Optimizing Finite Automata

We can improve the DFA created by `MakeDeterministic`.

Sometimes a DFA will have more states than necessary. For every DFA there is a unique *smallest* equivalent DFA (fewest states possible).

Some DFA's contain *unreachable states* that cannot be reached from the start state.

Other DFA's may contain *dead states* that cannot reach any accepting state.

It is clear that neither unreachable states nor dead states can participate in scanning any valid token. We therefore eliminate all such states as part of our optimization process.

We optimize a DFA by *merging together* states we know to be equivalent.

For example, two accepting states that have no transitions at all out of them are equivalent.

Why? Because they behave exactly the same way—they accept the string read so far, but will accept no additional characters.

If two states, s_1 and s_2, are equivalent, then all transitions to s_2 can be replaced with transitions to s_1. In effect, the two states are merged together into one common state.

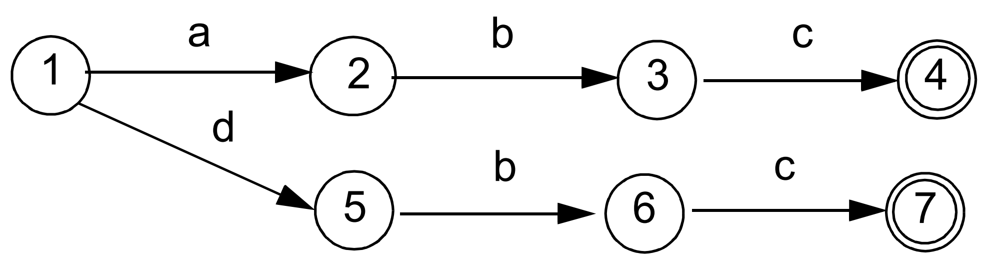
How do we decide what states to merge together?

We take a *greedy* approach and try the most optimistic merger of states. By definition, accepting and non- accepting states are distinct, so we initially try to create only two states: one representing the merger of all accepting states and the other representing the merger of all non- accepting states.

This merger into only two states is almost certainly too optimistic. In particular, all the constituents of a merged state must agree on the same transition for each possible character. That is, for character c all the merged states must have no successor under c or they must all go to a single (possibly merged) state.

If all constituents of a merged state do not agree on the transition to follow for some character, the merged state is *split* into two or more smaller states that do agree.

As an example, assume we start with the following automaton:



Initially we have a merged non-accepting state {1,2,3,5,6} and a merged accepting state {4,7}.

A merger is legal if and only if all constituent states agree on the same successor state for all characters. For example, states 3 and 6 would go to an accepting state given character c; states 1, 2, 5 would not, so a split must occur.

We will add an error state s_E to the original DFA that is the successor state under any illegal character. (Thus reaching s_E becomes equivalent to detecting an illegal token.) s_E is not a real state; rather it allows us to assume every state has a successor under every character. s_E is never merged with any real state.

Algorithm **`Split`**, shown below, splits merged states whose constituents do not agree on a common successor state for all characters. When **`Split`** terminates, we know that the states that remain merged are equivalent in that they always agree on common successors.

```
Split(FASet StateSet) {
 repeat
  for(each merged state S in StateSet) {
    Let S correspond to {s1,…,sn}
    for(each char c in Alphabet){
      Let t1,…,tn be the successor
       states to s1,…,sn under c
      if(t1,…,tn do not all belong to
           the same merged state){
            Split S into two or more new
            states such that si and sj
            remain in the same merged
            state if and only if ti and tj
            are in the same merged state}
      }
 until no more splits are possible
}
```

Returning to our example, we initially have states {1,2,3,5,6} and {4,7}. Invoking **`Split`**, we first observe that states 3 and 6 have a common successor under c, and states 1, 2, and 5 have no successor under c (equivalently, have the error state s_E as a successor).

This forces a split, yielding {1,2,5}, {3,6} and {4,7}.

Now, for character b, states 2 and 5 would go to the merged state {3,6}, but state 1 would not, so another split occurs.

We now have: {1}, {2,5}, {3,6} and {4,7}.

At this point we are done, as all constituents of merged states agree on the same successor for each input symbol.

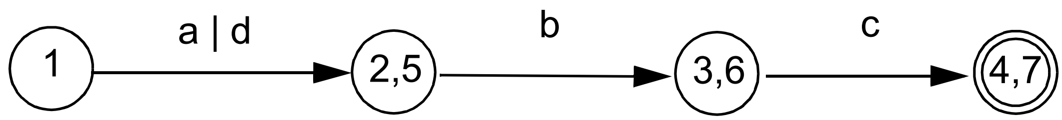
Once `Split` is executed, we are essentially done.

Transitions between merged states are the same as the transitions between states in the original DFA.

Thus, if there was a transition between state s_i and s_j under character c, there is now a transition under c from the merged state containing s_i to the merged state containing s_j. The start state is that merged state containing the original start state.

Accepting states are those merged states containing accepting states (recall that accepting and non- accepting states are never merged).

Returning to our example, the minimum state automaton we obtain is



Properties of Regular Expressions and Finite Automata

- Some token patterns *can't* be defined as regular expressions or finite automata. Consider the set of balanced brackets of the form [[[…]]]. This set is defined formally as
  $\{ [^m]^m \mid m \geq 1 \}$.
  This set is *not* regular.
  No finite automaton that recognizes *exactly* this set can exist.
  Why? Consider the inputs [, [[, [[[, ...
  For two different counts (call them i and j) $[^i$ and $[^j$ must reach the same state of a given FA! (Why?)
  Once that happens, we know that if $[^i]^i$ is accepted (as it should be), the $[^j]^i$ will also be accepted (and that should not happen).

- $\overline{R}$ = V* - R is regular if R is.
  Why?
  Build a finite automaton for R. Be careful to include transitions to an "error state" s_E for illegal characters. Now invert final and non- final states. What was previously accepted is now rejected, and what was rejected is now accepted. That is, $\overline{R}$ is accepted by the modified automaton.

- Not all subsets of a regular set are themselves regular. The regular expression $[^+]^+$ has a subset that isn't regular. (What is that subset?)

- Let R be a set of strings. Define R^{rev} as all strings in R, in reversed (backward) character order.
  Thus if R = {abc, def}
  then R^{rev} = {cba, fed}.
  If R is regular, then R^{rev} is too.
  Why? Build a finite automaton for R. Make sure the automaton has only one final state. Now *reverse* the direction of all transitions, and interchange the start and final states. What does the modified automation accept?

- If R_1 and R_2 are both regular, then $R_1 \cap R_2$ is also regular. We can show this two different ways:
  1. Build two finite automata, one for R1 and one for R2. Pair together states of the two automata to match R1 and R2 simultaneously. The paired-state automaton accepts only if both R1 and R2 would, so R1 $\cap$ R2 is matched.
  2. We can use the fact that R1 $\cap$ R2 is = $\overline{\overline{R_1} \cup \overline{R_2}}$ We already know union and complementation are regular.

Reading Assignment

- Read Chapter 4 of
  Crafting a Compiler

Context Free Grammars

A context- free grammar (CFG) is defined as:

- A finite terminal set V_t;
  these are the tokens produced by the scanner.
- A set of intermediate symbols, called non- terminals, V_n.
- A start symbol, a designated non- terminal, that starts all derivations.
- A set of productions (sometimes called rewriting rules) of the form

  $A \rightarrow X_1 \ldots X_m$

  X_1 to X_m may be any combination of terminals and non- terminals.

  If $m = 0$ we have $A \rightarrow \lambda$
  which is a valid production.

Example

Prog → { Stmts }

Stmts →Stmts ; Stmt

Stmts →Stmt

Stmt →id = Expr

Expr →id

Expr →Expr + id

Often more than one production shares the same left- hand side.

Rather than repeat the left hand side, an “or notation” is used:

Prog → { Stmts }

Stmts →Stmts ; Stmt

| Stmt

Stmt →id = Expr

Expr →id

| Expr + id

Derivations

Starting with the start symbol, non- terminals are rewritten using productions until only terminals remain.

Any terminal sequence that can be generated in this manner is syntactically valid.

If a terminal sequence can't be generated using the productions of the grammar it is invalid (has syntax errors).

The set of strings derivable from the start symbol is the *language* of the grammar (sometimes denoted L(G)).

For example, starting at **Prog** we generate a terminal sequence, by repeatedly applying productions:

Prog

{ Stmts }

{ Stmts ; Stmt }

{ Stmt ; Stmt }

{ id = Expr ; Stmt }

{ id = id ; Stmt }

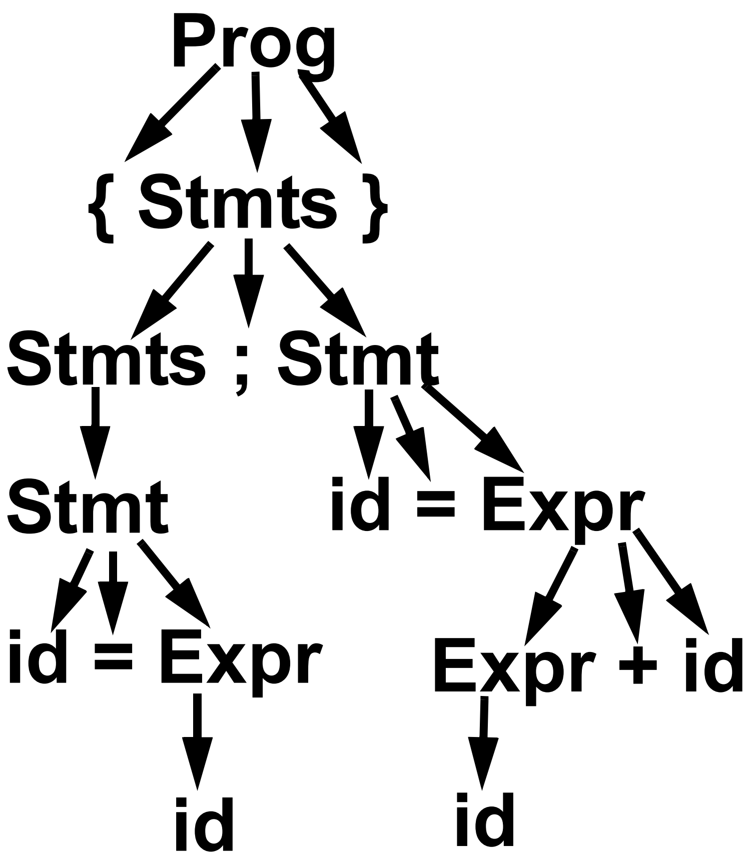
{ id = id ; id = Expr }

{ id = id ; id = Expr + id}

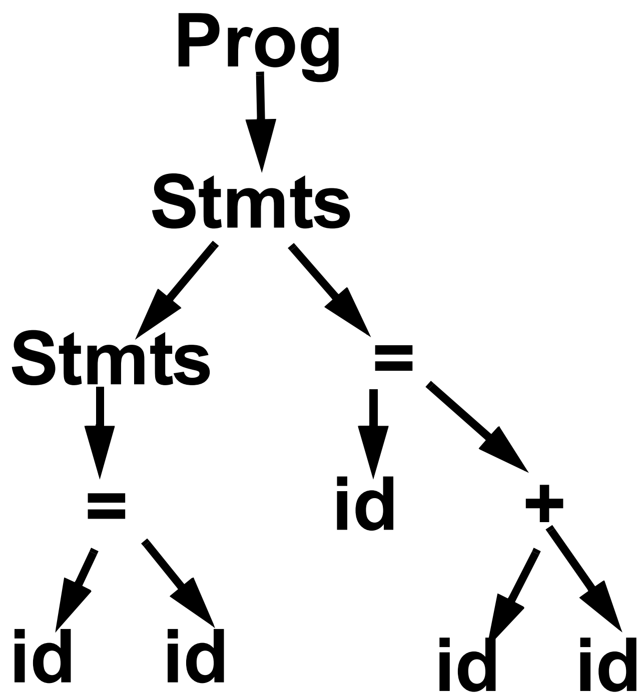
{ id = id ; id = id + id}

Parse Trees

To illustrate a derivation, we can draw a *derivation tree* (also called a *parse tree*):



An *abstract syntax tree* (AST) shows essential structure but eliminates unnecessary delimiters and intermediate symbols:



If $A \to \gamma$ is a production then

$\alpha A \beta \Rightarrow \alpha \gamma \beta$

where $\Rightarrow$ denotes a one step derivation (using production $A \to \gamma$).

We extend $\Rightarrow$ to $\Rightarrow^+$ (derives in one or more steps), and $\Rightarrow^*$ (derives in zero or more steps).

We can show our earlier derivation as

Prog $\Rightarrow$

{ Stmts } $\Rightarrow$

{ Stmts ; Stmt } $\Rightarrow$

{ Stmt ; Stmt } $\Rightarrow$

{ id = Expr ; Stmt } $\Rightarrow$

{ id = id ; Stmt } $\Rightarrow$

{ id = id ; id = Expr } $\Rightarrow$

{ id = id ; id = Expr + id} $\Rightarrow$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

When deriving a token sequence, if more than one non- terminal is present, we have a choice of which to expand next.

We must specify, at each step, which non- terminal is expanded, and what production is applied.

For simplicity we adopt a convention on what non- terminal is expanded at each step.

We can choose the leftmost possible non- terminal at each step.

A derivation that follows this rule is a *leftmost derivation*.

If we know a derivation is leftmost, we need only specify what productions are used; the choice of non- terminal is always fixed.

To denote derivations that are leftmost,

we use $\Rightarrow_L$, $\Rightarrow^+_L$, and $\Rightarrow^*_L$

The production sequence discovered by a large class of parsers (the top- down parsers) is a leftmost derivation, hence these parsers produce a *leftmost parse*.

Prog $\Rightarrow_L$

{ Stmts } $\Rightarrow_L$

{ Stmts ; Stmt } $\Rightarrow_L$

{ Stmt ; Stmt } $\Rightarrow_L$

{ id = Expr ; Stmt } $\Rightarrow_L$

{ id = id ; Stmt } $\Rightarrow_L$

{ id = id ; id = Expr } $\Rightarrow_L$

{ id = id ; id = Expr + id} $\Rightarrow_L$

{ id = id ; id = id + id}

Prog $\Rightarrow^+_L$ **{ id = id ; id = id + id}**

Rightmost Derivations

A rightmost derivation is an alternative to a leftmost derivation. Now the rightmost non- terminal is always expanded.

This derivation sequence may seem less intuitive given our normal left- to- right bias, but it corresponds to an important class of parsers (the bottom- up parsers, including CUP).

As a bottom- up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in *reverse order*.

The last production applied in a rightmost derivation is the first that is discovered. The first production used, involving the start symbol, is discovered last.

The sequence of productions recognized by a bottom- up parser is a rightmost parse.

It is the exact reverse of the production sequence that represents a rightmost derivation.

For rightmost derivations, we use the notation $\Rightarrow_R$, $\Rightarrow^+_R$, and $\Rightarrow^*_R$

Prog $\Rightarrow_R$

{ Stmts } $\Rightarrow_R$

{ Stmts ; Stmt } $\Rightarrow_R$

{ Stmts ; id = Expr } $\Rightarrow_R$

{ Stmts ; id = Expr + id } $\Rightarrow_R$

{ Stmts ; id = id + id } $\Rightarrow_R$

{ Stmt ; id = id + id } $\Rightarrow_R$

{ id = Expr ; id = id + id } $\Rightarrow_R$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

You can derive the same set of tokens using leftmost and rightmost derivations; the only difference is the order in which productions are used.

Ambiguous Grammars

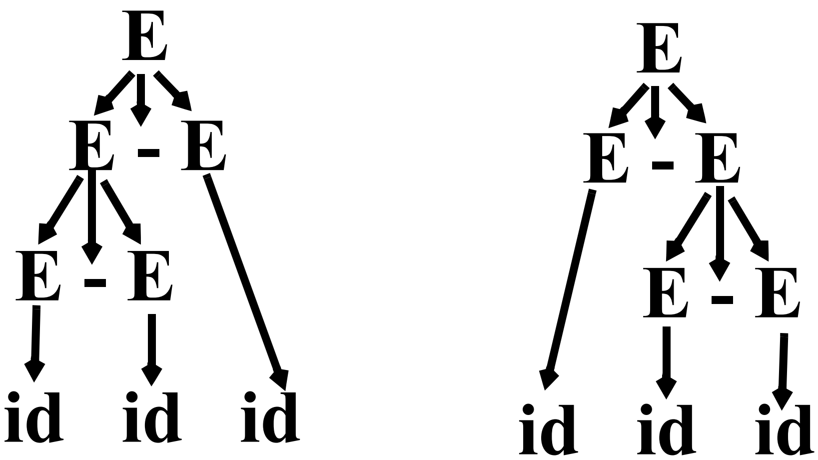
Some grammars allow more than one parse tree for the same token sequence. Such grammars are *ambiguous*. Because compilers use syntactic structure to drive translation, ambiguity is undesirable—it may lead to an unexpected translation.

Consider

E $\rightarrow$ **E - E**

| **id**

When parsing the input a- b- c (where a, b and c are scanned as identifiers) we can build the following two parse trees:



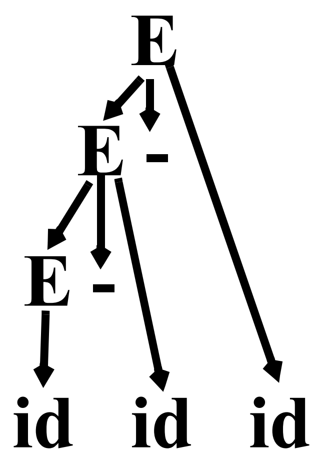
The effect is to parse a- b- c as either (a- b)- c or a- (b- c). These two groupings are certainly not equivalent.

Ambiguous grammars are usually voided in building compilers; the tools we use, like Yacc and CUP, strongly prefer unambiguous grammars.

To correct this ambiguity, we use

$$E \rightarrow E \text{ - id}$$
$$\quad\mid \ \text{id}$$

Now a- b- c can only be parsed as:



Operator Precedence

Most programming languages have *operator precedence* rules that state the order in which operators are applied (in the absence of explicit parentheses). Thus in C and Java and CSX, **`a+b*c`** means compute **`b*c`**, then add in **`a`**.

These operators precedence rules can be incorporated directly into a CFG.

Consider

E $\rightarrow$ **E + T**

  | T

T $\rightarrow$ **T * P**

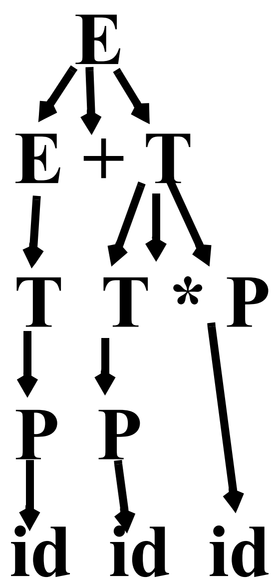
  | P

P $\rightarrow$ **id**

  | (E)

Does **`a+b*c`** mean **`(a+b)*c`** or **`a+(b*c)`**?

The grammar tells us! Look at the derivation tree:



The other grouping can't be obtained unless explicit parentheses are used.

(Why?)