## Properties of Regular Expressions and Finite Automata

 Some token patterns can't be defined as regular expressions or finite automata. Consider the set of balanced brackets of the form [[[...]]]. This set is defined formally as  $\{ [^m]^m \mid m \ge 1 \}.$ This set is *not* regular. No finite automaton that recognizes exactly this set can exist. Why? Consider the inputs [, [[, [[[, ... For two different counts (call them i and j) [i and [j must reach the same state of a given FA! (Why?) Once that happens, we know that if [1]1 is accepted (as it should be), the [J]1 will also be accepted (and that should

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not happen).

- R = V\* R is regular if R is.
  Why?
  Build a finite automaton for R. Be careful to include transitions to an "error state" s<sub>E</sub> for illegal characters.
  Now invert final and non-final states.
  What was previously accepted is now rejected, and what was rejected is now accepted. That is, R is accepted by the modified automaton.
- Not all subsets of a regular set are themselves regular. The regular expression [+]+ has a subset that isn't regular. (What is that subset?)

 Let R be a set of strings. Define R<sup>rev</sup> as all strings in R, in reversed (backward) character order.

Thus if  $R = \{abc, def\}$ 

then  $R^{rev} = \{cba, fed\}.$ 

If R is regular, then R<sup>rev</sup> is too.

Why? Build a finite automaton for R. Make sure the automaton has only one final state. Now *reverse* the direction of all transitions, and interchange the start and final states. What does the modified automation accept?

- If  $R_1$  and  $R_2$  are both regular, then  $R_1 \cap R_2$  is also regular. We can show this two different ways:
  - 1. Build two finite automata, one for R1 and one for R2. Pair together states of the two automata to match R1 and R2 simultaneously. The paired-state automaton accepts only if both R1 and R2 would, so R1 ∩ R2 is matched.
  - 2. We can use the fact that R1  $\cap$  R2 is =  $\overline{R_1} \cup \overline{R_2}$  We already know union and complementation are regular.

#### Reading Assignment

 Read Chapter 4 of Crafting a Compiler

#### CONTEXT FREE GRAMMARS

A context-free grammar (CFG) is defined as:

- A finite terminal set V<sub>t</sub>; these are the tokens produced by the scanner.
- A set of intermediate symbols, called non-terminals,  $V_n$ .
- A start symbol, a designated nonterminal, that starts all derivations.
- A set of productions (sometimes called rewriting rules) of the form

 $A \rightarrow X_1 \dots X_m$ 

 $X_1$  to  $X_m$  may be any combination of terminals and non-terminals.

If m =0 we have A  $\rightarrow \lambda$  which is a valid production.

#### Example

```
Prog → { Stmts }
Stmts → Stmts ; Stmt
Stmts → Stmt
Stmt → id = Expr
Expr → id
Expr → Expr + id
```

Often more than one production shares the same left-hand side. Rather than repeat the left hand side, an "or notation" is used:

#### **Derivations**

Starting with the start symbol, non-terminals are rewritten using productions until only terminals remain.

Any terminal sequence that can be generated in this manner is syntactically valid.

If a terminal sequence can't be generated using the productions of the grammar it is invalid (has syntax errors).

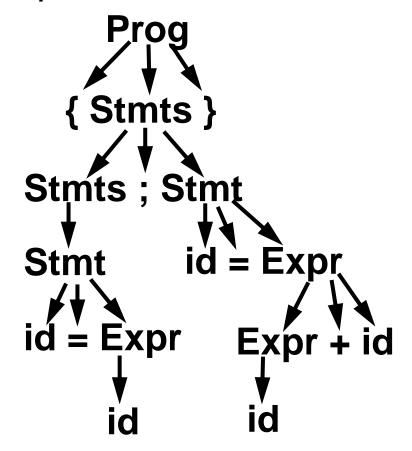
The set of strings derivable from the start symbol is the *language* of the grammar (sometimes denoted L(G)).

For example, starting at **Prog** we generate a terminal sequence, by repeatedly applying productions:

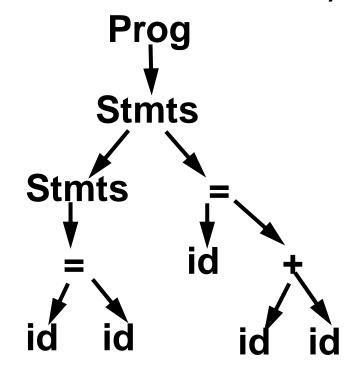
# Prog { Stmts } { Stmts; Stmt } { Stmt; Stmt } { id = Expr; Stmt } { id = id; Stmt } { id = id; id = Expr } { id = id; id = Expr + id} { id = id; id = id + id}

#### Parse Trees

To illustrate a derivation, we can draw a *derivation tree* (also called a *parse tree*):



An abstract syntax tree (AST) shows essential structure but eliminates unnecessary delimiters and intermediate symbols:



```
If A \rightarrow \gamma is a production then
  \alpha A\beta \Rightarrow \alpha \gamma \beta
where \Rightarrow denotes a one step
derivation (using production
 A \rightarrow \gamma).
We extend \Rightarrow to \Rightarrow<sup>+</sup> (derives in
one or more steps), and \Rightarrow^*
(derives in zero or more steps).
We can show our earlier
derivation as
Prog \Rightarrow
\{ Stmts \} \Rightarrow
\{ Stmts ; Stmt \} \Rightarrow
\{ Stmt ; Stmt \} \Rightarrow
\{ id = Expr ; Stmt \} \Rightarrow
\{ id = id ; Stmt \} \Rightarrow
\{ id = id ; id = Expr \} \Rightarrow
\{ id = id ; id = Expr + id \} \Rightarrow
\{ id = id ; id = id + id \}
\mathsf{Prog} \Rightarrow^+ \{ \mathsf{id} = \mathsf{id} \; ; \; \mathsf{id} = \mathsf{id} + \mathsf{id} \}
```

When deriving a token sequence, if more than one non-terminal is present, we have a choice of which to expand next.

We must specify, at each step, which non-terminal is expanded, and what production is applied.

For simplicity we adopt a convention on what non-terminal is expanded at each step.

We can choose the leftmost possible non-terminal at each step.

A derivation that follows this rule is a *leftmost derivation*.

If we know a derivation is leftmost, we need only specify what productions are used; the choice of non-terminal is always fixed.

To denote derivations that are leftmost,

we use  $\Rightarrow_L$ ,  $\Rightarrow_L^+$ , and  $\Rightarrow_L^*$ 

The production sequence discovered by a large class of parsers (the top-down parsers) is a leftmost derivation, hence these parsers produce a *leftmost parse*.

```
Prog \Rightarrow_{\perp}

{ Stmts } \Rightarrow_{\perp}

{ Stmts ; Stmt } \Rightarrow_{\perp}

{ Stmt ; Stmt } \Rightarrow_{\perp}

{ id = Expr ; Stmt } \Rightarrow_{\perp}

{ id = id ; Stmt } \Rightarrow_{\perp}

{ id = id ; id = Expr } \Rightarrow_{\perp}

{ id = id ; id = Expr + id} \Rightarrow_{\perp}

{ id = id ; id = id + id}

Prog \Rightarrow_{\perp}^{+} { id = id ; id = id + id}
```

#### Rightmost Derivations

A rightmost derivation is an alternative to a leftmost derivation. Now the rightmost non-terminal is always expanded.

This derivation sequence may seem less intuitive given our normal left-to-right bias, but it corresponds to an important class of parsers (the bottom-up parsers, including CUP).

As a bottom-up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in reverse order.

The last production applied in a rightmost derivation is the first that is discovered. The first production used, involving the start symbol, is discovered last.

The sequence of productions recognized by a bottom-up parser is a rightmost parse.

It is the exact reverse of the production sequence that represents a rightmost derivation.

For rightmost derivations, we use the notation  $\Rightarrow_R$ ,  $\Rightarrow_R^+$ , and  $\Rightarrow_R^*$ 

```
Prog \Rightarrow_R

{ Stmts } \Rightarrow_R

{ Stmts ; Stmt } \Rightarrow_R

{ Stmts ; id = Expr } \Rightarrow_R

{ Stmts ; id = Expr + id } \Rightarrow_R

{ Stmts ; id = id + id } \Rightarrow_R

{ Stmt ; id = id + id } \Rightarrow_R

{ id = Expr ; id = id + id } \Rightarrow_R

{ id = id ; id = id + id}

Prog \Rightarrow^+ { id = id ; id = id + id}
```

You can derive the same set of tokens using leftmost and rightmost derivations; the only difference is the order in which productions are used.

#### Ambiquous Grammars

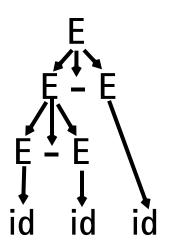
Some grammars allow more than one parse tree for the same token sequence. Such grammars are ambiguous. Because compilers use syntactic structure to drive translation, ambiguity is undesirable—it may lead to an unexpected translation.

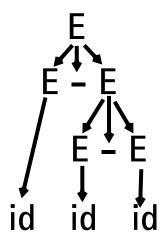
Consider

$$\mathsf{E} \to \mathsf{E} - \mathsf{E}$$

| id

When parsing the input a-b-c (where a, b and c are scanned as identifiers) we can build the following two parse trees:





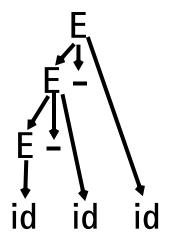
The effect is to parse a-b-c as either (a-b)-c or a-(b-c). These two groupings are certainly not equivalent.

Ambiguous grammars are usually voided in building compilers; the tools we use, like Yacc and CUP, strongly prefer unambiguous grammars.

To correct this ambiguity, we use

$$\mathsf{E} o \mathsf{E}$$
 - id  $|$  id

#### Now a-b-c can only be parsed as:



#### Operator Precedence

Most programming languages have operator precedence rules that state the order in which operators are applied (in the absence of explicit parentheses). Thus in C and Java and CSX, a+b\*c means compute b\*c, then add in a.

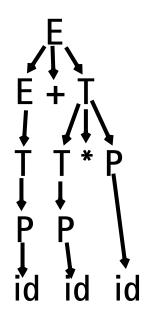
These operators precedence rules can be incorporated directly into a CFG.

Consider

$$\mathbf{E} 
ightarrow \mathbf{E} + \mathbf{T}$$
 $\mid \mathbf{T} 
ightarrow \mathbf{T} * \mathbf{P}$ 
 $\mid \mathbf{P} 
ightarrow \mathbf{id}$ 
 $\mid (\mathbf{E})$ 

Does a+b\*c mean (a+b)\*c or a+(b\*c)?

The grammar tells us! Look at the derivation tree:



The other grouping can't be obtained unless explicit parentheses are used. (Why?)

#### JAVA CUP

Java CUP is a parser-generation tool, similar to Yacc.

CUP builds a Java parser for LALR(1) grammars from production rules and associated Java code fragments.

When a particular production is recognized, its associated code fragment is executed (typically to build an AST).

CUP generates a Java source file parser.java. It contains a class parser, with a method Symbol parse()

The symbol returned by the parser is associated with the grammar's start symbol and contains the AST for the whole source program.

The file sym. java is also built for use with a JLex-built scanner (so that both scanner and parser use the same token codes).

If an unrecovered syntax error occurs, **Exception()** is thrown by the parser.

CUP and Yacc accept exactly the same class of grammars—all LL(1) grammars, plus many useful non-LL(1) grammars.

CUP is called as

java java\_cup.Main < file.cup</pre>

#### JAVA CUP Specifications

Java CUP specifications are of the form:

- Package and import specifications
- User code additions
- Terminal and non-terminal declarations
- A context-free grammar, augmented with Java code fragments

#### Package and Import Specifications

```
You define a package name as:
package name ;
You add imports to be used as:
import java_cup.runtime.*;
```

#### **User Code Additions**

You may define Java code to be included within the generated parser:

action code {: /\*java code \*/ :} This code is placed within the generated action class (which holds user-specified production actions).

parser code {: /\*java code \*/ :}
This code is placed within the
generated parser class.

init with{: /\*java code \*/ :}
This code is used to initialize the
generated parser.

scan with{: /\*java code \*/ :}
This code is used to tell the
generated parser how to get
tokens from the scanner.

### Terminal and Non-terminal Declarations

You define terminal symbols you will use as:

classname is a class used by the scanner for tokens (CSXToken, CSXIdentifierToken, etc.)

You define non-terminal symbols you will use as:

classname is the class for the AST node associated with the non-terminal (stmtNode, exprNode, etc.)

#### **Production Rules**

Production rules are of the form

name ::= name<sub>1</sub> name<sub>2</sub> ... action;

or

name ::= name<sub>1</sub> name<sub>2</sub> ...

action<sub>1</sub>

| name<sub>3</sub> name<sub>4</sub> ... action<sub>2</sub>

| ...

Names are the names of terminals or non-terminals, as declared earlier.

Actions are Java code fragments, of the form

```
{: /*java code */ :}
```

The Java object assocated with a symbol (a token or AST node) may be named by adding a :id suffix to a terminal or non-terminal in a rule.

**RESULT** names the left-hand side non-terminal.

The Java classes of the symbols are defined in the terminal and non-terminal declaration sections. For example,

This corresponds to the production  $prog \rightarrow \{ stmts \}$ 

The left brace is named 1; the stmts non-terminal is called s.

In the action code, a new **CSXLiteNode** is created and assigned to **prog**. It is constructed from the AST node associated with **s**. Its line and column numbers are those given to the left brace, **1** (by the scanner).

To tell CUP what non-terminal to use as the start symbol (**prog** in our example), we use the directive:

start with prog;