Reading Assignment

- Read pages 31-62 of "Automatic Program Optimization," by Ron Cytron. (Linked from the class Web page.)

Very Busy Expressions

This is an interesting variant of available expression analysis.

An expression is *very busy* at a point if it is *guaranteed* that the expression will be computed at some time in the future.

Thus starting at the point in question, the expression must be reached before its value changes.

Very busy expression analysis is a backward flow analysis, since it propagates information about future evaluations backward to "earlier" points in the computation.

The meet lattice is:

T (Expression is Very Busy)

|

F (Expression is Not Very Busy)

As initial values, at the end of all exit nodes, nothing is very busy. Hence, for a given expression,

VeryBusyOut(b_{last}) = F

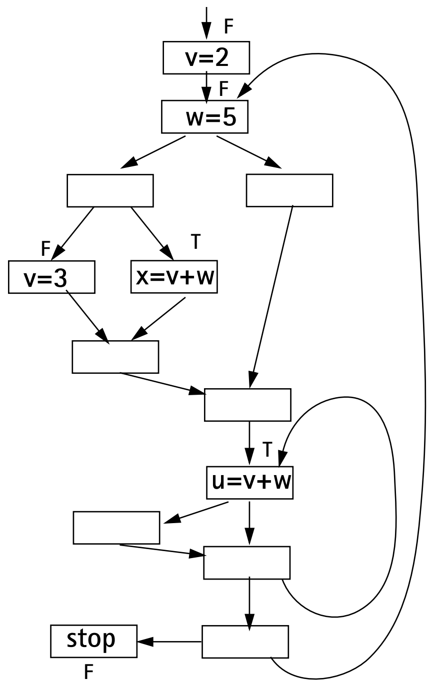
The transfer function for e_1 in block b is defined as:

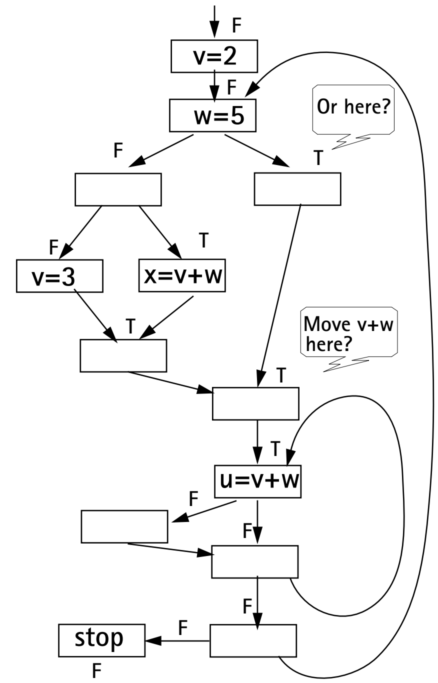
If e_1 is computed in b before any of its operands
Then VeryBusyIn(b) = T
Elsif any of e_1's operands are changed before e_1 is computed
Then VeryBusyIn(b) = F
Else VeryBusyIn(b) = VeryBusyOut(b)

The meet operation (to combine solutions) is:

$$\text{VeryBusyOut}(b) = \underset{s \in \text{Succ}(b)}{\text{AND}} \text{VeryBusyIn}(s)$$

Example: e_1=v+w





Identifying Identical Expressions

We can hash expressions, based on hash values assigned to operands and operators. This makes recognizing potentially redundant expressions straightforward.

For example, if H(a) = 10, H(b) = 21 and H(+) = 5, then (using a simple product hash),
H(a+b) = $10 \times 21 \times 5$ Mod TableSize

Effects of Aliasing and Calls

When looking for assignments to operands, we must consider the effects of pointers, formal parameters and calls.

An assignment through a pointer (e.g, `*p = val`) *kills* all expressions dependent on variables `p` might point too. Similarly, an assignment to a formal parameter kills all expressions dependent on variables the formal might be bound to.

A call kills all expressions dependent on a variable changeable during the call.

Lacking careful alias analysis, pointers, formal parameters and calls can kill all (or most) expressions.

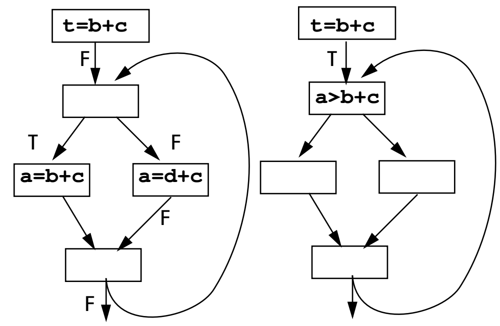
Very Busy Expressions and Loop Invariants

Very busy expressions are ideal candidates for invariant loop motion.

If an expression, invariant in a loop, is also very busy, we know it must be used in the future, and hence evaluation outside the loop must be worthwhile.


```
for (...) {
 if (...)
     a=b+c;
 else a=d+c;}
```

```
for (...) {
 if (a>b+c)
     x=1;
 else x=0;}
```



b+c is not very busy at loop entrance

b+c is very busy at loop entrance

Reaching Definitions

We have seen reaching definition analysis formulated as a set-valued problem. It can also be formulated on a per-definition basis.

That is, we ask "What blocks does a particular definition to v reach?"

This is a boolean-valued, forward flow data flow problem.

Initially, $\text{DefIn}(b_0)$ = false.

For basic block b:

DefOut(b) =
  If the definition being analyzed is
    the last definition to v in b
  Then True
  Elsif any other definition to v occurs
    in b
  Then False
  Else DefIn(b)

The meet operation (to combine solutions) is:

$$\text{DefIn}(b) = \underset{p \in \text{Pred}(b)}{\text{OR}} \text{DefOut}(p)$$

To get all reaching definition, we do a series of single definition analyses.

Live Variable Analysis

This is a boolean-valued, backward flow data flow problem.

Initially, LiveOut(b_{last}) = false.

For basic block b:

LiveIn(b) =
  If the variable is used before it is
    defined in b
  Then True
  Elsif it is defined before it is used
    in b
  Then False
  Else LiveOut(b)

The meet operation (to combine solutions) is:

$$\text{LiveOut}(b) = \underset{s \in \text{Succ}(b)}{\text{OR}} \text{LiveIn}(s)$$

Bit Vectoring Data Flow Problems

The four data flow problems we have just reviewed all fit within a *single* framework.

Their solution values are Booleans (bits).

The meet operation is And or OR.

The transfer function is of the general form

$\text{Out}(b) = (\text{In}(b) - \text{Kill}_b) \cup \text{Gen}_b$

or

$\text{In}(b) = (\text{Out}(b) - \text{Kill}_b) \cup \text{Gen}_b$

where Kill_b is true if a value is “killed” within b and Gen_b is true if a value is “generated” within b.

In Boolean terms:

$\text{Out}(b) = (\text{In}(b) \text{ AND Not } \text{Kill}_b) \text{ OR } \text{Gen}_b$

or

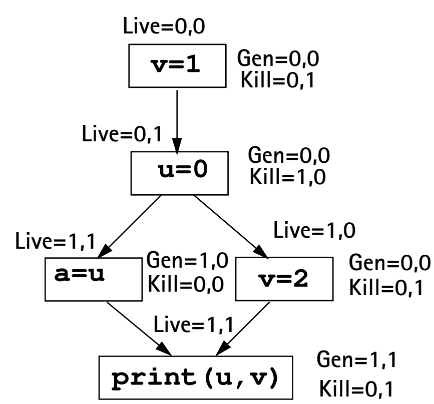
$\text{In}(b) = (\text{Out}(b) \text{ AND Not } \text{Kill}_b) \text{ OR } \text{Gen}_b$

An advantage of a bit vectoring data flow problem is that we can do a series of data flow problems “in parallel” using a bit vector.

Hence using ordinary word-level ANDs, ORs, and NOTs, we can solve 32 (or 64) problems simultaneously.

Example

Do live variable analysis for u and v, using a 2 bit vector:



We expect no variable to be live at the start of b_0. (Why?)

Depth-First Spanning Trees

Sometimes we want to “cover” the nodes of a control flow graph with an acyclic structure.

This allows us to visit nodes once, without worrying about cycles or infinite loops.

Also, a careful visitation order can approximate forward control flow (very useful in solving forward data flow problems).

A Depth-First Spanning Tree (DFST) is a tree structure that covers the nodes of a control flow graph, with the start node serving as root of the DFST.

Building a DFST

We will visit CFG nodes in depth-first order, keeping arcs if the visited node hasn’t be reached before.

To create a DFST, T, from a CFG, G:

1. T $\leftarrow$ empty tree
2. Mark all nodes in G as “unvisited.”
3. Call DF(start node)

DF (node) {

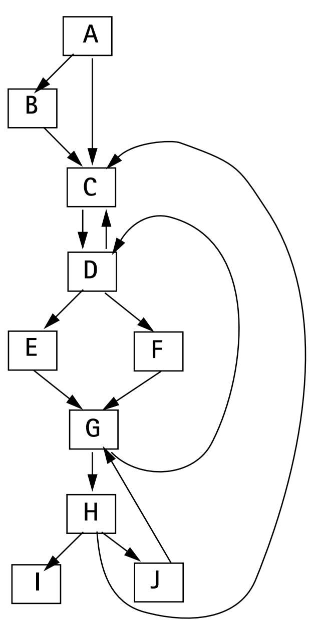
1. Mark node as visited.
2. For each successor, s, of node in G:

   If s is unvisited

   (a) Add node $\rightarrow$ s to T

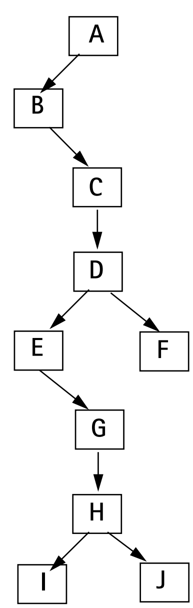
   (b) Call DF(s)

Example



Visit order is A, B, C, D, E, G, H, I, J, F

The DFST is



Categorizing Arcs using a DFST

Arcs in a CFG can be categorized by examining the corresponding DFST.

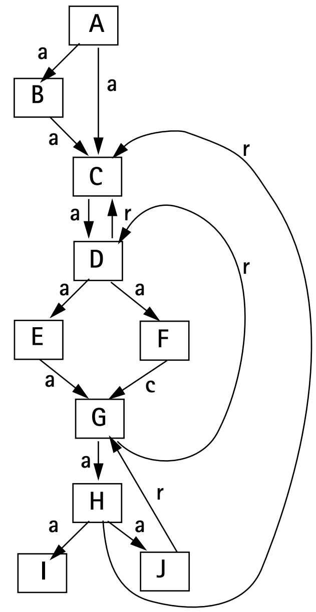
An arc A→B in a CFG is

(a) An *Advancing Edge* if B is a proper descendent of A in the DFST.

(b) A *Retreating Edge* if B is an ancestor of A in the DFST. (This includes the A→A case.)

(c) A *Cross Edge* if B is neither a descendent nor an ancestor of A in the DFST.

Example



Depth-First Order

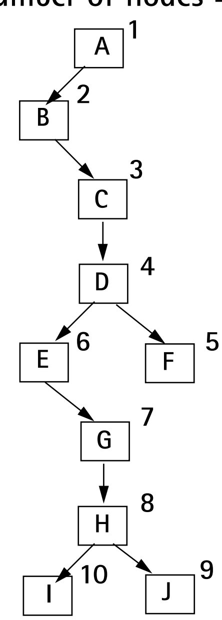
Once we have a DFST, we can label nodes with a *Depth-First Ordering* (DFO).

Let i = the number of nodes in a CFG (= the number of nodes in its DFST).

```
DFO(node) {
   For (each successor s of node) do
       DFO(s);
   Mark node with i;
   i--;
}
```


Example

The number of nodes = 10.



Application of Depth-First Ordering

- *Retreating edges* (a necessary component of loops) are easy to identify:
  a→b is a retreating edge if and only if dfo(b) ≤ dfo(a)
- A depth-first ordering in an excellent *visit order* for solving forward data flow problems. We want to visit nodes in essentially topological order, so that all predecessors of a node are visited (and evaluated) before the node itself is.