Properties of Regular Expressions and Finite Automata

- Some token patterns *can't* be defined as regular expressions or finite automata. Consider the set of balanced brackets of the form [[[...]]].
  This set is defined formally as
  $\{ [^m]^m \mid m \geq 1 \}$.
  This set is *not* regular.
  No finite automaton that recognizes *exactly* this set can exist.
  Why? Consider the inputs [, [[, [[[, ...
  For two different counts (call them i and j) $[^i$ and $[^j$ must reach the same state of a given FA! (Why?)
  Once that happens, we know that if $[^i]^i$ is accepted (as it should be), the $[^j]^i$ will also be accepted (and that should not happen).

- $\overline{R} = V^* - R$ is regular if R is.
  Why?
  Build a finite automaton for R. Be careful to include transitions to an "error state" s_E for illegal characters.
  Now invert final and non-final states. What was previously accepted is now rejected, and what was rejected is now accepted. That is, $\overline{R}$ is accepted by the modified automaton.

- Not all subsets of a regular set are themselves regular. The regular expression $[^+]^+$ has a subset that isn't regular. (What is that subset?)

- Let R be a set of strings. Define R^{rev} as all strings in R, in reversed (backward) character order.
  Thus if R = {abc, def}
  then R^{rev} = {cba, fed}.
  If R is regular, then R^{rev} is too.
  Why? Build a finite automaton for R. Make sure the automaton has only one final state. Now *reverse* the direction of all transitions, and interchange the start and final states. What does the modified automation accept?

- If R_1 and R_2 are both regular, then $R_1 \cap R_2$ is also regular. We can show this two different ways:
  1. Build two finite automata, one for R1 and one for R2. Pair together states of the two automata to match R1 and R2 simultaneously. The paired-state automaton accepts only if both R1 and R2 would, so R1 $\cap$ R2 is matched.
  2. We can use the fact that R1 $\cap$ R2 is = $\overline{\overline{R_1} \cup \overline{R_2}}$ We already know union and complementation are regular.

Reading Assignment

- Read Chapter 4 of **Crafting a Compiler**

Context Free Grammars

A context-free grammar (CFG) is defined as:

- A finite terminal set V_t; these are the tokens produced by the scanner.
- A set of intermediate symbols, called non-terminals, V_n.
- A start symbol, a designated non-terminal, that starts all derivations.
- A set of productions (sometimes called rewriting rules) of the form
  $A \to X_1 \ldots X_m$
  X_1 to X_m may be any combination of terminals and non-terminals.
  If m =0 we have $A \to \lambda$ which is a valid production.

Example

Prog → { Stmts }
Stmts →Stmts ; Stmt
Stmts →Stmt
Stmt →id = Expr
Expr →id
Expr →Expr + id

Often more than one production shares the same left-hand side.

Rather than repeat the left hand side, an “or notation” is used:

Prog → { Stmts }
Stmts →Stmts ; Stmt
| Stmt
Stmt →id = Expr
Expr →id
| Expr + id

Derivations

Starting with the start symbol, non-terminals are rewritten using productions until only terminals remain.

Any terminal sequence that can be generated in this manner is syntactically valid.

If a terminal sequence can't be generated using the productions of the grammar it is invalid (has syntax errors).

The set of strings derivable from the start symbol is the *language* of the grammar (sometimes denoted L(G)).

For example, starting at **Prog** we generate a terminal sequence, by repeatedly applying productions:

Prog

{ Stmts }

{ Stmts ; Stmt }

{ Stmt ; Stmt }

{ id = Expr ; Stmt }

{ id = id ; Stmt }

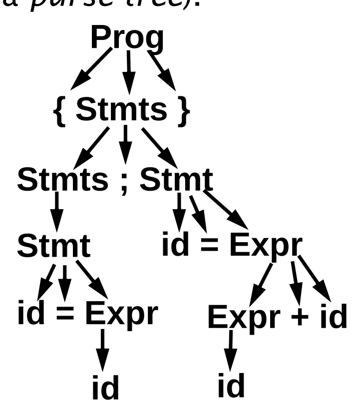
{ id = id ; id = Expr }

{ id = id ; id = Expr + id}

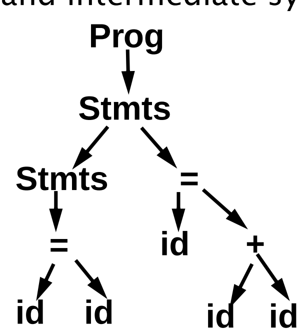
{ id = id ; id = id + id}

Parse Trees

To illustrate a derivation, we can draw a *derivation tree* (also called a *parse tree*):



An *abstract syntax tree* (AST) shows essential structure but eliminates unnecessary delimiters and intermediate symbols:



If $A \to \gamma$ is a production then
$\alpha A \beta \Rightarrow \alpha\gamma\beta$
where $\Rightarrow$ denotes a one step derivation (using production $A \to \gamma$).

We extend $\Rightarrow$ to $\Rightarrow^+$ (derives in one or more steps), and $\Rightarrow^*$ (derives in zero or more steps).

We can show our earlier derivation as

Prog $\Rightarrow$

{ Stmts } $\Rightarrow$

{ Stmts ; Stmt } $\Rightarrow$

{ Stmt ; Stmt } $\Rightarrow$

{ id = Expr ; Stmt } $\Rightarrow$

{ id = id ; Stmt } $\Rightarrow$

{ id = id ; id = Expr } $\Rightarrow$

{ id = id ; id = Expr + id} $\Rightarrow$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

When deriving a token sequence, if more than one non-terminal is present, we have a choice of which to expand next.

We must specify, at each step, which non-terminal is expanded, and what production is applied.

For simplicity we adopt a convention on what non-terminal is expanded at each step.

We can choose the leftmost possible non-terminal at each step.

A derivation that follows this rule is a *leftmost derivation*.

If we know a derivation is leftmost, we need only specify what productions are used; the choice of non-terminal is always fixed.

To denote derivations that are leftmost,
we use $\Rightarrow_L$, $\Rightarrow^+_L$, and $\Rightarrow^*_L$

The production sequence discovered by a large class of parsers (the top-down parsers) is a leftmost derivation, hence these parsers produce a *leftmost parse*.

Prog $\Rightarrow_L$

{ Stmts } $\Rightarrow_L$

{ Stmts ; Stmt } $\Rightarrow_L$

{ Stmt ; Stmt } $\Rightarrow_L$

{ id = Expr ; Stmt } $\Rightarrow_L$

{ id = id ; Stmt } $\Rightarrow_L$

{ id = id ; id = Expr } $\Rightarrow_L$

{ id = id ; id = Expr + id} $\Rightarrow_L$

{ id = id ; id = id + id}

Prog $\Rightarrow^+_L$ **{ id = id ; id = id + id}**

Rightmost Derivations

A rightmost derivation is an alternative to a leftmost derivation. Now the rightmost non-terminal is always expanded.

This derivation sequence may seem less intuitive given our normal left-to-right bias, but it corresponds to an important class of parsers (the bottom-up parsers, including CUP).

As a bottom-up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in *reverse order*.

The last production applied in a rightmost derivation is the first that is discovered. The first production used, involving the start symbol, is discovered last.

The sequence of productions recognized by a bottom-up parser is a rightmost parse.

It is the exact reverse of the production sequence that represents a rightmost derivation.

For rightmost derivations, we use the notation $\Rightarrow_R$, $\Rightarrow^+_R$, and $\Rightarrow^*_R$

Prog $\Rightarrow_R$

{ Stmts } $\Rightarrow_R$

{ Stmts ; Stmt } $\Rightarrow_R$

{ Stmts ; id = Expr } $\Rightarrow_R$

{ Stmts ; id = Expr + id } $\Rightarrow_R$

{ Stmts ; id = id + id } $\Rightarrow_R$

{ Stmt ; id = id + id } $\Rightarrow_R$

{ id = Expr ; id = id + id } $\Rightarrow_R$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

You can derive the same set of tokens using leftmost and rightmost derivations; the only difference is the order in which productions are used.

Ambiguous Grammars

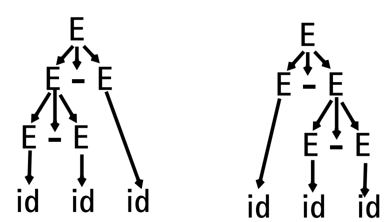
Some grammars allow more than one parse tree for the same token sequence. Such grammars are *ambiguous*. Because compilers use syntactic structure to drive translation, ambiguity is undesirable—it may lead to an unexpected translation.

Consider

E $\rightarrow$ **E - E**

  | **id**

When parsing the input a-b-c (where a, b and c are scanned as identifiers) we can build the following two parse trees:



The effect is to parse a-b-c as either (a-b)-c or a-(b-c). These two groupings are certainly not equivalent.

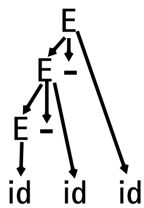
Ambiguous grammars are usually voided in building compilers; the tools we use, like Yacc and CUP, strongly prefer unambiguous grammars.

To correct this ambiguity, we use

E $\rightarrow$ **E - id**

  | **id**

Now a-b-c can only be parsed as:



Operator Precedence

Most programming languages have *operator precedence* rules that state the order in which operators are applied (in the absence of explicit parentheses). Thus in C and Java and CSX, **`a+b*c`** means compute **`b*c`**, then add in **`a`**.

These operators precedence rules can be incorporated directly into a CFG.

Consider

E → E + T

  | T

T → T * P

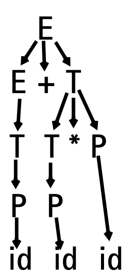
  | P

P → id

  | (E)

Does **`a+b*c`** mean **`(a+b)*c`** or **`a+(b*c)`**?

The grammar tells us! Look at the derivation tree:



The other grouping can't be obtained unless explicit parentheses are used.

(Why?)

Java CUP

Java CUP is a parser-generation tool, similar to Yacc.

CUP builds a Java parser for LALR(1) grammars from production rules and associated Java code fragments.

When a particular production is recognized, its associated code fragment is executed (typically to build an AST).

CUP generates a Java source file **`parser.java`**. It contains a class **`parser`**, with a method

```
Symbol parse()
```

The **`Symbol`** returned by the parser is associated with the grammar's start symbol and contains the AST for the whole source program.

The file **`sym.java`** is also built for use with a JLex-built scanner (so that both scanner and parser use the same token codes).

If an unrecovered syntax error occurs, **`Exception()`** is thrown by the parser.

CUP and Yacc accept exactly the same class of grammars—all LL(1) grammars, plus many useful non-LL(1) grammars.

CUP is called as

```
java java_cup.Main < file.cup
```


Java CUP Specifications

Java CUP specifications are of the form:

- Package and import specifications
- User code additions
- Terminal and non-terminal declarations
- A context-free grammar, augmented with Java code fragments

Package and Import Specifications

You define a package name as:

```
package name ;
```

You add imports to be used as:

```
import java_cup.runtime.*;
```


User Code Additions

You may define Java code to be included within the generated parser:

```
action code {: /*java code */ :}
```

This code is placed within the generated action class (which holds user-specified production actions).

```
parser code {: /*java code */ :}
```

This code is placed within the generated parser class .

```
init with{: /*java code */ :}
```

This code is used to initialize the generated parser.

```
scan with{: /*java code */ :}
```

This code is used to tell the generated parser how to get tokens from the scanner.

Terminal and Non-terminal Declarations

You define terminal symbols you will use as:

terminal classname name$_1$, name$_2$, ...

`classname` is a class used by the scanner for tokens (**`CSXToken`**, **`CSXIdentifierToken`**, etc.)

You define non-terminal symbols you will use as:

non terminal classname name$_1$, name$_2$, ...

`classname` is the class for the AST node associated with the non-terminal (**`stmtNode`**, **`exprNode`**, etc.)

Production Rules

Production rules are of the form

```
name ::= name1 name2 ... action ;
```

or

```
name ::= name1 name2 ...
action1
    |  name3 name4 ... action2
    |   ...
    ;
```

Names are the names of terminals or non-terminals, as declared earlier.

Actions are Java code fragments, of the form

```
{: /*java code */ :}
```

The Java object assocated with a symbol (a token or AST node) may be named by adding a **:id** suffix to a terminal or non-terminal in a rule.

RESULT names the left-hand side non-terminal.

The Java classes of the symbols are defined in the terminal and non-terminal declaration sections.

For example,

```
prog ::= LBRACE:l stmts:s RBRACE
   {: RESULT =
        new csxLiteNode(s,
          l.linenum,l.colnum); :}
```

This corresponds to the production

prog → { stmts }

The left brace is named **l**; the stmts non-terminal is called **s**.

In the action code, a new **CSXLiteNode** is created and assigned to **prog**. It is constructed from the AST node associated with **s**. Its line and column numbers are those given to the left brace, **l** (by the scanner).

To tell CUP what non-terminal to use as the start symbol (**prog** in our example), we use the directive:

```
start with prog;
```