calls Pending</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Prog()$</td>
<td>{ a = b + c; } Eof</td>
</tr>
<tr>
<td>Match(&quot;{&quot;);</td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Stmt();</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td>Calls Pending</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Match(id);</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>Match(&quot;=&quot;);</td>
<td></td>
</tr>
<tr>
<td>Expr();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;;&quot;);</td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;=&quot;);</td>
<td>= b + c; } Eof</td>
</tr>
<tr>
<td>Expr();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;;&quot;);</td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td>Expr();</td>
<td>b + c; } Eof</td>
</tr>
<tr>
<td>Match(&quot;;&quot;);</td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td>Match(id);</td>
<td>b + c; } Eof</td>
</tr>
<tr>
<td>Etail();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;;&quot;);</td>
<td></td>
</tr>
<tr>
<td>Stmts();</td>
<td></td>
</tr>
<tr>
<td>Match(&quot;}&quot;);</td>
<td></td>
</tr>
<tr>
<td>Match(Eof);</td>
<td></td>
</tr>
<tr>
<td>Calls Pending</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td><code>Etail(); Match(&quot;;&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</code></td>
<td><code>+ c; } Eof</code></td>
</tr>
<tr>
<td><code>Match(&quot;+&quot;); Expr(); Match(&quot;;&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</code></td>
<td><code>+ c; } Eof</code></td>
</tr>
<tr>
<td><code>Expr(); Match(&quot;;&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</code></td>
<td><code>c; } Eof</code></td>
</tr>
<tr>
<td><code>Match(id); Etail(); Match(&quot;;&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</code></td>
<td><code>c; } Eof</code></td>
</tr>
<tr>
<td>Calls Pending</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-----------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
</tbody>
</table>
| `Etail();
Match(";");
Stmts();
Match("}");
Match(Eof);`                                                          | `; } Eof`       |
| `/* null */
Match(";");
Stmts();
Match("}");
Match(Eof);`                                                          | `; } Eof`       |
| `Match(";");
Stmts();
Match("}");
Match(Eof);`                                                          | `; } Eof`       |
| `Stmts();
Match("}");
Match(Eof);`                                                          | `{ } Eof`       |
| `/* null */
Match("}");
Match(Eof);`                                                          | `{ } Eof`       |
| `Match("}");
Match(Eof);`                                                          | `{ } Eof`       |
<table>
<thead>
<tr>
<th>Calls Pending</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>Match(Eof);</code></td>
<td><code>Eof</code></td>
</tr>
<tr>
<td><code>Done!</code></td>
<td><code>All input matched</code></td>
</tr>
</tbody>
</table>
Syntax Errors in Recursive Descent Parsing

In recursive descent parsing, syntax errors are automatically detected. In fact, they are detected as soon as possible (as soon as the first illegal token is seen).

How? When an illegal token is seen by the parser, either it fails to predict any valid production or it fails to match an expected token in a call to Match.

Let’s see how the following illegal CSX-lite program is parsed:

{ b + c = a; } Eof

(Where should the first syntax error be detected?)
<table>
<thead>
<tr>
<th>Calls Pending</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prog()</td>
<td>{ b + c = a; } Eof</td>
</tr>
<tr>
<td>Match(&quot;{&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</td>
<td>{ b + c = a; } Eof</td>
</tr>
<tr>
<td>Stmts(); Match(&quot;}&quot;); Match(Eof);</td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>Stmt(); Stmts(); Match(&quot;}&quot;); Match(Eof);</td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>Match(id); Match(&quot;=&quot;); Expr(); Match(&quot;;&quot;); Stmts(); Match(&quot;}&quot;); Match(Eof);</td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>Calls Pending</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td><code>Match(&quot;=&quot;); Expr(); Match(&quot;;&quot;); Stmts(); Match(&quot;{&quot;); Match(Eof);</code></td>
<td><code>+ c = a; } Eof</code></td>
</tr>
<tr>
<td>Call to Match fails!</td>
<td><code>+ c = a; } Eof</code></td>
</tr>
</tbody>
</table>
Table-Driven Top-Down Parsers

Recursive descent parsers have many attractive features. They are actual pieces of code that can be read by programmers and extended. This makes it fairly easy to understand how parsing is done. Parsing procedures are also convenient places to add code to build ASTs, or to do type-checking, or to generate code.

A major drawback of recursive descent is that it is quite inconvenient to change the grammar being parsed. Any change, even a minor one, may force parsing procedures to be reprogrammed, as
productions and predict sets are modified.

To a less extent, recursive descent parsing is less efficient than it might be, since subprograms are called just to match a single token or to recognize a righthand side.

An alternative to parsing procedures is to encode all prediction in a parsing table. A pre-programmed driver program can use a parse table (and list of productions) to parse any LL(1) grammar.

If a grammar is changed, the parse table and list of productions will change, but the driver need not be changed.
Late Penalties Suspended

There will be no late penalties for Project 3 (Parser) and Homework #1. Final accept dates (November 4 and October 29) are still in effect.
LL(1) Parse Tables

An LL(1) parse table, T, is a two-dimensional array. Entries in T are production numbers or blank (error) entries.

T is indexed by:

- A, a non-terminal. A is the non-terminal we want to expand.
- CT, the current token that is to be matched.
- T[A][CT] = A → X₁...Xₙ
  if CT is in Predict(A → X₁...Xₙ)
- T[A][CT] = error
  if CT predicts no production with A as its lefthand side
CSX-lite Example

<table>
<thead>
<tr>
<th>Production</th>
<th>Predict Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Prog → { Stmts } Eof</td>
<td>{</td>
</tr>
<tr>
<td>2 Stmts → Stmt Stmts</td>
<td>id if</td>
</tr>
<tr>
<td>3 Stmts → λ</td>
<td>}</td>
</tr>
<tr>
<td>4 Stmt → id = Expr ;</td>
<td>id</td>
</tr>
<tr>
<td>5 Stmt → if ( Expr ) Stmt</td>
<td>if</td>
</tr>
<tr>
<td>6 Expr → id Etail</td>
<td>id</td>
</tr>
<tr>
<td>7 Etail → + Expr</td>
<td>+</td>
</tr>
<tr>
<td>8 Etail → - Expr</td>
<td>-</td>
</tr>
<tr>
<td>9 Etail → λ</td>
<td>) ;</td>
</tr>
</tbody>
</table>

|                     | { | } | if | ( | ) | id | = | + | - | ; | eof |
|---------------------|----|----|----|----|----|----|----|----|----|----|
| Prog                | 1  |    |    |    |    |    |    |    |    |    |
| Stmts               | 3  | 2  | 2  |    |    |    |    |    |    |    |
| Stmt                | 5  | 4  |    |    |    |    |    |    |    |    |
| Expr                |    | 6  |    |    |    |    |    |    |    |    |
| Etail               |    | 9  | 7  | 8  | 9  |    |    |    |    |    |
LL(1) Parser Driver

Here is the driver we’ll use with the LL(1) parse table. We’ll also use a parse stack that remembers symbols we have yet to match.

```c
void LLDriver(){
    Push(StartSymbol);
    while(! stackEmpty()){ //Let X=Top symbol on parse stack
        //Let CT = current token to match
        if (isTerminal(X)) {
            match(X); //CT is updated
            pop();    //X is updated
        } else if (T[X][CT] != Error){
            //Let T[X][CT] = X->Y₁...Yₘ
            Replace X with
            Y₁...Yₘ on parse stack
        } else SyntaxError(CT);
    }
}
```
Example of LL(1) Parsing

We’ll again parse
\[ \{ \ a = b + c; \ \} \ Eof \]
We start by placing Prog (the start symbol) on the parse stack.

<table>
<thead>
<tr>
<th>Parse Stack</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prog</td>
<td>{ a = b + c; } Eof</td>
</tr>
<tr>
<td>{</td>
<td></td>
</tr>
<tr>
<td>Stmts</td>
<td></td>
</tr>
<tr>
<td>}</td>
<td>{ a = b + c; } Eof</td>
</tr>
<tr>
<td>Eof</td>
<td></td>
</tr>
<tr>
<td>Stmts</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>}</td>
<td></td>
</tr>
<tr>
<td>Eof</td>
<td></td>
</tr>
<tr>
<td>Stmt</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>Stmts</td>
<td></td>
</tr>
<tr>
<td>}</td>
<td></td>
</tr>
<tr>
<td>Eof</td>
<td></td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>id = Expr ; Stmts } Eof</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td>= Expr ; Stmts } Eof</td>
<td></td>
</tr>
<tr>
<td>Expr ; Stmts } Eof</td>
<td>= b + c; } Eof</td>
</tr>
<tr>
<td>id Etail ; Stmts } Eof</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b + c; } Eof</td>
</tr>
</tbody>
</table>

Parse Stack Remaining Input
<table>
<thead>
<tr>
<th>Parse Stack</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etail ; Stmts } Eof</td>
<td>+ c; } Eof</td>
</tr>
<tr>
<td>+ Expr ; Stmts } Eof</td>
<td>+ c; } Eof</td>
</tr>
<tr>
<td>Expr ; Stmts } Eof</td>
<td>c; } Eof</td>
</tr>
<tr>
<td>id Etail ; Stmts } Eof</td>
<td>c; } Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td><code>Etail ; Stmts } Eof</code></td>
<td><code>; } Eof</code></td>
</tr>
<tr>
<td><code>; Stmts } Eof</code></td>
<td><code>; } Eof</code></td>
</tr>
<tr>
<td><code>Stmts } Eof</code></td>
<td><code>} Eof</code></td>
</tr>
<tr>
<td><code>} Eof</code></td>
<td><code>} Eof</code></td>
</tr>
<tr>
<td><code>Eof</code></td>
<td><code>Eof</code></td>
</tr>
<tr>
<td><strong>Done!</strong></td>
<td>All input matched</td>
</tr>
</tbody>
</table>
Syntax Errors in LL(1) Parsing

In LL(1) parsing, syntax errors are automatically detected as soon as the first illegal token is seen.

How? When an illegal token is seen by the parser, either it fetches an error entry from the LL(1) parse table or it fails to match an expected token.

Let’s see how the following illegal CSX-lite program is parsed:

{ b + c = a; } Eof

(Where should the first syntax error be detected?)
<table>
<thead>
<tr>
<th>Parse Stack</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prog</td>
<td>{ b + c = a; } Eof</td>
</tr>
<tr>
<td></td>
<td>{ b + c = a; } Eof</td>
</tr>
<tr>
<td></td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td></td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>id = Expr ; Stmts</td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td></td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
</tr>
<tr>
<td><code>=</code> <code>Expr</code> <code>;</code> <code>Stmts</code> <code>}</code> <code>Eof</code></td>
<td><code>+ c = a; } Eof</code></td>
</tr>
</tbody>
</table>

Current token (+) fails to match expected token (=)!

| Current token (+) fails to match expected token (=)! | `+ c = a; } Eof` |
How do LL(1) Parsers Build Syntax Trees?

So far our LL(1) parser has acted like a recognizer. It verifies that input tokens are syntactically correct, but it produces no output.

Building complete (concrete) parse trees automatically is fairly easy. As tokens and non-terminals are matched, they are pushed onto a second stack, the semantic stack.

At the end of each production, an action routine pops off n items from the semantic stack (where n is the length of the production’s right-hand side). It then builds a syntax tree whose root is the lefthand side, and
whose children are the n items just popped off.

For example, for production

$$\textit{Stmt} \rightarrow \text{id} = \textit{Expr} ;$$

the parser would include an action symbol after the “;” whose actions are:

- \( P4 = \text{pop}() \); // Semicolon token
- \( P3 = \text{pop}() \); // Syntax tree for \( \textit{Expr} \)
- \( P2 = \text{pop}() \); // Assignment token
- \( P1 = \text{pop}() \); // Identifier token
- \( \text{Push(new StmtNode(P1,P2,P3,P4))} \);
Creating Abstract Syntax Trees

Recall that we prefer that parsers generate abstract syntax trees, since they are simpler and more concise. Since a parser generator can’t know what tree structure we want to keep, we must allow the user to define “custom” action code, just as Java CUP does.

We allow users to include “code snippets” in Java or C. We also allow labels on symbols so that we can refer to the tokens and trees we wish to access. Our production and action code will now look like this:

stmt → id:i = Expr:e ;

{ : RESULT = new StmtNode(i,e); : }
How do We Make Grammars LL(1)?

Not all grammars are LL(1); sometimes we need to modify a grammar’s productions to create the disjoint Predict sets LL1) requires.

There are two common problems in grammars that make unique prediction difficult or impossible:

   Two or more productions with the same lefthand side begin with the same symbol(s).
   For example,
   
   \[
   \text{Stmt} \rightarrow \text{id} = \text{Expr} \ ; \\
   \text{Stmt} \rightarrow \text{id} ( \text{Args} ) \ ;
   \]

2. Left-Recursion

A production of the form

\[ A \rightarrow A \ldots \]

is said to be left-recursive. When a left-recursive production is used, a non-terminal is immediately replaced by itself (with additional symbols following).

Any grammar with a left-recursive production can never be LL(1).

Why?

Assume a non-terminal A reaches the top of the parse stack, with CT as the current token. The LL(1) parse table entry, \( T[A][CT] \), predicts \( A \rightarrow A \ldots \)

We expand A again, and \( T[A][CT] \), so we predict \( A \rightarrow A \ldots \) again. We are in an infinite prediction loop!
Eliminating Common Prefixes

Assume we have two or more productions with the same left-hand side and a common prefix on their right-hand sides:

\[
A \rightarrow \alpha \beta | \alpha \gamma | ... | \alpha \delta
\]

We create a new non-terminal, \( X \).

We then rewrite the above productions into:

\[
A \rightarrow \alpha X \quad X \rightarrow \beta | \gamma | ... | \delta
\]

For example,

\[
\text{Stmt} \rightarrow \text{id} = \text{Expr} ; \\
\text{Stmt} \rightarrow \text{id ( Args )} ;
\]

becomes

\[
\text{Stmt} \rightarrow \text{id StmtSuffix} \\
\text{StmtSuffix} \rightarrow = \text{Expr} ; \\
\text{StmtSuffix} \rightarrow ( \text{Args} ) ;
\]
Eliminating Left Recursion

Assume we have a non-terminal that is left recursive:

\[ A \rightarrow A\alpha \quad A \rightarrow \beta | \gamma | ... | \delta \]

To eliminate the left recursion, we create two new non-terminals, \( N \) and \( T \).

We then rewrite the above productions into:

\[ A \rightarrow N \ T 
N \rightarrow \beta | \gamma | ... | \delta 
T \rightarrow \alpha \ T | \lambda \]
For example,

\[
\begin{align*}
\text{Expr} & \rightarrow \text{Expr} + \text{id} \\
\text{Expr} & \rightarrow \text{id}
\end{align*}
\]

becomes

\[
\begin{align*}
\text{Expr} & \rightarrow \text{N T} \\
\text{N} & \rightarrow \text{id} \\
\text{T} & \rightarrow + \text{id} \text{T} | \lambda
\end{align*}
\]

This simplifies to:

\[
\begin{align*}
\text{Expr} & \rightarrow \text{id} \text{T} \\
\text{T} & \rightarrow + \text{id} \text{T} | \lambda
\end{align*}
\]
Reading Assignment

Get Chapter 6 of Crafting a Compiler featuring Java. Read Sections 6.1 to 6.5.1.

(Available from Dolt Tech Store)
How does JavaCup Work?

The main limitation of LL(1) parsing is that it must predict the correct production to use when it first starts to match the production’s righthand side.

An improvement to this approach is the LALR(1) parsing method that is used in JavaCUP (and Yacc and Bison too).

The LALR(1) parser is bottom-up in approach. It tracks the portion of a righthand side already matched as tokens are scanned. It may not know immediately which is the correct production to choose, so it tracks sets of possible matching productions.
Configurations

We’ll use the notation

\[ X \rightarrow A \mathbf{B} \cdot C D \]

to represent the fact that we are trying to match the production

\[ X \rightarrow A B \cdot C D \]

with A and B matched so far.

A production with a “·” somewhere in its righthand side is called a configuration.

Our goal is to reach a configuration with the “dot” at the extreme right:

\[ X \rightarrow A B C D \cdot \]

This indicates that an entire production has just been matched.

Since we may not know which production will eventually be fully
matched, we may need to track a configuration set. A configuration set is sometimes called a state.

When we predict a production, we place the “dot” at the beginning of a production:

\[ X \rightarrow \cdot A B C D \]

This indicates that the production may possibly be matched, but no symbols have actually yet been matched.

We may predict a \( \lambda \)-production:

\[ X \rightarrow \lambda \cdot \]

When a \( \lambda \)-production is predicted, it is immediately matched, since \( \lambda \) can be matched at any time.
Starting the Parse

At the start of the parse, we know some production with the start symbol must be used initially. We don’t yet know which one, so we predict them all:

\[ S \rightarrow \cdot A B C D \]
\[ S \rightarrow \cdot e F g \]
\[ S \rightarrow \cdot h I \]

...
Closure

When we encounter a configuration with the dot to the left of a non-terminal, we know we need to try to match that non-terminal.

Thus in

\[ X \rightarrow \cdot A B C D \]

we need to match some production with A as its left hand side.

Which production?

We don’t know, so we predict all possibilities:

\[ A \rightarrow \cdot P Q R \]
\[ A \rightarrow \cdot s T \]

... 

The newly added configurations may predict other non-terminals, forcing
additional productions to be included. We continue this process until no additional configurations can be added.

This process is called closure (of the configuration set).

Here is the closure algorithm:

```
ConfigSet Closure(ConfigSet C) {
    repeat
        if (X → α • B δ is in C &&
            B is a non-terminal)
            Add all configurations of
            the form B → •γ to C)
        until (no more configurations
            can be added);
    return C;
}
```
Example of Closure

Assume we have the following grammar:

\[
\begin{align*}
S & \rightarrow A \: b \\
A & \rightarrow C \: D \\
C & \rightarrow D \\
C & \rightarrow c \\
D & \rightarrow d
\end{align*}
\]

To compute Closure(S \rightarrow \cdot A \: b) we first include all productions that rewrite A:

\[
A \rightarrow \cdot C \: D
\]

Now C productions are included:

\[
C \rightarrow \cdot D \\
C \rightarrow \cdot c
\]

Finally, the D production is added:
D → • d

The complete configuration set is:

S → • A b
A → • C D
C → • D
C → • c
D → • d

This set tells us that if we want to match an A, we will need to match a C, and this is done by matching a c or d token.
Shift Operations

When we match a symbol (a terminal or non-terminal), we shift the “dot” past the symbol just matched. Configurations that don’t have a dot to the left of the matched symbol are deleted (since they didn’t correctly anticipate the matched symbol).

The \texttt{GoTo} function computes an updated configuration set after a symbol is shifted:

\begin{verbatim}
ConfigSet GoTo(ConfigSet C, Symbol X) {
    B = \emptyset;
    for each configuration f in C{
        if (f is of the form A → α•X δ)
            Add A → α X •δ to B;
        return Closure(B);
    }
}
\end{verbatim}
For example, if $c$ is

\begin{align*}
S & \rightarrow \cdot A \ b \\
A & \rightarrow \cdot C \ D \\
C & \rightarrow \cdot D \\
C & \rightarrow \cdot c \\
D & \rightarrow \cdot d
\end{align*}

and $x$ is $C$, then $\text{GoTo}$ returns

\begin{align*}
A & \rightarrow \ C \cdot D \\
D & \rightarrow \cdot d
\end{align*}
Reduce Actions

When the dot in a configuration reaches the rightmost position, we have matched an entire right-hand side. We are ready to replace the right-hand side symbols with the left-hand side of the production. The left-hand side symbol can now be considered matched.

If a configuration set can shift a token and also reduce a production, we have a potential shift/reduce error.

If we can reduce more than one production, we have a potential reduce/reduce error.

How do we decide whether to do a shift or reduce? How do we choose among more than one reduction?
We examine the next token to see if it is consistent with the potential reduce actions.

The simplest way to do this is to use Follow sets, as we did in LL(1) parsing.

If we have a configuration

\[ A \rightarrow \alpha \cdot \]

we will reduce this production only if the current token, CT, is in Follow(A).

This makes sense since if we reduce \( \alpha \) to A, we can’t correctly match CT if CT can’t follow A.
Shift/Reduce and Reduce/Reduce Errors

If we have a parse state that contains the configurations

\[ A \rightarrow \alpha \cdot \]
\[ B \rightarrow \beta \cdot a \gamma \]

and \( a \) in \( \text{Follow}(A) \) then there is an unresolvable shift/reduce conflict. This grammar can’t be parsed.

Similarly, if we have a parse state that contains the configurations

\[ A \rightarrow \alpha \cdot \]
\[ B \rightarrow \beta \cdot \]

and \( \text{Follow}(A) \cap \text{Follow}(B) \neq \emptyset \), then the parser has an unresolvable reduce/reduce conflict. This grammar can’t be parsed.
Building Parse States

All the manipulations needed to build and complete configuration sets suggest that parsing may be slow—configuration sets need to be updated after each token is matched.

Fortunately, all the configuration sets we ever will need can be computed and tabled in advance, when a tool like Java Cup builds a parser.

The idea is simple. We first compute an initial parse state, \( s_0 \), that corresponds to predicting productions that expand the start symbol. We then just compute successor states for each token that might be scanned. A complete set of states can be computed. For typical programming
language grammars, only a few hundred states are needed.

Here is the algorithm that builds a complete set of parse states for a grammar:

StateSet BuildStates()
{
    Let $s_0$ = Closure($\{S \rightarrow \cdot \alpha, S \rightarrow \cdot \beta, \ldots\}$);
    C = $\{s_0\}$;
    while (not all states in C are marked)
    {
        Choose any unmarked state, s, in C
        Mark s;
        For each X in terminals U nonterminals {
            if (GoTo(s,X) is not in C)
            Add GoTo(s,X) to C;
        }
    }
    return C;
}
### Configuration Sets for CSX-Lite

<table>
<thead>
<tr>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
</tr>
<tr>
<td>$s_1$</td>
</tr>
<tr>
<td>$s_2$</td>
</tr>
<tr>
<td>$s_3$</td>
</tr>
<tr>
<td>$s_4$</td>
</tr>
<tr>
<td>$s_5$</td>
</tr>
</tbody>
</table>

#### Configuration Set

- **$s_0$**
  
  \[
  \text{Prog} \rightarrow \bullet \{ \text{Stmts} \} \ Eof
  \]

- **$s_1$**
  
  \[
  \begin{align*}
  \text{Prog} & \rightarrow \{ \bullet \text{Stmts} \} \ Eof \\
  \text{Stmts} & \rightarrow \bullet \text{Stmt} \ \text{Stmts} \\
  \text{Stmts} & \rightarrow \lambda \bullet \\
  \text{Stmt} & \rightarrow \bullet \text{id} = \text{Expr} ; \\
  \text{Stmt} & \rightarrow \bullet \text{if} \ ( \text{Expr} ) \ \text{Stmt}
  \end{align*}
  \]

- **$s_2$**
  
  \[
  \text{Prog} \rightarrow \{ \text{Stmts} \bullet \} \ Eof
  \]

- **$s_3$**
  
  \[
  \begin{align*}
  \text{Stmts} & \rightarrow \text{Stmt} \bullet \text{Stmts} \\
  \text{Stmts} & \rightarrow \bullet \text{Stmt} \ \text{Stmts} \\
  \text{Stmts} & \rightarrow \lambda \bullet \\
  \text{Stmt} & \rightarrow \bullet \text{id} = \text{Expr} ; \\
  \text{Stmt} & \rightarrow \bullet \text{if} \ ( \text{Expr} ) \ \text{Stmt}
  \end{align*}
  \]

- **$s_4$**
  
  \[
  \text{Stmt} \rightarrow \text{id} \bullet = \text{Expr} ;
  \]

- **$s_5$**
  
  \[
  \text{Stmt} \rightarrow \text{if} \bullet ( \text{Expr} ) \ \text{Stmt}
  \]
<table>
<thead>
<tr>
<th>State</th>
<th>Configuration Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_6$</td>
<td>$\text{Prog} \rightarrow { \text{Stmts} } \cdot \text{Eof}$</td>
</tr>
<tr>
<td>$s_7$</td>
<td>$\text{Stmts} \rightarrow \text{Stmt} ; \text{Stmts} \cdot$</td>
</tr>
</tbody>
</table>
| $s_8$ | $\begin{align*} 
\text{Stmt} & \rightarrow \text{id} = \cdot \text{Expr} \; ; \\
\text{Expr} & \rightarrow \cdot \text{Expr} + \text{id} \\
\text{Expr} & \rightarrow \cdot \text{Expr} - \text{id} \\
\text{Expr} & \rightarrow \cdot \text{id} 
\end{align*}$ |
| $s_9$ | $\begin{align*} 
\text{Stmt} & \rightarrow \text{if} \; ( \cdot \text{Expr} ) \; \text{Stmt} \\
\text{Expr} & \rightarrow \cdot \text{Expr} + \text{id} \\
\text{Expr} & \rightarrow \cdot \text{Expr} - \text{id} \\
\text{Expr} & \rightarrow \cdot \text{id} 
\end{align*}$ |
| $s_{10}$ | $\text{Prog} \rightarrow \{ \text{Stmts} \} \; \text{Eof} \cdot$ |
| $s_{11}$ | $\begin{align*} 
\text{Stmt} & \rightarrow \text{id} = \text{Expr} \cdot \; ; \\
\text{Expr} & \rightarrow \text{Expr} \cdot + \text{id} \\
\text{Expr} & \rightarrow \text{Expr} \cdot - \text{id} 
\end{align*}$ |
| $s_{12}$ | $\text{Expr} \rightarrow \text{id} \cdot$ |
| $s_{13}$ | $\begin{align*} 
\text{Stmt} & \rightarrow \text{if} \; ( \cdot \text{Expr} \cdot ) \; \text{Stmt} \\
\text{Expr} & \rightarrow \text{Expr} \cdot + \text{id} \\
\text{Expr} & \rightarrow \text{Expr} \cdot - \text{id} 
\end{align*}$ |
<table>
<thead>
<tr>
<th>State</th>
<th>Configuration Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{14}$</td>
<td>Stmt $\rightarrow$ id = Expr ; $\cdot$</td>
</tr>
<tr>
<td>$s_{15}$</td>
<td>Expr $\rightarrow$ Expr + $\cdot$ id</td>
</tr>
<tr>
<td>$s_{16}$</td>
<td>Expr $\rightarrow$ Expr - $\cdot$ id</td>
</tr>
</tbody>
</table>
| $s_{17}$ | Stmt $\rightarrow$ if ( Expr ) $\cdot$ Stmt  
Stmt $\rightarrow$ $\cdot$ id = Expr ;  
Stmt $\rightarrow$ $\cdot$ if ( Expr ) Stmt |
| $s_{18}$ | Expr $\rightarrow$ Expr + id $\cdot$                                       |
| $s_{19}$ | Expr $\rightarrow$ Expr - id $\cdot$                                       |
| $s_{20}$ | Stmt $\rightarrow$ if ( Expr ) Stmt $\cdot$       |
Late Penalties Suspended

There will be no late penalties for the remainder of the semester. Final accept dates are still in effect.
Parser Action Table

We will table possible parser actions based on the current state (configuration set) and token.

Given configuration set C and input token T four actions are possible:

- Reduce i: The i-th production has been matched.
- Shift: Match the current token.
- Accept: Parse is correct and complete.
- Error: A syntax error has been discovered.
We will let $A[C][T]$ represent the possible parser actions given configuration set $C$ and input token $T$.

$$A[C][T] = \{ \text{Reduce } i \mid i\text{-th production is } A \rightarrow \alpha \text{ and } A \rightarrow \alpha \cdot \text{ is in } C \text{ and } T \text{ in } \text{Follow}(A) \}$$

$$\cup \text{ (If } (B \rightarrow \beta \cdot T \gamma \text{ is in } C) \{ \text{Shift} \} \text{ else } \phi)$$

This rule simply collects all the actions that a parser might do given $C$ and $T$.

But we want parser actions to be unique so we require that the parser action always be unique for any $C$ and $T$. 
If the parser action isn’t unique, then we have a shift/reduce error or reduce/reduce error. The grammar is then rejected as unparsable.

If parser actions are always unique then we will consider a shift of EOF to be an accept action.

An empty (or undefined) action for C and T will signify that token T is illegal given configuration set C. A syntax error will be signaled.
LALR Parser Driver

Given the GoTo and parser action tables, a Shift/Reduce (LALR) parser is fairly simple:

```c
void LALRDriver()
{
    Push(S0);
    while(true){
        //Let S = Top state on parse stack
        //Let CT = current token to match
        switch (A[S][CT]) {
            case error:
                SyntaxError(CT);return;
            case accept:
                return;
            case shift:
                push(GoTo[S][CT]);
                CT= Scanner();
                break;
            case reduce i:
                //Let prod i = A→Y1...Ym
                pop m states;
                //Let S' = new top state
                push(GoTo[S'][A]);
                break;
        }
    }
}
```
## Action Table for CSX-Lite

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>{</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>}</td>
<td>R3</td>
<td>S</td>
<td>R3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>if</td>
<td>S</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R4</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(</td>
<td></td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>id</td>
<td>S</td>
<td>S</td>
<td></td>
<td>S</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R4</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>=</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eof</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

{S} R 3 S R 3 S R 4 S R 5

if S S R 4 S R 5

( S

) R 8 S R 6 R 7

id S S S S S S R 4 S S S

= S

+ S R 8 S R 6 R 7

- S R 8 S R 6 R 7

; S R 8 R 6 R 7 R 5

The table represents the action table for the CSX-Lite parser, showing the actions to be taken for different symbols and states in the parsing process.
## GoTo Table for CSX-Lite

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>`{</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>}</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>if</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>id</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>12</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>=</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eof</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stmts</td>
<td>2</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stmt</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>expr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>
# Example of LALR(1) Parsing

We’ll again parse

\{ a = b + c; } Eof

We start by pushing state 0 on the parse stack.

<table>
<thead>
<tr>
<th>Parse Stack</th>
<th>Top State</th>
<th>Action</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Prog → •{ Stmts } Eof</td>
<td>Shift</td>
<td>{ a = b + c; } Eof</td>
</tr>
<tr>
<td>1 0</td>
<td>Prog → { • Stmts } Eof Stmts Stmts → • Stmt Stmts</td>
<td>Shift</td>
<td>a = b + c; } Eof</td>
</tr>
<tr>
<td></td>
<td>Stmt → • id = Expr ; Stmt → • if ( Expr )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 1 0</td>
<td>Stmt → id • = Expr ;</td>
<td></td>
<td>= b + c; } Eof</td>
</tr>
<tr>
<td>8 4 1 0</td>
<td>Stmt → id = • Expr ; Expr → • Expr + id Expr → • Expr - id Expr → • id</td>
<td>Shift</td>
<td>b + c; } Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Top State</td>
<td>Action</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------</td>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
<td>12 8 4 1 0</td>
<td>Expr → id •</td>
<td>Reduce 8</td>
<td>+ c; } Eof</td>
</tr>
<tr>
<td>11 8 4 1 0</td>
<td>Stmt → id = Expr • ;</td>
<td>Shift</td>
<td>+ c; } Eof</td>
</tr>
<tr>
<td></td>
<td>Expr → Expr • + id</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Expr → Expr • - id</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 11 8 4 1 0</td>
<td>Expr → Expr + • id</td>
<td>Shift</td>
<td>c; } Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Top State</td>
<td>Action</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------</td>
<td>--------</td>
<td>-----------------</td>
</tr>
<tr>
<td>18 15 11 8 4 1 0</td>
<td>Expr → Expr + id •</td>
<td>Reduce 6</td>
<td>; } Eof</td>
</tr>
<tr>
<td>11 8 4 1 0</td>
<td>Stmt → id =Expr • ; &lt;br&gt; Expr → Expr • + id &lt;br&gt; Expr → Expr • - id</td>
<td>Shift</td>
<td>; } Eof</td>
</tr>
<tr>
<td>14 11 8 4 1 0</td>
<td>Stmt → id = Expr ; •</td>
<td>Reduce 4</td>
<td>} Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Top State</td>
<td>Action</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------------------------------------</td>
<td>----------</td>
<td>-----------------</td>
</tr>
<tr>
<td>3 1 0</td>
<td>$\text{Stmts} \rightarrow \text{Stmt} \cdot \text{Stmts}$  $\text{Stmts} \rightarrow \cdot \text{Stmt} \text{Stmts}$  $\text{Stmts} \rightarrow \lambda \cdot$  $\text{Stmt} \rightarrow \cdot \text{id} = \text{Expr} ;$  $\text{Stmt} \rightarrow \cdot \text{if ( Expr )}$ $\text{Stmt}$</td>
<td>Reduce 3</td>
<td>{ Eof }</td>
</tr>
<tr>
<td>7 3 1 0</td>
<td>$\text{Stmts} \rightarrow \text{Stmt} \text{Stmts} \cdot$</td>
<td>Reduce 2</td>
<td>{ Eof }</td>
</tr>
<tr>
<td>2 1 0</td>
<td>$\text{Prog} \rightarrow { \text{Stmts} \cdot }$ $\text{Eof}$</td>
<td>Shift</td>
<td>{ Eof }</td>
</tr>
<tr>
<td>6 2 1 0</td>
<td>$\text{Prog} \rightarrow { \text{Stmts} \cdot }$ $\text{Eof}$</td>
<td>Accept</td>
<td>Eof</td>
</tr>
</tbody>
</table>
Error Detection in LALR Parsers

In bottom-up, LALR parsers syntax errors are discovered when a blank (error) entry is fetched from the parser action table.

Let’s again trace how the following illegal CSX-lite program is parsed:

\[
\{ \ b + c = a; \ } \ Eof
\]

<table>
<thead>
<tr>
<th>Parse Stack</th>
<th>Top State</th>
<th>Action</th>
<th>Remaining Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Prog → •{ Stmts } Eof</td>
<td>Shift</td>
<td>{ b + c = a; } Eof</td>
</tr>
<tr>
<td>Parse Stack</td>
<td>Top State</td>
<td>Action</td>
<td>Remaining Input</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------</td>
<td>--------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1 0</td>
<td>Prog → { • Stmts } Eof Stmts → • Stmt Stmts Stmts → λ • Stmt → • id = Expr ; Stmt → • if ( Expr )</td>
<td>Shift</td>
<td>b + c = a; } Eof</td>
</tr>
<tr>
<td>4 1 0</td>
<td>Stmt → id • = Expr ; Error (blank)</td>
<td>+ c = a; } Eof</td>
<td></td>
</tr>
</tbody>
</table>
**LALR is More Powerful**

Essentially all LL(1) grammars are LALR(1) plus many more. Grammar constructs that confuse LL(1) are readily handled.

- Common prefixes are no problem. Since sets of configurations are tracked, more than one prefix can be followed. For example, in

\[
\begin{align*}
\text{Stmt} &\rightarrow \text{id} = \text{Expr} ; \\
\text{Stmt} &\rightarrow \text{id} (\text{Args}) ;
\end{align*}
\]

after we match an id we have

\[
\begin{align*}
\text{Stmt} &\rightarrow \text{id} \cdot = \text{Expr} ; \\
\text{Stmt} &\rightarrow \text{id} \cdot (\text{Args}) ;
\end{align*}
\]

The next token will tell us which production to use.
- Left recursion is also not a problem. Since sets of configurations are tracked, we can follow a left-recursive production and all others it might use. For example, in

\[
\begin{align*}
\text{Expr} & \rightarrow \cdot \text{Expr} + \text{id} \\
\text{Expr} & \rightarrow \cdot \text{id} \\
\text{Expr} & \rightarrow \cdot \text{id}
\end{align*}
\]

we can first match an id:

\[
\begin{align*}
\text{Expr} & \rightarrow \text{id} \cdot \\
\text{Expr} & \rightarrow \text{Expr} \cdot + \text{id}
\end{align*}
\]

Then the Expr is recognized:

The left-recursion is handled!
But ambiguity will still block construction of an LALR parser. Some shift/reduce or reduce/reduce conflict must appear. (Since two or more distinct parses are possible for some input).

Consider our original productions for if-then and if-then-else statements:

\[
\text{Stmt} \rightarrow \text{if ( Expr ) Stmt} \quad \cdot \\
\text{Stmt} \rightarrow \text{if ( Expr ) Stmt} \quad \cdot \quad \text{else Stmt}
\]

Since else can follow Stmt, we have an unresolvable shift/reduce conflict.
Grammar Engineering

Though LALR grammars are very general and inclusive, sometimes a reasonable set of productions is rejected due to shift/reduce or reduce/reduce conflicts.

In such cases, the grammar may need to be “engineered” to allow the parser to operate.

A good example of this is the definition of MemberDecls in CSX. A straightforward definition is

\[
\begin{align*}
\text{MemberDecls} & \rightarrow \text{FieldDecls MethodDecls} \\
\text{FieldDecls} & \rightarrow \text{FieldDecl FieldDecls} \\
\text{FieldDecls} & \rightarrow \lambda \\
\text{MethodDecls} & \rightarrow \text{MethodDecl MethodDecls} \\
\text{MethodDecls} & \rightarrow \lambda \\
\text{FieldDecl} & \rightarrow \text{int id ;} \\
\text{MethodDecl} & \rightarrow \text{int id ( ) ; Body}
\end{align*}
\]
When we predict \texttt{MemberDecls} we get:

\begin{align*}
\text{MemberDecls} & \rightarrow \cdot \text{FieldDecls} \text{ MethodDecls} \\
\text{FieldDecls} & \rightarrow \cdot \text{FieldDecl} \text{ FieldDecls} \\
\text{FieldDecls} & \rightarrow \lambda \cdot \\
\text{FieldDecl} & \rightarrow \cdot \text{int id ;}
\end{align*}

Now int follows FieldDecls since MethodDecls $\Rightarrow^{+} \text{int ...}$

Thus an unresolvable shift/reduce conflict exists.

The problem is that int is derivable from both FieldDecls and MethodDecls, so when we see an int, we can’t tell which way to parse it (and FieldDecls $\rightarrow \lambda$ requires we make an immediate decision!).
If we rewrite the grammar so that we can delay deciding from where the int was generated, a valid LALR parser can be built:

\[
\begin{align*}
\text{MemberDecls} & \rightarrow \text{FieldDecl} \; \text{MemberDecls} \\
\text{MemberDecls} & \rightarrow \text{MethodDecls} \\
\text{MethodDecls} & \rightarrow \text{MethodDecl} \; \text{MethodDecls} \\
\text{MethodDecls} & \rightarrow \lambda \\
\text{FieldDecl} & \rightarrow \text{int id ;} \\
\text{MethodDecl} & \rightarrow \text{int id ( ) ; Body}
\end{align*}
\]

When MemberDecls is predicted we have

\[
\begin{align*}
\text{MemberDecls} & \rightarrow \ast \; \text{FieldDecl} \; \text{MemberDecls} \\
\text{MemberDecls} & \rightarrow \ast \; \text{MethodDecls} \\
\text{MethodDecls} & \rightarrow \ast \text{MethodDecl} \; \text{MethodDecls} \\
\text{MethodDecls} & \rightarrow \lambda \ast \\
\text{FieldDecl} & \rightarrow \ast \text{int id ;} \\
\text{MethodDecl} & \rightarrow \ast \text{int id ( ) ; Body}
\end{align*}
\]
Now \text{Follow(}\text{MethodDecls}) = \text{Follow(}\text{MemberDecls}) = "\{"", so we have no shift/reduce conflict. After \text{int id} is matched, the next token (a ";") or a "(" will tell us whether a FieldDecl or a MethodDecl is being matched.
Properties of LL and LALR Parsers

- Each prediction or reduce action is guaranteed correct. Hence the entire parse (built from LL predictions or LALR reductions) must be correct.

This follows from the fact that LL parsers allow only one valid prediction per step. Similarly, an LALR parser never skips a reduction if it is consistent with the current token (and all possible reductions are tracked).
• LL and LALR parsers detect an syntax error as soon as the first invalid token is seen.

Neither parser can match an invalid program prefix. If a token is matched it must be part of a valid program prefix. In fact, the prediction made or the stacked configuration sets show a possible derivation of the token accepted so far.

• All LL and LALR grammars are unambiguous.

LL predictions are always unique and LALR shift/reduce or reduce/reduce conflicts are disallowed. Hence only one valid derivation of any token sequence is possible.
• All LL and LALR parsers require only linear time and space (in terms of the number of tokens parsed).

The parsers do only fixed work per node of the concrete parse tree, and the size of this tree is linear in terms of the number of leaves in it (even with $\lambda$-productions included!).
Reading Assignment

Get and read Chapter 8 of Crafting a Compiler featuring Java.
(Available from Dolt Tech Store)

Midterm Exam

Wednesday, November 13, 7-9 pm
Symbol Tables in CSX

CSX is designed to make symbol tables easy to create and use.

There are three places where a new scope is opened:

- In the class that represents the program text. The scope is opened as soon as we begin processing the `classNode` (that roots the entire program). The scope stays open until the entire class (the whole program) is processed.

- When a `methodDeclNode` is processed. The name of the method is entered in the top-level (global) symbol table. Declarations of parameters and locals are placed in the method’s symbol table. A
method’s symbol table is closed after all the statements in its body are type checked.

- When a `blockNode` is processed. Locals are placed in the block’s symbol table. A block’s symbol table is closed after all the statements in its body are type checked.
CSX Allows no Forward References

This means we can do type-checking in one pass over the AST. As declarations are processed, their identifiers are added to the current (innermost) symbol table. When a use of an identifier occurs, we do an ordinary block-structured lookup, always using the innermost declaration found. Hence in

```
int i = j;
int j = i;
```

the first declaration initializes \( i \) to the nearest non-local definition of \( j \). The second declaration initializes \( j \) to the current (local) definition of \( i \).
Midterm Exam

Wednesday, November 13, 7:15-915 pm, 105 Psychology.
Forward References Require Two Passes

If forward references are allowed, we can process declarations in two passes.

First we walk the AST to establish symbol tables entries for all local declarations. No uses (lookups) are handled in this passes.

On a second complete pass, all uses are processed, using the symbol table entries built on the first pass.

Forward references make type checking a bit trickier, as we may reference a declaration not yet fully processed.

In Java, forward references to fields within a class are allowed.
Thus in

class Duh {
    int i = j;
    int j = i;
}

a Java compiler must recognize that
the initialization of $i$ is to the $j$ field
and that the $j$ declaration is
incomplete (Java forbids uninitialized
fields or variables).

Forward references do allow methods
to be mutually recursive. That is, we
can let method $a$ call $b$, while $b$ calls $a$.

In CSX this is impossible!
(Why?)
Incomplete Declarations

Some languages, like C++, allow incomplete declarations.

First, part of a declaration (usually the header of a procedure or method) is presented.

Later, the declaration is completed.

For example (in C++):

```cpp
class C {
    int i;
    public:
        int f();
};
int C::f(){return i+1;}
```
Incomplete declarations solve potential forward reference problems, as you can declare method headers first, and bodies that use the headers later.

Headers support abstraction and separate compilation too.

In C and C++, it is common to use a `#include` statement to add the headers (but not bodies) of external or library routines you wish to use.

C++ also allows you to declare a class by giving its fields and method headers first, with the bodies of the methods declared later. This is good for users of the class, who don’t always want to see implementation details.
Classes, Structs and Records

The fields and methods declared within a class, struct or record are stored within an individual symbol table allocated for its declarations.

Member names must be unique within the class, record or struct, but may clash with other visible declarations. This is allowed because member names are qualified by the object they occur in.

Hence the reference \texttt{x.a} means look up \texttt{x}, using normal scoping rules. Object \texttt{x} should have a type that includes local fields. The type of \texttt{x} will include a pointer to the symbol table containing the field declarations. Field \texttt{a} is looked up in that symbol table.
Chains of field references are no problem.

For example, in Java

```
System.out.println
```

is commonly used.

`System` is looked up and found to be a class in one of the standard Java packages (`java.lang`). Class `System` has a static member `out` (of type `PrintStream`) and `PrintStream` has a member `println`. 
Internal and External Field Access

Within a class, members may be accessed without qualification. Thus in

```java
class C {
    static int i;
    void subr() {
        int j = i;
    }
}
```

field `i` is accessed like an ordinary non-local variable.

To implement this, we can treat member declarations like an ordinary scope in a block-structured symbol table.
When the class definition ends, its symbol table is popped and members are referenced through the symbol table entry for the class name. This means a simple reference to $i$ will no longer work, but $C.i$ will be valid.
In languages like C++ that allow incomplete declarations, symbol table references need extra care. In

```cpp
class C {
    int i;
    public:
        int f();
};
int C::f(){return i+1;}
```

when the definition of \( f() \) is completed, we must restore \( C \)'s field definitions as a containing scope so that the reference to \( i \) in \( i+1 \) is properly compiled.
Public and Private Access

C++ and Java (and most other object-oriented languages) allow members of a class to be marked public or private.

Within a class the distinction is ignored; all members may be accessed.

Outside of the class, when a qualified access like `c.i` is required, only public members can be accessed.

This means lookup of class members is a two-step process. First the member name is looked up in the symbol table of the class. Then, the public/private qualifier is checked. Access to private members from outside the class generates an error message.
C++ and Java also provide a `protected` qualifier that allows access from subclasses of the class containing the member definition.

When a subclass is defined, it “inherits” the member definitions of its ancestor classes. Local definitions may hide inherited definitions.

Moreover, inherited member definitions must be `public` or `protected`; private definitions may not be directly accessed (though they are still inherited and may be indirectly accessed through other inherited definitions).

Java also allows “blank” access qualifiers which allow `public` access by all classes within a package (a collection of classes).
Packages and Imports

Java allows packages which group class and interface definitions into named units.

A package requires a symbol table to access members. Thus a reference

\texttt{java.util.Vector}

locates the package \texttt{java.util} (typically using a \texttt{CLASSPATH}) and looks up \texttt{Vector} within it.

Java supports import statements that modify symbol table lookup rules.

A single class import, like

\texttt{import java.util.Vector;}

brings the name \texttt{Vector} into the current symbol table (unless a
definition of $\text{Vector}$ is already present).

An “import on demand” like

```java
import java.util.*;
```

will lookup identifiers in the named packages after explicit user declarations have been checked.
Classfiles and Object Files

Class files (".class" files, produced by Java compilers) and object files (".o" files, produced by C and C++ compilers) contain internal symbol tables.

When a field or method of a Java class is accessed, the JVM uses the classfile’s internal symbol table to access the symbol’s value and verify that type rules are respected.

When a C or C++ object file is linked, the object file’s internal symbol table is used to determine what external names are referenced, and what internally defined names will be exported.
C, C++ and Java all allow users to request that a more complete symbol table be generated for debugging purposes. This makes internal names (like local variable) visible so that a debugger can display source level information while debugging.
Overloading

A number of programming languages, including Java and C++, allow method and subprogram names to be overloaded.

This means several methods or subprograms may share the same name, as long as they differ in the number or types of parameters they accept. For example,

class C {
    int x;
    public static int sum(int v1,
                           int v2) {
        return v1 + v2;
    }
    public int sum(int v3) {
        return x + v3;
    }
}
For overloaded identifiers the symbol table must return a list of valid definitions of the identifier. Semantic analysis (type checking) then decides which definition to use.

In the above example, while checking 
(new C()).sum(10);
both definitions of sum are returned when it is looked up. Since one argument is provided, the definition that uses one parameter is selected and checked.

A few languages (like Ada) allow overloading to be disambiguated on the basis of a method’s result type. Algorithms that do this analysis are known, but are fairly complex.
Overloaded Operators

A few languages, like C++, allow operators to be overloaded. This means users may add new definitions for existing operators, though they may not create new operators or alter existing precedence and associativity rules. (Such changes would force changes to the scanner or parser.)

For example,

class complex{
    float re, im;
    complex operator+(complex d) {
        complex ans;
        ans.re = d.re+re;
        ans.im = d.im+im;
        return ans;
    }
}
complex c,d; c=c+d;
During type checking of an operator, all visible definitions of the operator (including predefined definitions) are gathered and examined. Only one definition should successfully pass type checks. Thus in the above example, there may be many definitions of $+$, but only one is defined to take complex operands.
Contextual Resolution

Overloading allows multiple definitions of the same kind of object (method, procedure or operator) to co-exist.

Programming languages also sometimes allow reuse of the same name in defining different kinds of objects. Resolution is by context of use.

For example, in Java, a class name may be used for both the class and its constructor. Hence we see

```java
C cvar = new C(10);
```

In Pascal, the name of a function is also used for its return value.

Java allows rather extensive reuse of an identifier, with the same identifier
potentially denoting a class (type), a class constructor, a package name, a method and a field.

For example,

class C {
    double v;
    C(double f) {v=f;}
}
class D {
    int C;
    double C() {return 1.0;}
    C cval = new C(C+C());
}

At type-checking time we examine all potential definitions and use that definition that is consistent with the context of use. Hence new C() must be a constructor, +C() must be a function call, etc.
Allowing multiple definitions to co-exist certainly makes type checking more complicated than in other languages.

Whether such reuse benefits programmers is unclear; it certainly violates Java’s “keep it simple” philosophy.
Type and Kind Information in CSX

In CSX symbol table entries and in AST nodes for expressions, it is useful to store type and kind information. This information is created and tested during type checking. In fact, most of type checking involves deciding whether the type and kind values for the current construct and its components are valid.

Possible values for type include:

- Integer (int)
- Boolean (bool)
- Character (char)
- String
- **Void**
  Void is used to represent objects that have no declared type (e.g., a label or procedure).

- **Error**
  Error is used to represent objects that should have a type, but don’t (because of type errors). Error types suppress further type checking, preventing cascaded error messages.

- **Unknown**
  Unknown is used as an initial value, before the type of an object is determined.
Possible values for kind include:

- **Var** (a local variable or field that may be assigned to)
- **Value** (a value that may be read but not changed)
- **Array**
- **ScalarParm** (a by-value scalar parameter)
- **ArrayParm** (a by-reference array parameter)
- **Method** (a procedure or function)
- **Label** (on a while loop)
Most combinations of type and kind represent something in CSX. Hence type==Boolean and kind==Value is a bool constant or expression.

type==Void and kind==Method is a procedure (a method that returns no value).

Type checking procedure and function declarations and calls requires some care.

When a method is declared, you should build a linked list of (type,kind) pairs, one for each declared parameter.

When a call is type checked you should build a second linked list of (type,kind) pairs for the actual parameters of the call.
You compare the lengths of the list of formal and actual parameters to check that the correct number of parameters has been passed.

You then compare corresponding formal and actual parameter pairs to check if each individual actual parameter correctly matches its corresponding formal parameter.

For example, given

\[ p(\text{int } a, \text{ bool } b[]) \{ \ldots \} \]

and the call

\[ p(1, \text{false}); \]

you create the parameter list

\[(\text{Integer, ScalarParm}), (\text{Boolean, Array Parm})\]

for \( p \)'s declaration and the parameter list

\[(\text{Integer, Value}), (\text{Boolean, Value})\]
for p’s call.
Since a Value can’t match an ArrayParm, you know that the second parameter in p’s call is incorrect.
Reading Assignment

Get and read Chapter 9 of Crafting a Compiler featuring Java.
(Available from Dolt Tech Store)
Type Checking Simple Variable Declarations

Type checking steps:
1. Check that identNode.idname is not already in the symbol table.
2. Enter identNode.idname into symbol table with type=typeNode.type and kind = Variable.
Type Checking Initialized Variable Declarations

Type checking steps:
1. Check that identNode.idname is not already in the symbol table.
2. Type check initial value expression.
3. Check that the initial value’s type is typeNode.type
4. Check that the initial value’s kind is scalar (Variable, Value or ScalarParm).
5. Enter identNode.idname into symbol table with type=typeNode.type and kind = Variable.
Type Checking Constant Declarations

Type checking steps:
1. Check that identNode.idname is not already in the symbol table.
2. Type check the const value expression.
3. Check that the const value’s kind is scalar (Variable, Value or ScalarParm).
4. Enter identNode.idname into symbol table with type = constValue.type and kind = Value.
Type Checking IdentNodes

Type checking steps:
1. Lookup identNode.idname in the symbol table; error if absent.
2. Copy symbol table entry’s type and kind information into the identNode.
3. Store a link to the symbol table entry in the identNode (in case we later need to access symbol table information).
Type Checking NameNodes

Type checking steps:
1. Type check the identNode.
2. If the subscriptVal is null, copy the identNode’s type and kind values into the nameNode and Return.
3. Type check the subscriptVal.
4. Check that identNode’s kind is an array.
5. Check that subscriptVal’s kind is scalar and type is integer or character.
6. Set the nameNode’s type to the identNode’s type and the nameNode’s kind to Variable.
Type Checking Binary Operators

Type checking steps:
1. Type check left and right operands.
2. Check that left and right operands are both scalars.
3. binaryOpNode.kind = Value.
4. If binaryOpNode.operator is a plus, minus, star or slash:
   (a) Check that left and right operands have an arithmetic type (integer or
character).
(b) binaryOpNode.type = Integer

5. If binaryOpNode.operator is an and or is an or:
   (a) Check that left and right operands have a boolean type.
   (b) binaryOpNode.type = Boolean.

6. If binaryOpNode.operator is a relational operator:
   (a) Check that both left and right operands have an arithmetic type or both have a boolean type.
   (b) binaryOpNode.type = Boolean.
Type Checking Assignments

Type checking steps:
1. Type check the nameNode.
2. Type check the expression tree.
3. Check that the nameNode’s kind is assignable (Variable, Array, ScalarParm, or ArrayParm).
4. If the nameNode’s kind is scalar then check the expression tree’s kind is also scalar and that both have the same type. Then return.
5. If the nameNode’s and the expression tree’s kinds are both arrays and both have the same type, check that the arrays have the same length. (Lengths of array parms are checked at run-time). Then return.

6. If the nameNode’s kind is array and its type is character and the expression tree’s kind is string, check that both have the same length. (Lengths of array parms are checked at run-time). Then return.

7. Otherwise, the expression may not be assigned to the nameNode.
Type Checking While Loops

Type checking steps:
1. Type check the condition (an expr tree).
2. Check that the condition’s type is Boolean and kind is scalar.
3. If the label is null (no identNode is present) then type check the stmtNode (the loop body) and return.
4. If there is a label (an identNode) then:
   (a) Check that the label is not already present in the symbol table.
   (b) If it isn’t, enter label in the symbol table with kind=VisibleLabel and type=void.
   (c) Type check the stmtNode (the loop body).
   (d) Change the label’s kind (in the symbol table) to HiddenLabel.
Type Checking Breaks and Continues

Type checking steps:
1. Check that the identNode is declared in the symbol table.
2. Check that identNode’s kind is VisibleLabel. If identNode’s kind is HiddenLabel issue a special error message.
Type Checking Returns

It is convenient to arrange that a static field named `currentMethod` will always point to the methodDeclNode of the method we are currently checking.

Type checking steps:

1. If `returnVal` (an expr) is null, check that `currentMethod.returnType` is `Void`.

2. If `returnVal` (an expr) is not null then check that `returnVal`’s kind is scalar and `returnVal`’s type is `currentMethod.returnType`. 
Type Checking Method Declarations

Type checking steps:
1. Create a new symbol table entry \( m \), with type \( = \) typeNode.type and kind \( = \) Method.
2. Check that identNode.idname is not already in the symbol table; if it isn’t, enter \( m \) using identNode.idname.
3. Create a new scope in the symbol table.
4. Set currentMethod = this methodDeclNode.
5. Type check the args subtree.
6. Build a list of the symbol table nodes corresponding to the args subtree; store it in m.
7. Type check the decls subtree.
8. Type check the stmts subtree.
9. Close the current scope at the top of the symbol table.
We consider calls of procedures in a statement. Calls of functions in an expression are very similar.

Type checking steps:

1. Check that identNode.idname is declared in the symbol table. Its type should be Void and kind should be Method.

2. Type check the args subtree.
3. Build a list of the expression nodes found in the args subtree.

4. Get the list of parameter symbols declared for the method (stored in the method’s symbol table entry).

5. Check that the arguments list and the parameter symbols list both have the same length.

6. Compare each argument node with its corresponding parameter symbol:
   (a) Both should have the same type.
   (b) A Variable, Value, or ScalarParm kind in an argument node matches a ScalarParm parameter. An Array or ArrayParm kind in an argument node matches an ArrayParm parameter.
Reading Assignment

Get and read Chapter 11 of Crafting a Compiler featuring Java.
(Available from DoIt Tech Store)
Virtual Memory & Run-Time Memory Organization

The compiler decides how data and instructions are placed in memory. It uses an address space provided by the hardware and operating system. This address space is usually virtual—the hardware and operating system map instruction-level addresses to “actual” memory addresses.

Virtual memory allows:

- Multiple processes to run in private, protected address spaces.
- Paging can be used to extend address ranges beyond actual memory limits.
For static structures, a fixed address is used throughout execution.
This is the oldest and simplest memory organization.
In current compilers, it is used for:

- Program code (often read-only & sharable).
- Data literals (often read-only & sharable).
- Global variables.
- Static variables.
Stack Allocation

Modern programming languages allow recursion, which requires dynamic allocation.

Each recursive call allocates a new copy of a routine’s local variables.

The number of local data allocations required during program execution is not known at compile-time.

To implement recursion, all the data space required for a method is treated as a distinct data area that is called a frame or activation record.

Local data, within a frame, is accessible only while a subprogram is active.
In mainstream languages like C, C++ and Java, subprograms must return in a stack-like manner—the most recently called subprogram will be the first to return.

A frame is pushed onto a run-time stack when a method is called (activated).

When it returns, the frame is popped from the stack, freeing the routine’s local data.
As an example, consider the following C subprogram:

```c
p(int a) {
    double b;
    double c[10];
    b = c[a] * 2.51;
}
```

Procedure `p` requires space for the parameter `a` as well as the local variables `b` and `c`.

It also needs space for control information, such as the return address.

The compiler records the space requirements of a method.

The offset of each data item relative to the beginning of the frame is stored in the symbol table.
The total amount of space needed, and thus the size of the frame, is also recorded.

Assume \( p \)'s control information requires 8 bytes (this size is usually the same for all methods).

Assume parameter \( a \) requires 4 bytes, local variable \( b \) requires 8 bytes, and local array \( c \) requires 80 bytes.

Many machines require that word and doubleword data be aligned, so it is common to pad a frame so that its size is a multiple of 4 or 8 bytes.

This guarantees that at all times the top of the stack is properly aligned.
Here is p’s frame:

<table>
<thead>
<tr>
<th>Padding</th>
<th>Total size= 104</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space for c</td>
<td>Offset = 20</td>
</tr>
<tr>
<td>Space for b</td>
<td>Offset = 12</td>
</tr>
<tr>
<td>Space for a</td>
<td>Offset = 8</td>
</tr>
<tr>
<td>Control Information</td>
<td>Offset = 0</td>
</tr>
</tbody>
</table>

Within p, each local data object is addressed by its offset relative to the start of the frame.

This offset is a fixed constant, determined at compile-time.

We normally store the start of the frame in a register, so each piece of data can be addressed as a (Register, Offset) pair, which is a standard addressing mode in almost all computer architectures.
For example, if register R points to the beginning of p’s frame, variable b can be addressed as (R,12), with 12 actually being added to the contents of R at run-time, as memory addresses are evaluated.

Normally, the literal 2.51 of procedure p is not stored in p’s frame because the values of local data that are stored in a frame disappear with it at the end of a call.

It is easier and more efficient to allocate literals in a static area, often called a literal pool or constant pool. Java uses a constant pool to store literals, type, method and interface information as well as class and field names.
Accessing Frames at Run-Time

During execution there can be many frames on the stack. When a procedure A calls a procedure B, a frame for B’s local variables is pushed on the stack, covering A’s frame. A’s frame can’t be popped off because A will resume execution after B returns.

For recursive routines there can be hundreds or even thousands of frames on the stack. All frames but the topmost represent suspended subroutines, waiting for a call to return.
The topmost frame is active; it is important to be able to access it directly.

The active frame is at the top of the stack, so the stack top register could be used to access it.

The run-time stack may also be used to hold data other than frames.

It is unwise to require that the currently active frame always be at exactly the top of the stack.

Instead a distinct register, often called the frame pointer, is used to access the current frame.

This allows local variables to be accessed directly as offset + frame pointer, using the indexed addressing mode found on all modern machines.
Consider the following recursive function that computes factorials.

```c
int fact(int n) {
    if (n > 1)
        return n * fact(n-1);
    else return 1;
}
```

The run-time stack corresponding to the call `fact(3)` (when the call of `fact(1)` is about to return) is:

<table>
<thead>
<tr>
<th>Space for n = 3</th>
<th>Control Information</th>
<th>Return Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space for n = 2</td>
<td>Control Information</td>
<td>Return Value</td>
</tr>
<tr>
<td>Space for n = 1</td>
<td>Control Information</td>
<td>Return Value = 1</td>
</tr>
</tbody>
</table>

Top of Stack

Frame Pointer
We place a slot for the function’s return value at the very beginning of the frame.

Upon return, the return value is conveniently placed on the stack, just beyond the end of the caller’s frame. Often compilers return scalar values in specially designated registers, eliminating unnecessary loads and stores. For values too large to fit in a register (arrays or objects), the stack is used.

When a method returns, its frame is popped from the stack and the frame pointer is reset to point to the caller’s frame.

In simple cases this is done by adjusting the frame pointer by the size of the current frame.
Dynamic Links

Because the stack may contain more than just frames (e.g., function return values or registers saved across calls), it is common to save the caller’s frame pointer as part of the callee’s control information.

Each frame points to its caller’s frame on the stack. This pointer is called a dynamic link because it links a frame to its dynamic (run-time) predecessor.
The run-time stack corresponding to a call of \texttt{fact(3)}, with dynamic links included, is:

\begin{itemize}
  \item Space for $n = 1$
    \begin{itemize}
      \item Dynamic Link
      \item Return Value = 1
    \end{itemize}
  \item Space for $n = 2$
    \begin{itemize}
      \item Dynamic Link
      \item Return Value
    \end{itemize}
  \item Space for $n = 3$
    \begin{itemize}
      \item Dynamic Link = \texttt{Null}
      \item Return Value
    \end{itemize}
\end{itemize}

Top of Stack \hspace{1cm} Frame Pointer
Classes and Objects

C, C++ and Java do not allow procedures or methods to nest. A procedure may not be declared within another procedure. This simplifies run-time data access—all variables are either global or local. Global variables are statically allocated. Local variables are part of a single frame, accessed through the frame pointer.

Java and C++ allow classes to have member functions that have direct access to instance variables.
Consider:

class K {
    int a;
    int sum() {
        int b;
        return a+b;
    }
}

Each object that is an instance of class K contains a member function sum. Only one translation of sum is created; it is shared by all instances of K.

When sum executes it needs two pointers to access local and object-level data.

Local data, as usual, resides in a frame on the run-time stack.
Data values for a particular instance of \( \kappa \) are accessed through an object pointer (called the \texttt{this} pointer in Java and C++). When \texttt{obj.sum()} is called, it is given an extra implicit parameter that a pointer to \texttt{obj}.

When \( a + b \) is computed, \( b \), a local variable, is accessed directly through the frame pointer. \( a \), a member of object \texttt{obj}, is accessed indirectly through the object pointer that is stored in the frame (as all parameters to a method are).
C++ and Java also allow inheritance via subclassing. A new class can extend an existing class, adding new fields and adding or redefining methods.

A subclass $D$, of class $C$, maybe be used in contexts expecting an object of class $C$ (e.g., in method calls).

This is supported rather easily—objects of class $D$ always contain a class $C$ object within them.

If $C$ has a field $F$ within it, so does $D$. The fields $D$ declares are merely appended at the end of the allocations for $C$.

As a result, access to fields of $C$ within a class $D$ object works perfectly.
Handling Multiple Scopes

Many languages allow procedure declarations to nest. Java now allows classes to nest.

Procedure nesting can be very useful, allowing a subroutine to directly access another routine’s locals and parameters.

Run-time data structures are complicated because multiple frames, corresponding to nested procedure declarations, may need to be accessed.
To see the difficulties, assume that routines can nest in Java or C:

```java
int p(int a) {
    int q(int b) {
        if (b < 0)
            q(-b);
        else return a+b;
    }
    return q(-10);
}
```

When \( q \) executes, it can access not only its own frame, but also that of \( p \), in which it is nested.

If the depth of nesting is unlimited, so is the number of frames that must be made accessible. In practice, the level of nesting actually seen is modest—usually no greater than two or three.
Static Links

Two approaches are commonly used to support access to multiple frames. One approach generalizes the idea of dynamic links introduced earlier. Along with a dynamic link, we'll also include a static link in the frame's control information area. The static link points to the frame of the procedure that statically encloses the current procedure. If a procedure is not nested within any other procedure, its static link is null.
The following illustrates static links:

As usual, dynamic links always point to the next frame down in the stack. Static links always point down, but they may skip past many frames. They always point to the most recent frame of the routine that statically encloses the current routine.

As usual, dynamic links always point to the next frame down in the stack. Static links always point down, but they may skip past many frames. They always point to the most recent frame of the routine that statically encloses the current routine.
In our example, the static links of both of q’s frames point to p, since it is p that encloses q’s definition.

In evaluating the expression a+b that q returns, b, being local to q, is accessed directly through the frame pointer. Variable a is local to p, but also visible to q because q nests within p. a is accessed by extracting q’s static link, then using that address (plus the appropriate offset) to access a.
Displays

An alternative to using static links to access frames of enclosing routines is the use of a display.

A display generalizes our use of a frame pointer. Rather than maintaining a single register, we maintain a set of registers which comprise the display.

If procedure definitions nest $n$ deep (this can be easily determined by examining a program’s AST), we need $n+1$ display registers.

Each procedure definition is tagged with a nesting level. Procedures not nested within any other routine are at level 0. Procedures nested within only one routine are at level 1, etc.
Frames for routines at level 0 are always accessed using display register D0. Those at level 1 are always accessed using register D1, etc.

Whenever a procedure \( r \) is executing, we have direct access to \( r \)'s frame plus the frames of all routines that enclose \( r \). Each of these routines must be at a different nesting level, and hence will use a different display register.
The following illustrates the use of display registers:

Since $q$ is at nesting level 1, its frame is pointed to by D1. All of $q$'s local variables, including $b$, are at a fixed offset relative to D1.

Since $p$ is at nesting level 0, its frame and local variables are accessed via D0. Each frame’s control information area contains a slot for the previous value of the frame’s display register.
A display register is saved when a call begins and restored when the call ends. A dynamic link is still needed, because the previous display values doesn’t always point to the caller’s frame.

Not all compiler writers agree on whether static links or displays are better to use. Displays allow direct access to all frames, and thus make access to all visible variables very efficient. However, if nesting is deep, several valuable registers may need to be reserved. Static links are very flexible, allowing unlimited nesting of procedures. However, access to non-local procedure variables can be slowed by the need to extract and follow static links.
Heap Management

A very flexible storage allocation mechanism is heap allocation.

Any number of data objects can be allocated and freed in a memory pool, called a heap.

Heap allocation is enormously popular. Almost all non-trivial Java and C programs use new or malloc.
Heap Allocation

A request for heap space may be explicit or implicit.

An explicit request involves a call to a routine like `new` or `malloc`. An explicit pointer to the newly allocated space is returned.

Some languages allow the creation of data objects of unknown size. In Java, the `+` operator is overloaded to represent string catenation.

The expression `Str1 + Str2` creates a new string representing the catenation of strings `Str1` and `Str2`. There is no compile-time bound on the sizes of `Str1` and `Str2`, so heap space must be implicitly allocated to hold the newly created string.
Whether allocation is explicit or implicit, a heap allocator is needed. This routine takes a size parameter and examines unused heap space to find space that satisfies the request. A heap block is returned. This block must be big enough to satisfy the space request, but it may well be bigger.

Heaps blocks contain a header field that contains the size of the block as well as bookkeeping information.

The complexity of heap allocation depends in large measure on how deallocation is done.

Initially, the heap is one large block of unallocated memory. Memory requests can be satisfied by simply modifying an “end of heap” pointer,
very much as a stack is pushed by modifying a stack pointer.

Things get more involved when previously allocated heap objects are deallocated and reused.

Deallocated objects are stored for future reuse on a free space list.

When a request for \( n \) bytes of heap space is received, the heap allocator must search the free space list for a block of sufficient size. There are many search strategies that might be used:

- **Best Fit**
  The free space list is searched for the free block that matches most closely the requested size. This minimizes wasted heap space, the search may be quite slow.
• **First Fit**
  The first free heap block of sufficient size is used. Unused space within the block is split off and linked as a smaller free space block. This approach is fast, but may “clutter” the beginning of the free space list with a number of blocks too small to satisfy most requests.

• **Next Fit**
  This is a variant of first fit in which succeeding searches of the free space list begin at the position where the last search ended. The idea is to “cycle through” the entire free space list rather than always revisiting free blocks at the head of the list.
Segregated Free Space Lists
There is no reason why we must have only one free space list. An alternative is to have several, indexed by the size of the free blocks they contain.
Deallocation Mechanisms

Allocating heap space is fairly easy. But how do we deallocate heap memory no longer in use?

Sometimes we may never need to deallocate! If heaps objects are allocated infrequently or are very long-lived, deallocation is unnecessary. We simply fill heap space with “in use” objects.

Virtual memory & paging may allow us to allocate a very large heap area.

On a 64-bit machine, if we allocate heap space at 1 MB/sec, it will take 500,000 years to span the entire address space! Fragmentation of a very large heap space commonly forces us to include some form of reuse of heap space.
User-controlled Deallocation

Deallocation can be manual or automatic. Manual deallocation involves explicit programmer-initiated calls to routines like `free(p)` or `delete(p)`. The object is then added to a free-space list for subsequent reallocation. It is the programmer’s responsibility to free unneeded heap space by executing deallocation commands. The heap manager merely keeps track of freed space and makes it available for later reuse.

The really hard decision—when space should be freed—is shifted to the programmer, possibly leading to catastrophic dangling pointer errors.
Consider the following C program fragment

```c
q = p = malloc(1000);
free(p);
/* code containing more malloc’s */
q[100] = 1234;
```

After `p` is freed, `q` is a **dangling pointer**. `q` points to heap space that is no longer considered allocated.

Calls to `malloc` may reassign the space pointed to by `q`. Assignment through `q` is illegal, but this error is almost never detected.

Such an assignment may change data that is now part of another heap object, leading to very subtle errors. It may even change a header field or a free-space link, causing the heap allocator itself to fail!
Automatic Garbage Collection

The alternative to manual deallocation of heap space is garbage collection.

Compiler-generated code tracks pointer usage. When a heap object is no longer pointed to, it is garbage, and is automatically collected for subsequent reuse.

Many garbage collection techniques exist. Here are some of the most important approaches:
Reference Counting

This is one of the oldest and simplest garbage collection techniques.

A reference count field is added to each heap object. It counts how many references to the heap object exist. When an object’s reference count reaches zero, it is garbage and may collected.

The reference count field is updated whenever a reference is created, copied, or destroyed. When a reference count reaches zero and an object is collected, all pointers in the collected object are also be followed and corresponding reference counts decremented.
As shown below, reference counting has difficulty with circular structures.

If pointer $P$ is set to null, the object’s reference count is reduced to 1. Both objects have a non-zero count, but neither is accessible through any external pointer. The two objects are garbage, but won’t be recognized as such.

If circular structures are common, then an auxiliary technique, like mark-sweep collection, is needed to collect garbage that reference counting misses.
Mark-Sweep Collection

Mark & sweep does nothing until heap space is nearly exhausted.

Then it executes a marking phase that identifies all live heap objects.

Starting with global pointers and pointers in stack frames, it marks reachable heap objects. Pointers in marked heap objects are also followed, until all live heap objects are marked.

After the marking phase, any object not marked is garbage that may be freed. We then sweep through the heap, collecting all unmarked objects. During the sweep phase we also clear all marks from heap objects found to be still in use.
Mark-sweep garbage collection is illustrated below.

Objects 1 and 3 are marked because they are pointed to by global pointers. Object 5 is marked because it is pointed to by object 3, which is marked. Shaded objects are not marked and will be added to the free-space list.

In any mark-sweep collector, it is vital that we mark all accessible heap objects. If we miss a pointer, we may fail to mark a live heap object and later incorrectly free it. Finding all
pointers is a bit tricky in languages like Java, C and C++, that have pointers mixed with other types within data structures, implicit pointers to temporaries, and so forth. Considerable information about data structures and frames must be available at run-time for this purpose. In cases where we can’t be sure if a value is a pointer or not, we may need to do conservative garbage collection.

In mark-sweep garbage collection all heap objects must be swept. This is costly if most objects are dead. We’d prefer to examine only live objects.
Compaction

After the sweep phase, live heap objects are distributed throughout the heap space. This can lead to poor locality. If live objects span many memory pages, paging overhead may be increased. Cache locality may be degraded too.

We can add a compaction phase to mark-sweep garbage collection.

After live objects are identified, they are placed together at one end of the heap. This involves another tracing phase in which global, local and internal heap pointers are found and adjusted to reflect the object’s new location.

Pointers are adjusted by the total size of all garbage objects between the
Compaction merges together freed objects into one large block of free heap space. Small fragments are no longer a problem. Moreover, heap allocation is greatly simplified. Using an “end of heap” pointer, whenever a heap request is received, the end of heap pointer is adjusted, making heap allocation no more complex than stack allocation.
Because pointers are adjusted, compaction may not be suitable for languages like C and C++, in which it is difficult to unambiguously identify pointers.
Copying Collectors

Compaction provides many valuable benefits. Heap allocation is simple and efficient. There is no fragmentation problem, and because live objects are adjacent, paging and cache behavior is improved.

An entire family of garbage collection techniques, called copying collectors are designed to integrate copying with recognition of live heap objects. Copying collectors are very popular and are widely used.

Consider a simple copying collector that uses semispaces. We start with the heap divided into two halves—the “from” and “to” spaces.
Initially, we allocate heap requests from the from space, using a simple “end of heap” pointer. When the “from space” is exhausted, we stop and do garbage collection.

Actually, though we don’t collect garbage. We collect live heap objects—garbage is never touched.

We trace through global and local pointers, finding live objects. As each object is found, it is moved from its current position in the “from space” to the next available position in the “to space.”

The pointer is updated to reflect the object’s new location. A “forwarding pointer” is left in the object’s old location in case there are multiple pointers to the same object.
This is illustrated below:

The “from space” is completely filled. We trace global and local pointers, moving live objects to the “to space” and updating pointers. (Dashed arrows are forwarding pointers):
We now handle pointers internal to copied heap objects. Copied objects are traversed. Objects referenced are copied and internal pointers are updated.

Finally, the “to” and “from spaces” are interchanged. Heap allocation resumes just beyond the last copied object:
The biggest advantage of copying collectors is their speed. Only live objects are copied; deallocation of dead objects is essentially free. Garbage collection can be made, on average, as fast as you wish—simply make the heap bigger. As the heap gets bigger, the time between collections increases, reducing the number of times a live object must be copied. In the limit, objects are never copied, so garbage collection becomes free!

We can’t increase the size of heap memory to infinity. We don’t want to make the heap so large that paging is required, since swapping pages to disk is slow.
We aim to make the heap large enough that the lifetime of most heap objects is less than the time between collections, so that deallocation of short-lived objects will appear to be free, though longer-lived objects will still exact a cost.
Conservative Garbage Collection

The garbage collection techniques we’ve studied all require that we identify pointers to heap objects accurately. In strongly typed languages like Java or ML, this can be done.

Languages like C and C++ are weakly typed, and this makes identification of pointers much harder. Pointers may be type-cast into integers and then back into pointers. Pointer arithmetic allows pointers into the middle of an object. Pointers in frames and heap objects need not be initialized, and may contain unpredictable values. Pointers may overlay integers in unions, making the current type a dynamic property.
As a result, C and C++ have the reputation of being incompatible with garbage collection. Surprisingly, this belief is false. Using conservative garbage collection, C and C++ programs can be garbage collected.

The basic idea is simple—if we can’t be sure whether a value is a pointer or not, we’ll be conservative and assume it is a pointer. If what we think is a pointer isn’t, we may retain an object that’s really dead, but we’ll find all valid pointers, and never incorrectly collect a live object. We may mistake an integer (or a floating value, or even a string) as an pointer, so compaction in any form can’t be done. However, mark-sweep collection will work.