Data Flow Frameworks

- Data Flow Graph:

  Nodes of the graph are basic blocks or individual instructions.

  Arcs represent flow of control.

  Forward Analysis:

    Information flow is the same direction as control flow.

  Backward Analysis:

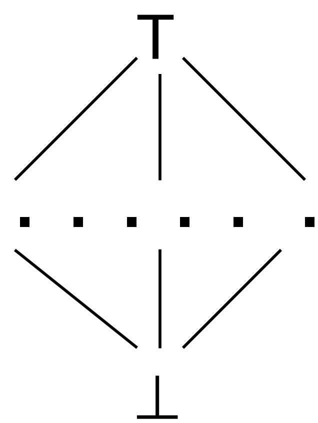
    Information flow is the opposite direction as control flow.

  Bi-directional Analysis:

    Information flow is in both directions. (Not too common.)

- Meet Lattice

  Represents solution space for the data flow analysis.



- Meet operation
  (And, Or, Union, Intersection, etc.)

  Combines solutions from predecessors or successors in the control flow graph.

- Transfer Function

  Maps a solution at the top of a node to a solution at the end of the node (forward flow)
  or
  Maps a solution at the end of a node to a solution at the top of the node (backward flow).

Example: Available Expressions

This data flow analysis determines whether an expression that has been previously computed may be reused.

Available expression analysis is a forward flow problem—computed expression values flow forward to points of possible reuse.

The best solution is True—the expression may be reused.

The worst solution is False—the expression may not be reused.

The Meet Lattice is:

T (Expression is Available)

|

F (Expression is Not Available)

As initial values, at the top of the start node, nothing is available. Hence, for a given expression,

$\text{AvailIn}(b_0) = F$

We choose an expression, and consider all the variables that contribute to its evaluation.

Thus for e_1=a+b-c, a, b and c are e_1's *operands*.

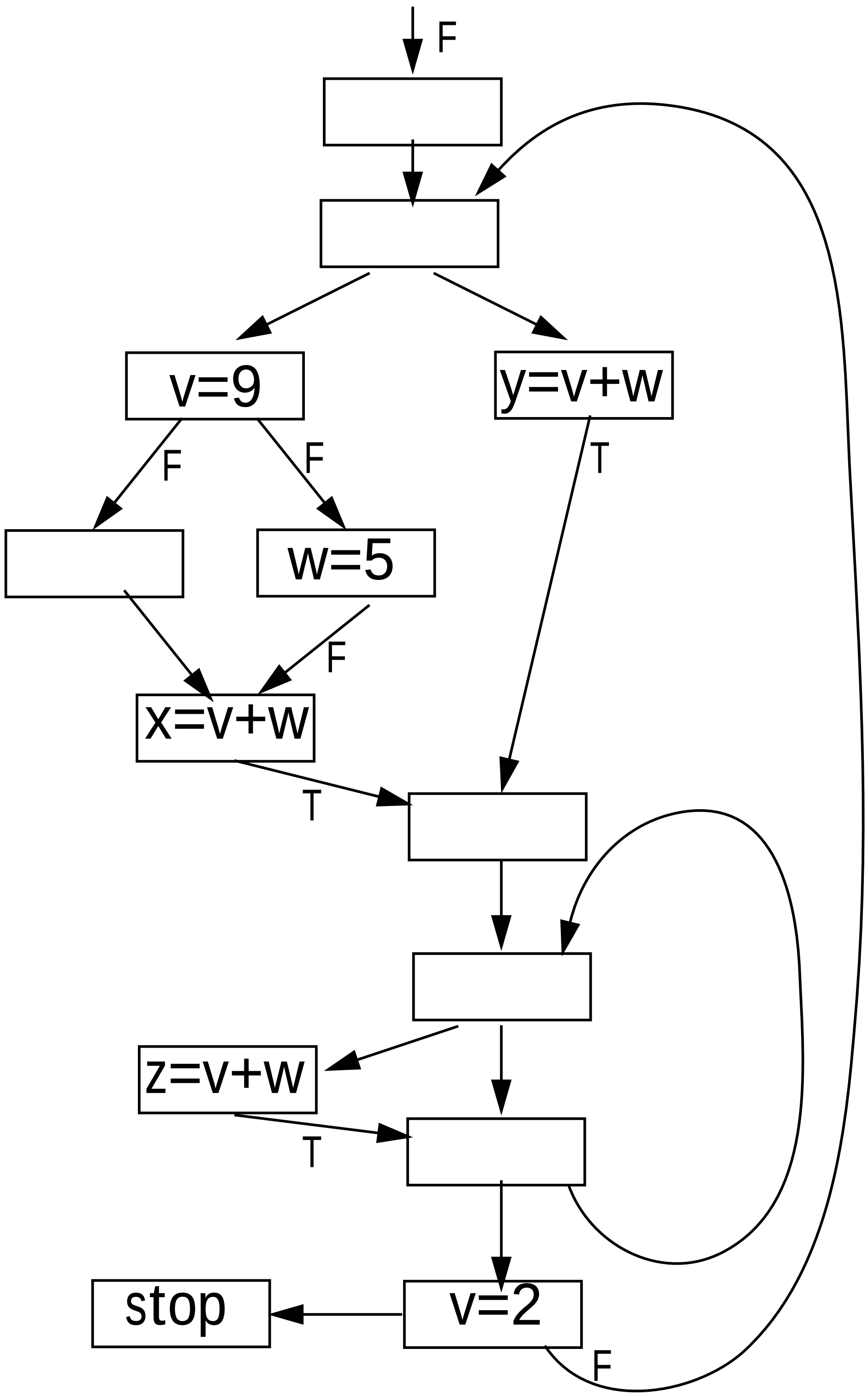
The transfer function for e_1 in block b is defined as:

If e_1 is computed in b after any assignments to e_1's operands in b
Then AvailOut(b) = T
Elsif any of e_1's operands are changed after the last computation of e_1 or e_1's operands are changed without any computation of e_1
Then AvailOut(b) = F
Else AvailOut(b) = AvailIn(b)

The meet operation (to combine solutions) is:

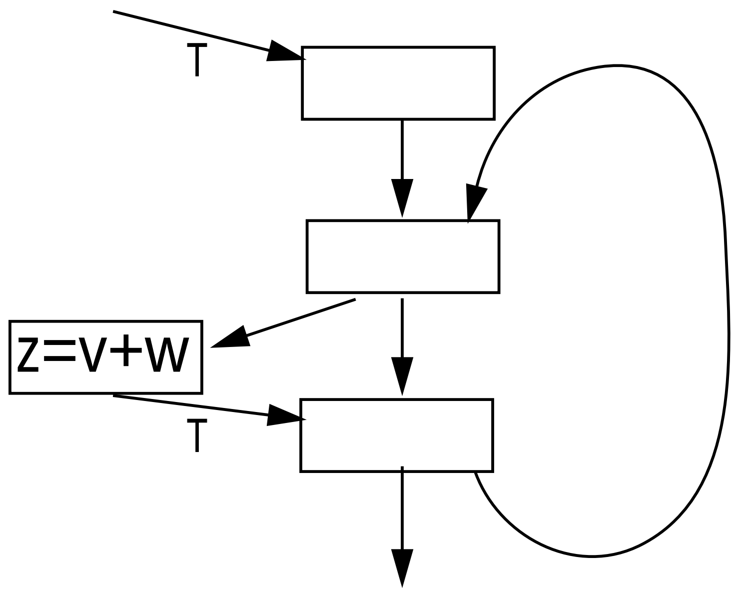
$$\text{AvailIn}(b) = \underset{p \in \text{Pred}(b)}{\text{AND}} \text{AvailOut}(p)$$

Example: e_1=v+w



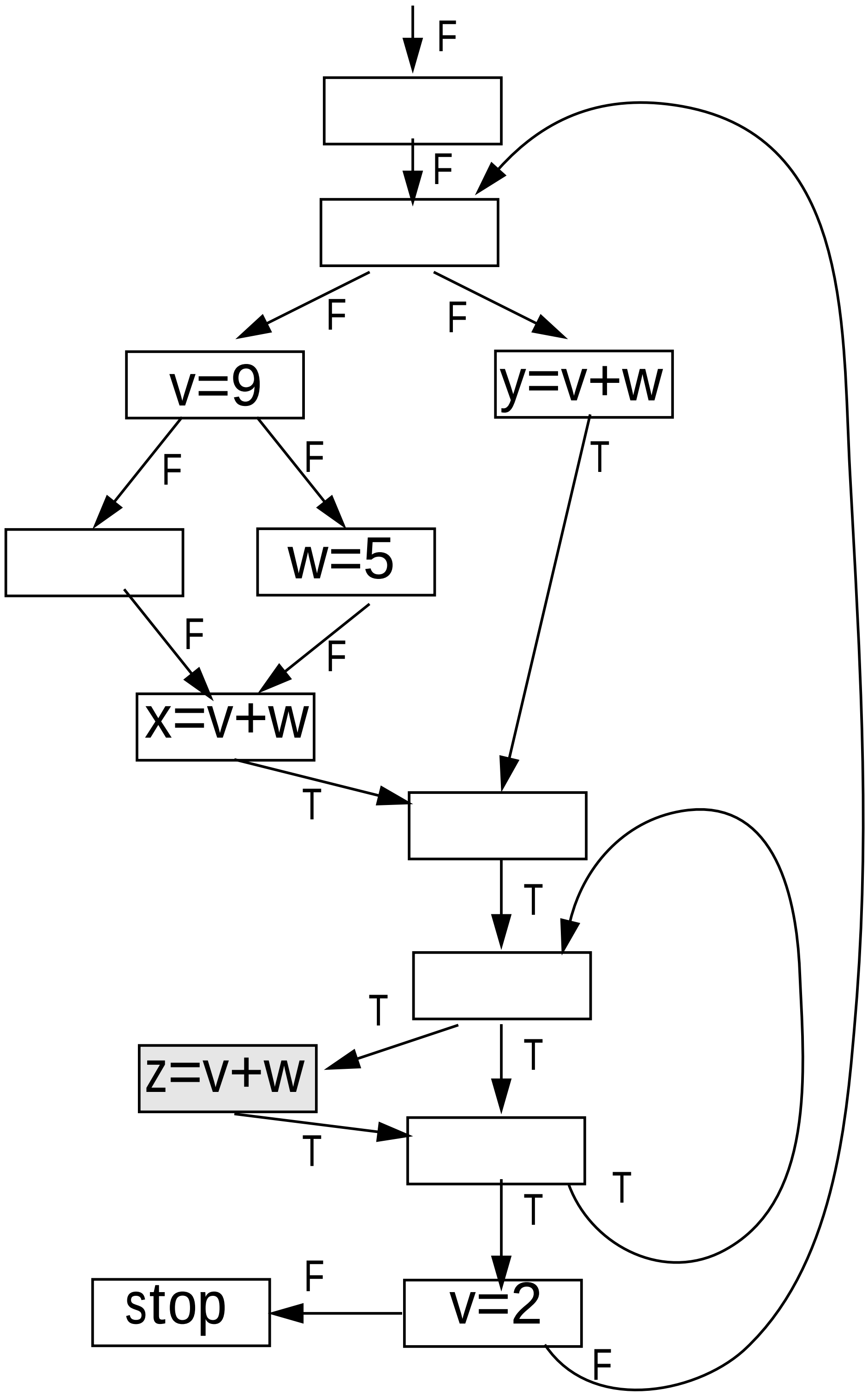
Circularities Require Care

Since data flow values can depend on themselves (because of loops), care is required in assigning initial “guesses” to unknown values. Consider



If the flow value on the loop backedge is initially set to false, it can never become true. (Why?)

Instead we should use True, the *identity* for the AND operation.


F
F
F
F
v=9
y=v+w
F
F
T
w=5
F
F
x=v+w
T
T
T
z=v+w
T
T
T
T
F
stop
v=2
F

Very Busy Expressions

This is an interesting variant of available expression analysis.

An expression is *very busy* at a point if it is *guaranteed* that the expression will be computed at some time in the future.

Thus starting at the point in question, the expression must be reached before its value changes.

Very busy expression analysis is a backward flow analysis, since it propagates information about future evaluations backward to “earlier” points in the computation.

The meet lattice is:

T (Expression is Very Busy)
|
F (Expression is Not Very Busy)

As initial values, at the end of all exit nodes, nothing is very busy. Hence, for a given expression,

$\text{VeryBusyOut}(b_{last}) = F$

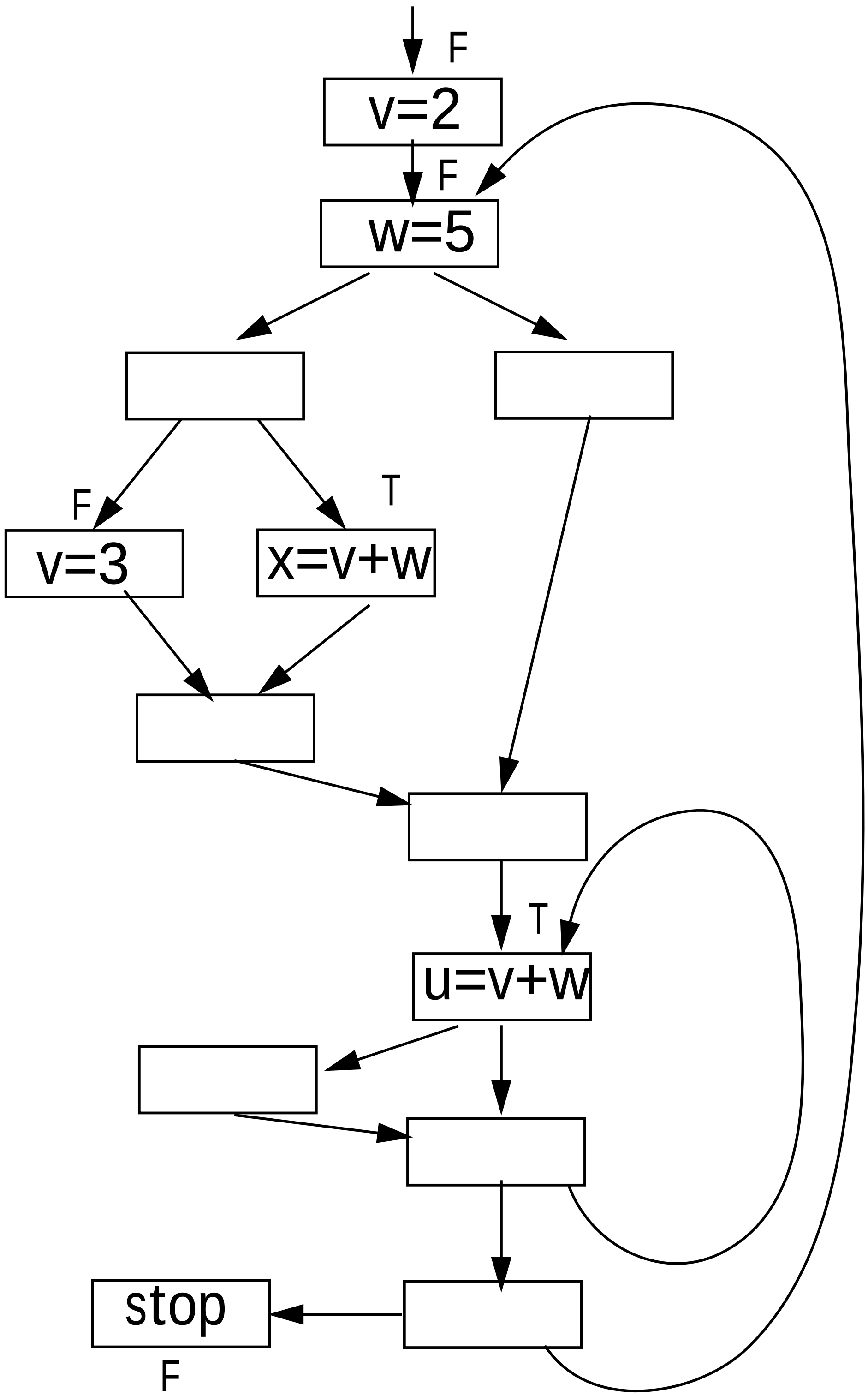
The transfer function for e_1 in block b is defined as:

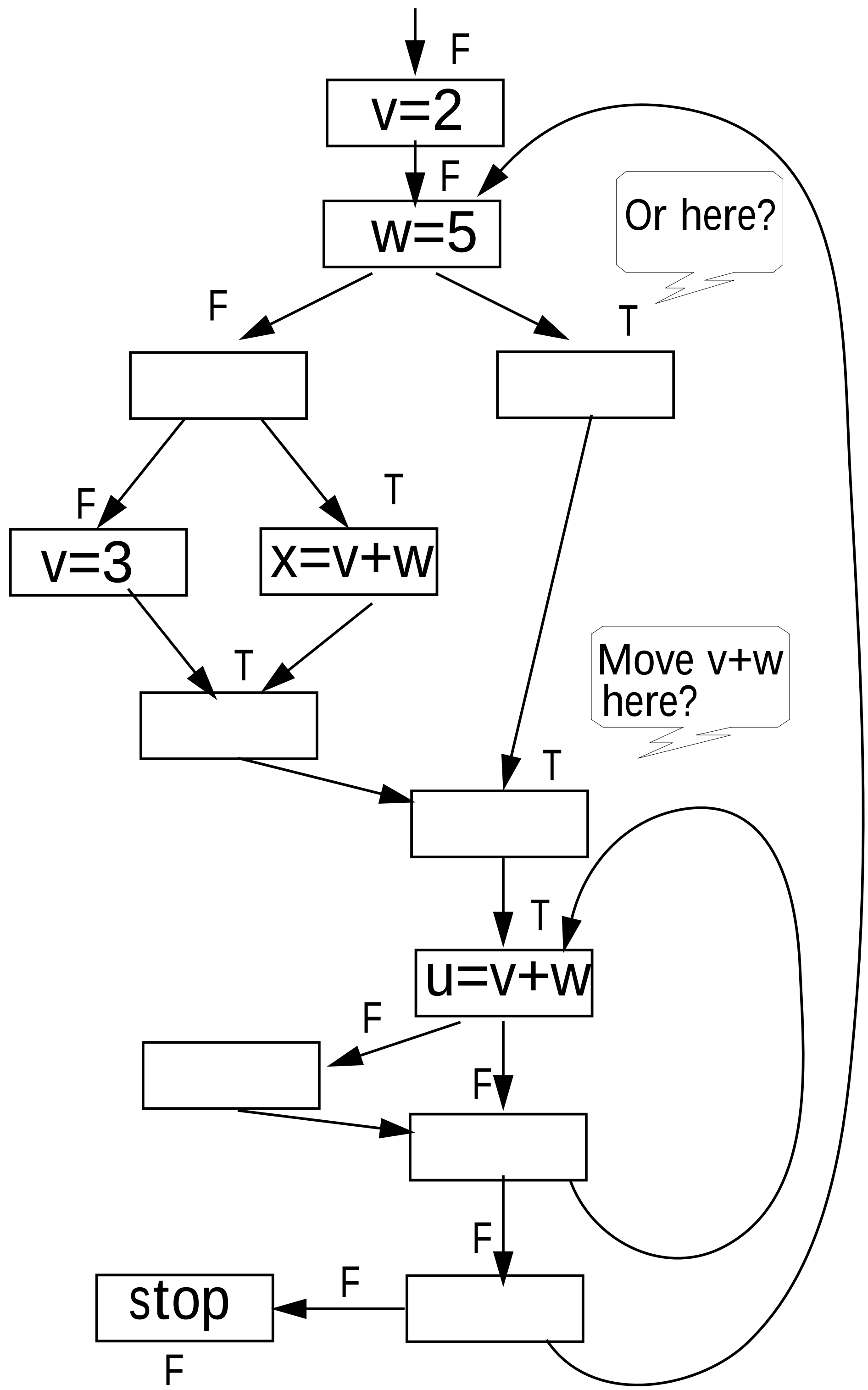
If e_1 is computed in b before any of its operands
Then VeryBusyIn(b) = T
Elsif any of e_1's operands are changed before e_1 is computed
Then VeryBusyIn(b) = F
Else VeryBusyIn(b) = VeryBusyOut(b)

The meet operation (to combine solutions) is:

$$\text{VeryBusyOut}(b) = \underset{s \in \text{Succ}(b)}{\text{AND}} \text{VeryBusyIn}(s)$$

Example: e_1=v+w




F
v=2
F
w=5
Or here?
F
T
F
T
v=3
x=v+w
Move v+w here?
T
T
T
u=v+w
F
F
F
F
stop
F

Identifying Identical Expressions

We can hash expressions, based on hash values assigned to operands and operators. This makes recognizing potentially redundant expressions straightforward.

For example, if H(a) = 10, H(b) = 21 and H(+) = 5, then (using a simple product hash),
$H(a+b) = 10 \times 21 \times 5 \text{ Mod TableSize}$

Effects of Aliasing and Calls

When looking for assignments to operands, we must consider the effects of pointers, formal parameters and calls.

An assignment through a pointer (e.g, **`*p = val`**) *kills* all expressions dependent on variables **`p`** might point too. Similarly, an assignment to a formal parameter kills all expressions dependent on variables the formal might be bound to.

A call kills all expressions dependent on a variable changeable during the call.

Lacking careful alias analysis, pointers, formal parameters and calls can kill all (or most) expressions.

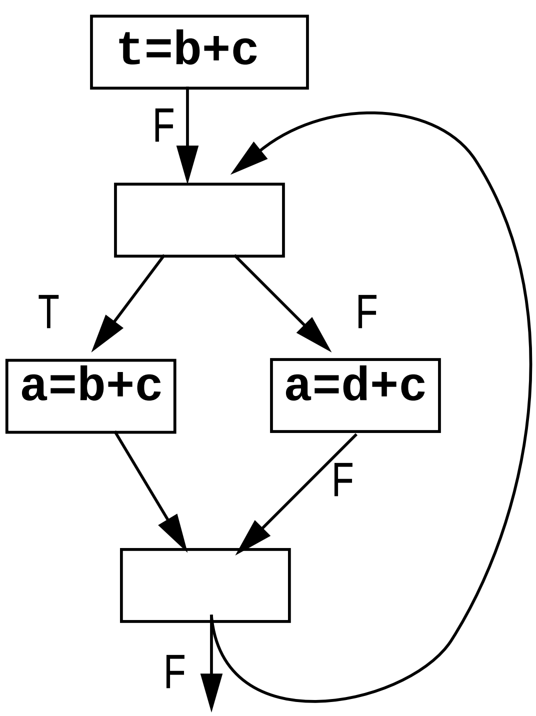
Very Busy Expressions and Loop Invariants

Very busy expressions are ideal candidates for invariant loop motion.

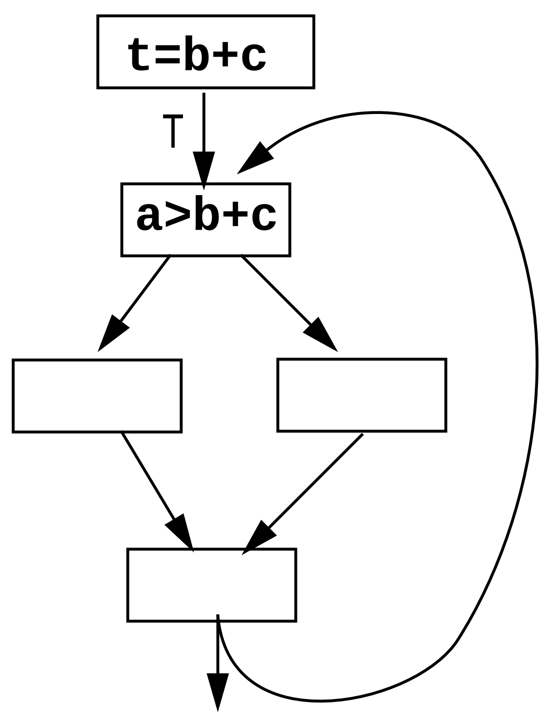
If an expression, invariant in a loop, is also very busy, we know it must be used in the future, and hence evaluation outside the loop must be worthwhile.

```
for (...) {
 if (...)
      a=b+c;
 else a=d+c;}
```

```
for (...) {
 if (a>b+c)
      x=1;
 else x=0;}
```



b+c is not very busy at loop entrance



b+c is very busy at loop entrance

Reaching Definitions

We have seen reaching definition analysis formulated as a set-valued problem. It can also be formulated on a per-definition basis.

That is, we ask "What blocks does a particular definition to v reach?"

This is a boolean-valued, forward flow data flow problem.

Initially, $\text{DefIn}(b_0)$ = false.

For basic block b:

DefOut(b) =
  If the definition being analyzed is
    the last definition to v in b
  Then True
  Elsif any other definition to v occurs
    in b
  Then False
  Else DefIn(b)

The meet operation (to combine solutions) is:

$$\text{DefIn}(b) = \underset{p \in \text{Pred}(b)}{\text{OR}} \text{DefOut}(p)$$

To get all reaching definition, we do a series of single definition analyses.

Live Variable Analysis

This is a boolean-valued, backward flow data flow problem.

Initially, LiveOut(b_{last}) = false.

For basic block b:

LiveIn(b) =
  If the variable is used before it is defined in b
  Then True
  Elsif it is defined before it is used in b
  Then False
  Else LiveOut(b)

The meet operation (to combine solutions) is:

$$\text{LiveOut}(b) = \underset{s \in \text{Succ}(b)}{\text{OR}} \text{LiveIn}(s)$$

Bit Vectoring Data Flow Problems

The four data flow problems we have just reviewed all fit within a *single* framework.

Their solution values are Booleans (bits).

The meet operation is And or OR.

The transfer function is of the general form

$$Out(b) = (In(b) - Kill_b) \cup Gen_b$$

or

$$In(b) = (Out(b) - Kill_b) \cup Gen_b$$

where $Kill_b$ is true if a value is "killed" within b and Gen_b is true if a value is "generated" within b.

In Boolean terms:

$Out(b) = (In(b)\ AND\ Not\ Kill_b)\ OR\ Gen_b$

or

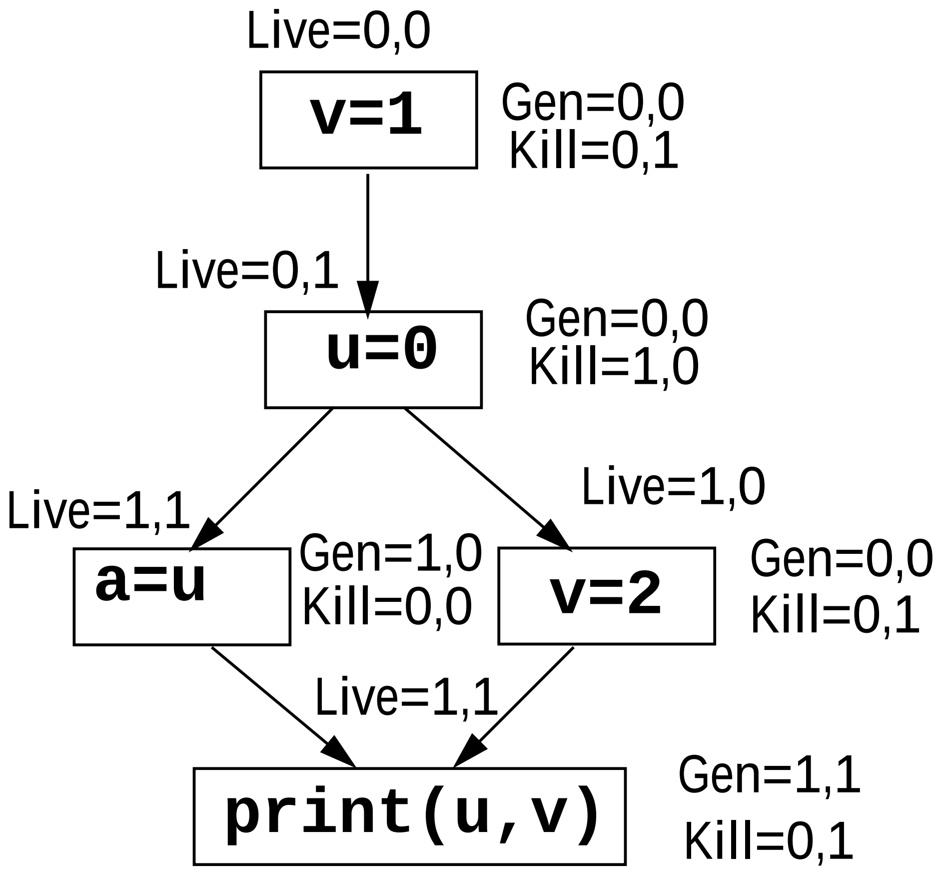
$In(b) = (Out(b)\ AND\ Not\ Kill_b)\ OR\ Gen_b$

An advantage of a bit vectoring data flow problem is that we can do a series of data flow problems “in parallel” using a bit vector.

Hence using ordinary word-level ANDs, ORs, and NOTs, we can solve 32 (or 64) problems simultaneously.

Example

Do live variable analysis for u and v, using a 2 bit vector:



We expect no variable to be live at the start of b_0. (Why?)

Reading Assignment

- Read pages 31-62 of “Automatic Program Optimization,” by Ron Cytron. (Linked from the class Web page.)

Depth-First Spanning Trees

Sometimes we want to “cover” the nodes of a control flow graph with an acyclic structure.

This allows us to visit nodes once, without worrying about cycles or infinite loops.

Also, a careful visitation order can approximate forward control flow (very useful in solving forward data flow problems).

A Depth-First Spanning Tree (DFST) is a tree structure that covers the nodes of a control flow graph, with the start node serving as root of the DFST.

Building a DFST

We will visit CFG nodes in depth-first order, keeping arcs if the visited node hasn't be reached before.

To create a DFST, T, from a CFG, G:

1. T $\leftarrow$ empty tree
2. Mark all nodes in G as "unvisited."
3. Call DF(start node)

DF (node) {

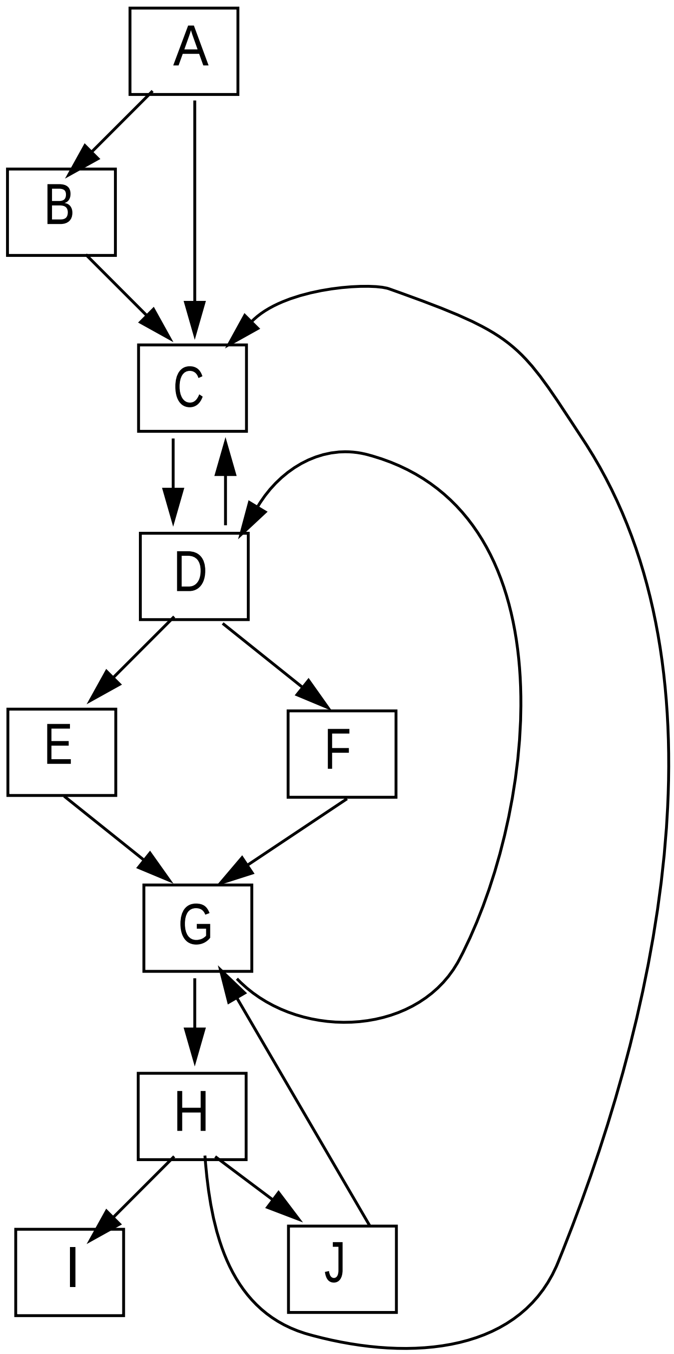
1. Mark node as visited.
2. For each successor, s, of node in G:

   If s is unvisited

   (a) Add node $\rightarrow$ s to T

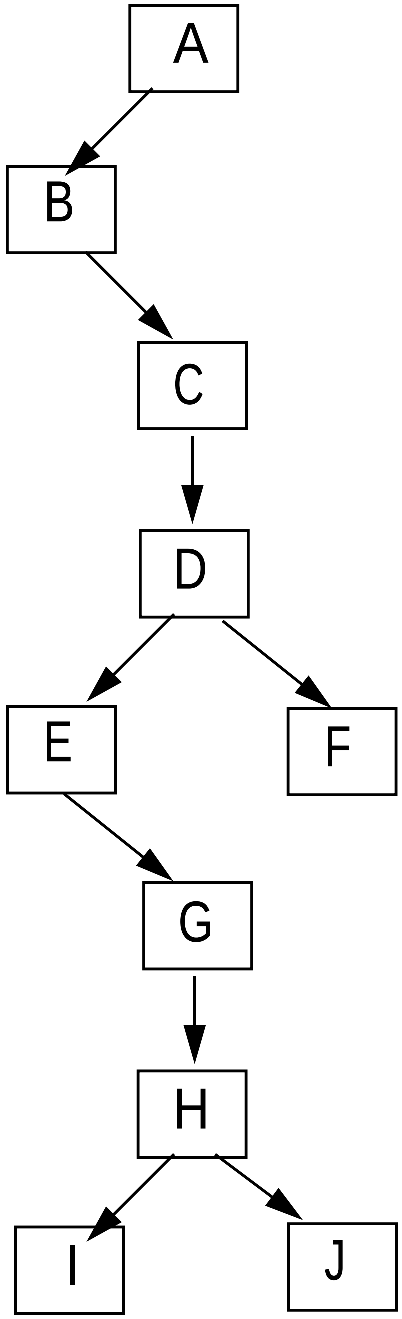
   (b) Call DF(s)

Example



Visit order is A, B, C, D, E, G, H, I, J, F

The DFST is



Categorizing Arcs using a DFST

Arcs in a CFG can be categorized by examining the corresponding DFST.

An arc A→B in a CFG is

(a) An *Advancing Edge* if B is a proper descendent of A in the DFST.

(b) A *Retreating Edge* if B is an ancestor of A in the DFST. (This includes the A→A case.)

(c) A *Cross Edge* if B is neither a descendent nor an ancestor of A in the DFST.

Example