CS 536 Announcements for Wednesday, March 6, 2024

Last Time

- wrap up CYK
- classes of grammars
- top-down parsing

Today

- review grammar transformations
- building a predictive parser
- FIRST and FOLLOW sets

Next Time

- predictive parsing and syntax-directed translation

Associativity

$a + b + c$

$(a+b)+c \qquad a+(b+c)$

$- - x$

$\cancel{(- -)x} \qquad -(-x)$

$a > b > c$

$\cancel{(a>b)>c} \qquad a>\cancel{(b>c)}$

LL(1) Predictive Parser

Predict the parse tree top-down

Parser structure

- 1 token lookahead
- parse/selector table
- stack tracking current parse tree's frontier

Necessary conditions

- left-factored
- free of left-recursion

Review of LL(1) grammar transformations

Necessary (but not sufficient conditions) for LL(1) parsing

- free of left recursion – no left-recursive rules
- left-factored – no rules with a common prefix, for any nonterminal

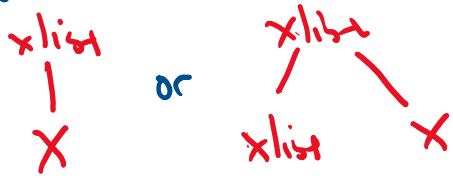
Why left-recursion is a problem

Outside/high-level view

CFG snippet: xlist → xlist X | X

Current parse tree: $\boxed{\text{xlist}}$ Current token: X

How to grow parse tree?



Depends on if there are more X's

→ need more lookahead

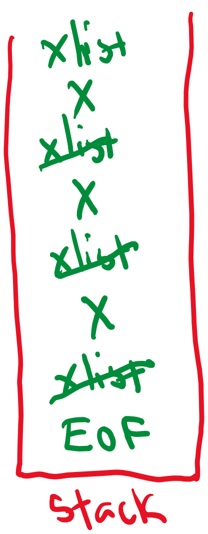
Inside/algorithmic-level view

CFG snippet: xlist → xlist X | ~~X~~ ε

Current parse tree: $\boxed{\text{xlist}}$ Current token: X

Parse table

	X	EOF
xlist	xlist X	ε



→ stack overflow

Removing left-recursion (review)

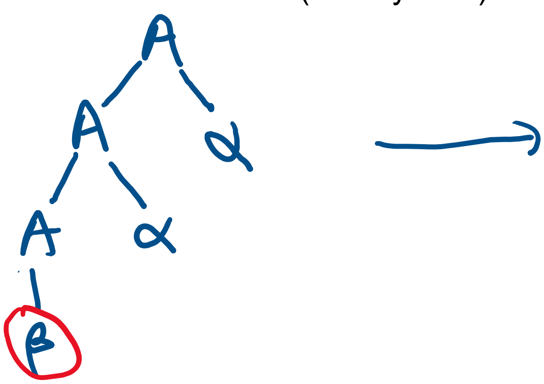
Replace

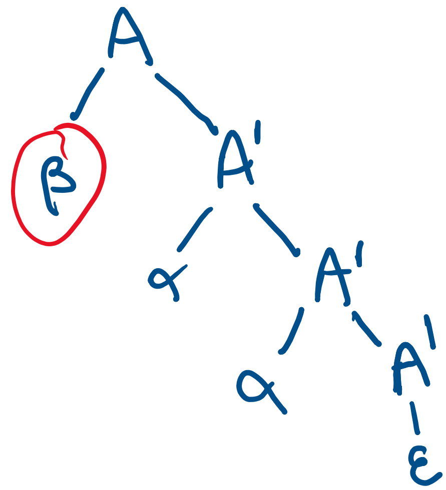
$$A \rightarrow A\ \alpha \mid \beta \quad \leftarrow \text{head of list}$$

with

$$A \rightarrow \beta\ A'$$
$$A' \rightarrow \alpha\ A' \mid \varepsilon$$

where β does not start with A (or may be ε)





Preserves the language (as a list of α's, starting with a β), but uses right recursion

Example

xlist → xlist X | ε

xlist → ε xlist'

xlist' → X xlist' | ε

$\underbrace{\text{xlist} \rightarrow \text{xlist}'}_{\text{remove \& change xlist' to xlist}}$ ⇒ xlist → X xlist | ε

Left factoring (review)

Removing a common prefix from a grammar

Replace

$$A \rightarrow \boxed{\alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n} \mid \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_m$$

with

$$A \rightarrow \alpha A' \mid \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_m$$
$$A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$$

where β_i and γ_i are sequence of symbols with no common prefix

Note: γ_i may not be present, and one of the β_i may be ε

Idea: combine all "problematic" rules that start with α into one rule $\alpha A'$
A' now represents the suffix of the problematic rules

Example 1

exp → < A > | < B > | < C > | D

exp → < exp' | D
exp' → A> | B> | C>

Example 2

stmt → ID ASSIGN exp | ID (elist) | return
exp → INTLIT | ID
elist → exp | exp COMMA elist

stmt → ID stmt' | return
stmt' → ASSIGN exp | (elist)
exp → INTLIT | ID
elist → exp elist'
elist' → ε | COMMA elist

Building the parse table

Goal: given production *lhs* → *rhs*, determine what terminals would lead us to choose that production

ie, figure out T such that table[lhs][T] = rhs

– also what terminals could indicate an error at this point?

- what terminals could *rhs* possibly start with?
- What terminals could possibly come after *lhs*?

Idea: FIRST(*rhs*) = set of terminals that begin sequences derivable from *rhs*

Suppose top-of-stack symbol is nonterminal *p* and the current token is **A** and we have

- Production 1: $p \to \alpha$
- Production 2: $p \to \beta$

FIRST lets us disambiguate:

- if **A** ∈ FIRST(α), then production 1 is a viable choice
- if **A** ∈ FIRST(β), then production 2 is a viable choice
- if **A** is in just one of them, then we can <u>predict</u> which production to use

FIRST sets

FIRST(α) is the set of terminals that begin the strings derivable from α, and also, if α can derive ε, then ε is in FIRST(α).

Formally,

terminals

FIRST(α) = $\{T \mid (T \in \Sigma \wedge \alpha \Rightarrow^* T\beta) \vee (T = \varepsilon \wedge \alpha \Rightarrow^* \varepsilon)\}$

For a symbol X

- if X is terminal: FIRST(X) = {X}
- if X is ε : FIRST(X) = {ε}
- if X is nonterminal : for each production $X \to Y_1Y_2Y_3..Y_n$
  - put FIRST(Y_1) – ε into FIRST(X)
  - if ε is in FIRST(Y_1), put FIRST(Y_2) – ε into FIRST(X)
  - if ε is in FIRST(Y_2), put FIRST(Y_3) – ε into FIRST(X)
  - ...
  - if ε is in FIRST(Y_i) for all i, put ε into FIRST(X)

repeat until there are no changes in any nonterminal's FIRST set

Example

<u>Original CFG</u>

expr → expr + term
| term
term → term * factor
| factor
factor → exponent ^ factor
| exponent
exponent → INTLIT
| (expr)

<u>Transformed CFG</u>

expr → term expr'
expr' → + term expr' | ε
term → factor term'
term' → * factor term' | ε
factor → exponent factor'
factor' → ^ factor | ε
exponent → INTLIT | (expr)

	FIRST	FOLLOW
expr	INTLIT (	EOF)
expr'	+ ε	= FOLLOW(expr) EOF)
term	INTLIT (	+ EOF)
term'	* ε	= FOLLOW(term) + EOF)
factor	INTLIT (	* + EOF)
factor'	^ ε	* + EOF)
exponent	INTLIT (	^ * + EOF)

	FIRST
expr → term expr'	INTLIT (
expr' → + term expr'	+
expr' → ε	ε
term → factor term'	INTLIT (
term' → * factor term'	*
term' → ε	ε
factor → exponent factor'	INTLIT (
factor' → ^ factor	^
factor' → ε	ε
exponent → INTLIT	INTLIT
exponent → (expr)	(

Computing FIRST(α) (continued)

Extend FIRST to strings of symbols α

– want to define FIRST for all RHS of productions

Let $\alpha = Y_1Y_2Y_3..Y_n$

- put FIRST(Y_1) – ε into FIRST(α)
  - if ε is in FIRST(Y_1), put FIRST(Y_2) – ε into FIRST(α)
  - if ε is in FIRST(Y_2), put FIRST(Y_3) – ε into FIRST(α)
  - ...
  - if ε is in FIRST(Y_i) for all i, put ε into FIRST(α)

Given two productions for nonterminal *p*

- Production 1: $p \rightarrow \alpha$ FIRST(α)
- Production 2: $p \rightarrow \beta$ FIRST(β)

look for current token

If only 1 has it, pick that production
If both have it, grammar is not LL(1)
If neither have it, if one FIRST set has ε in it,
look at what terminals can follow p

FOLLOW sets

For single nonterminal *a*, FOLLOW(*a*) is the set of terminals that can appear immediately to the right of *a*

Formally,

terminals

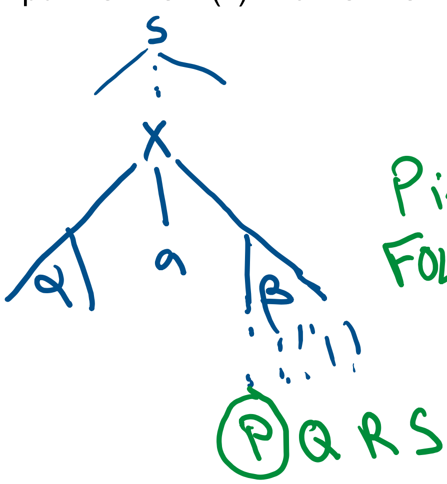
FOLLOW(*a*) = $\{ T \mid (T \in \Sigma \wedge S \Rightarrow^* \alpha a T \beta) \vee (T = EOF \wedge S \Rightarrow^* \alpha a) \}$

Computing FOLLOW sets

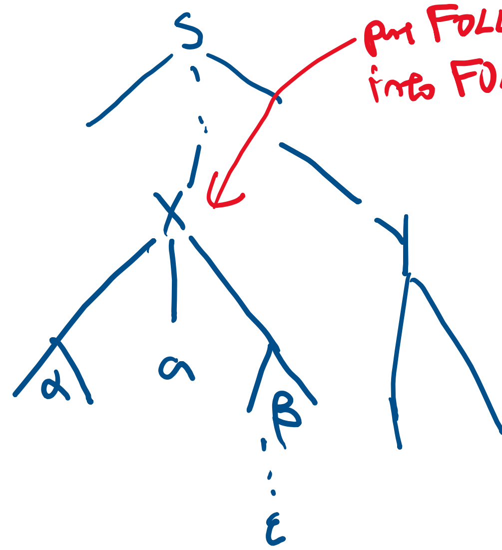
To build FOLLOW(a)

- if a is the start non-term, put EOF in FOLLOW(a)
- for each production x → α a β
  - put FIRST(β) – ε into FOLLOW(a)
  - if ε is in FIRST(β), put FOLLOW(x) into FOLLOW(a)
- for each production x → α a
  - put FOLLOW(x) into FOLLOW(a)

repeat until no changes



P is in FOLLOW(a)



put FOLLOW(x) into FOLLOW(a)

Building the parse table

```
for each production x → α {
     for each terminal T in FIRST(α) {
         put α in table[x][T]
     }
     if ε is in FIRST(α) {
         for each terminal T in FOLLOW(x) {
             put α in table[x][T]
         }
     }
}
```