Merging Live Ranges

It is possible that each Basic Block that contains a definition of v creates a *distinct* Live Range of V.

∀ Basic Blocks, b, that contain a definition of V:

$$\text{Range}(b) = \{b\} \cup \{k \mid b \in \text{DefsIn}(k) \;\&\; \text{LiveIn}(k)\}$$

This rule states that the Live Range of a definition to V in Basic Block b is b plus all other Basic Blocks that the definition of V reaches and in which V is live.

If two Live Ranges overlap (have one of more Basic Blocks in common), they *must* share the same register too. (Why?)

Therefore,

If $\text{Range}(b_1) \cap \text{Range}(b_2) \neq \phi$

Then replace

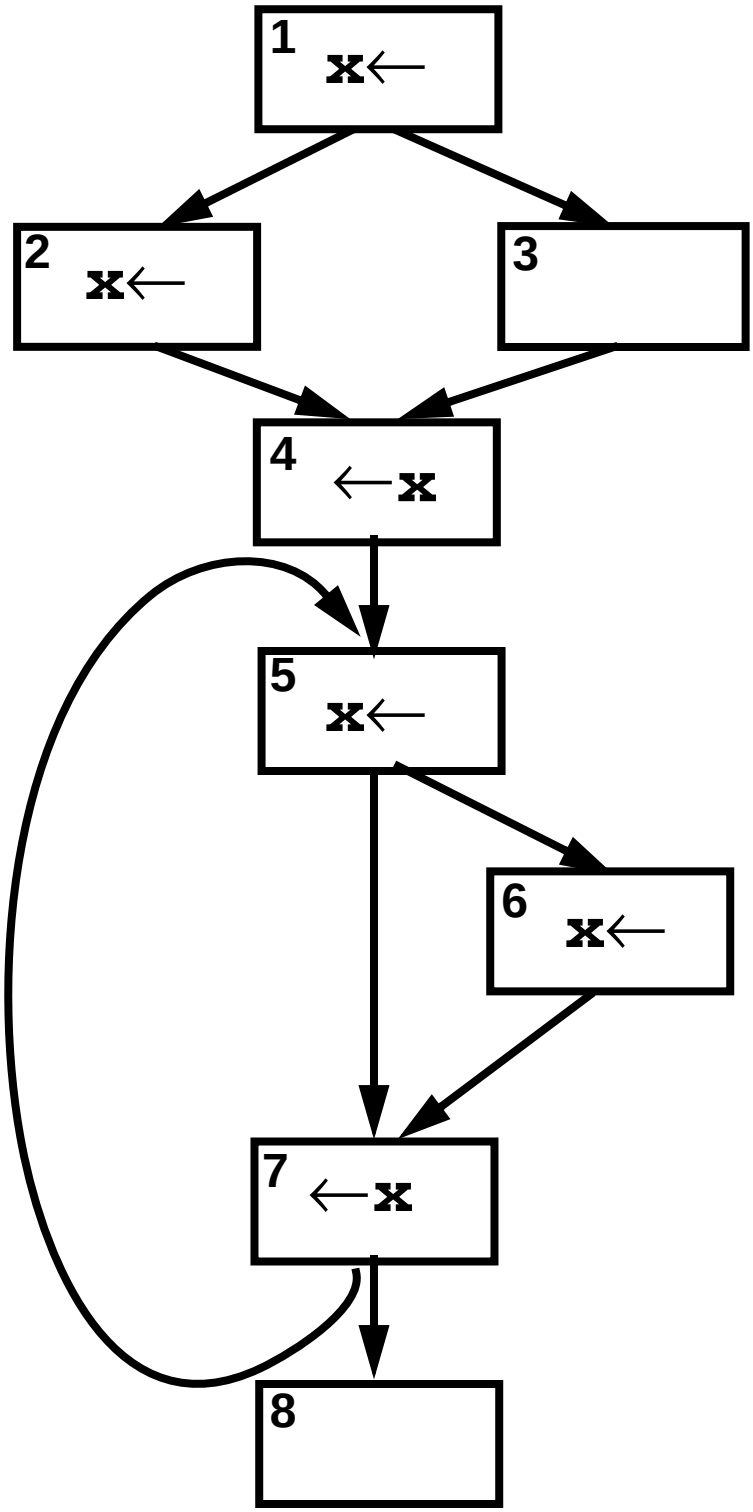
$\text{Range}(b_1)$ and $\text{Range}(b_2)$

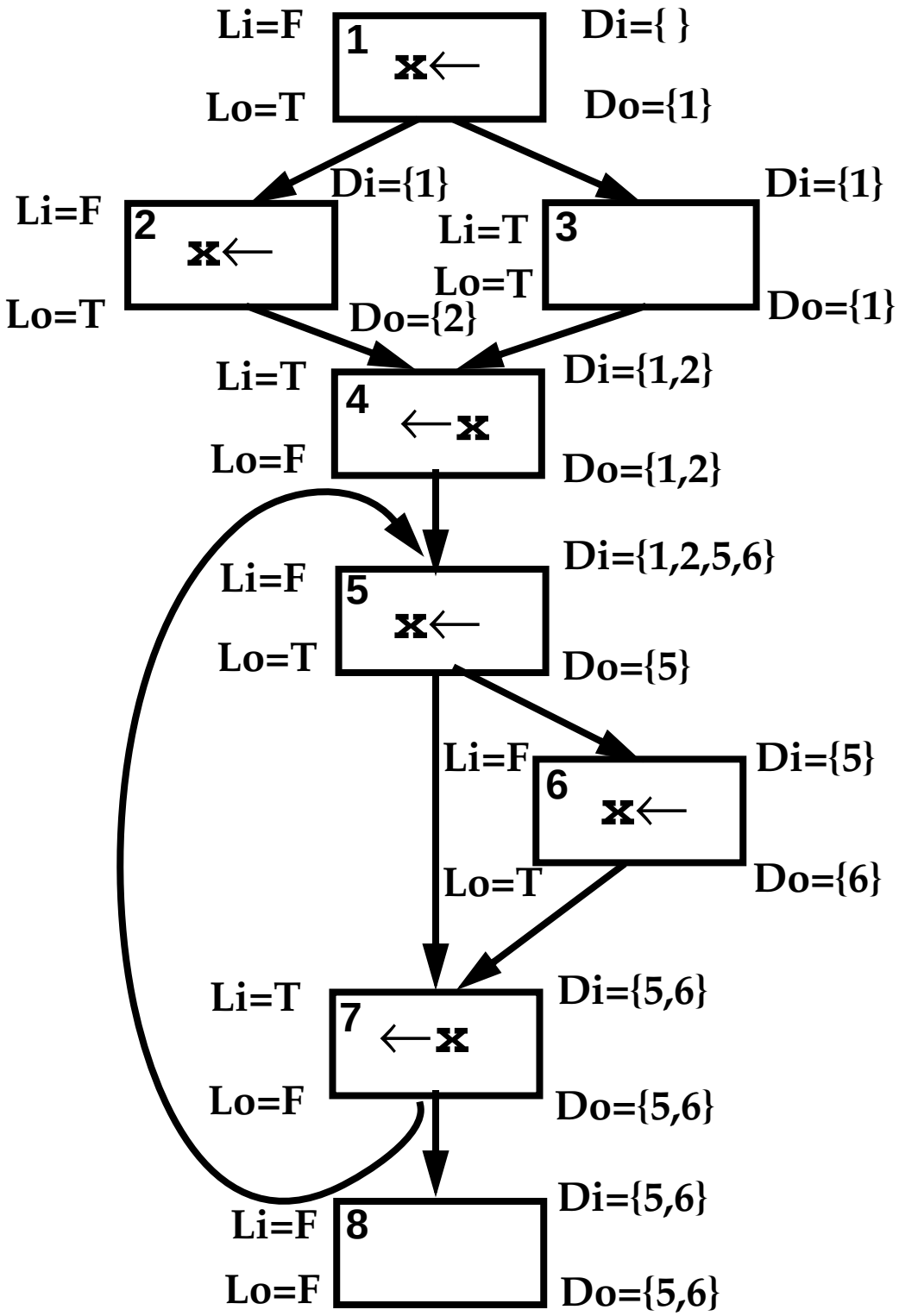
with $\text{Range}(b_1) \cup \text{Range}(b_2)$

Reading Assignment

- Read George and Appel's paper, "Iterated Register Coalescing." (Linked from Class Web page)
- Read Larus and Hilfinger's paper, "Register Allocation in the SPUR Lisp Compiler."

Example




Li=F
1
x←
Di={ }
Lo=T
Do={1}
Di={1}
Di={1}
Li=F
2
x←
Li=T
3
Lo=T
Lo=T
Do={1}
Do={2}
Li=T
Di={1,2}
4
←x
Lo=F
Do={1,2}
Di={1,2,5,6}
Li=F
5
x←
Lo=T
Do={5}
Li=F
Di={5}
6
x←
Lo=T
Do={6}
Li=T
Di={5,6}
7
←x
Lo=F
Do={5,6}
Di={5,6}
Li=F
8
Lo=F
Do={5,6}

The Live Ranges we Compute are

Range(1) = {1} U {3,4} = {1,3,4}

Range(2) = {2} U {4} = {2,4}

Range(5) = {5} U {7} = {5,7}

Range(6) = {6} U {7} = {6,7}

Ranges 1 and 2 overlap, so

Range(1) = Range(2) = {1,2,3,4}

Ranges 5 and 6 overlap, so

Range(5) = Range(6) = {5,6,7}

Interference Graph

An *Interference Graph* represents interferences between Live Ranges.

Two Live Ranges *interfere* if they share one or more Basic Blocks in common.

Live Ranges that interfere *must* be allocated different registers.

In an Interference Graph:

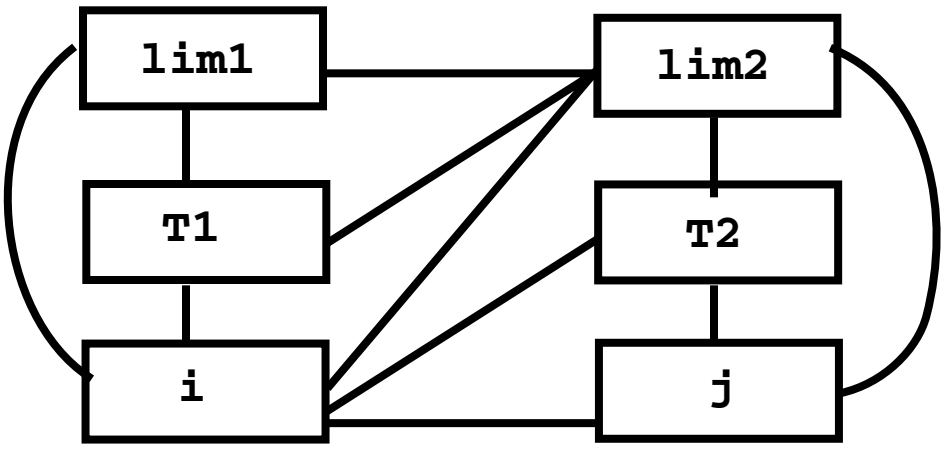
- Nodes are Live Ranges
- An undirected arc connects two Live Ranges if and only if they interfere

Example

```
int p(int lim1, int lim2) {
  for (i=0; i<lim1 && A[i]>0;i++){}
  for (j=0; j<lim2 && B[j]>0;j++){}
  return i+j;
}
```

We optimize array accesses by placing `&A[0]` and `&B[0]` in temporaries:

```
int p(int lim1, int lim2) {
  int *T1 = &A[0];
  for (i=0; i<lim1 && *(T1+i)>0;i++){}
  int *T2 = &B[0];
  for (j=0; j<lim2 && *(T2+j)>0;j++){}
  return i+j;
}
```



Register Allocation via Graph Coloring

We model global register allocation as a Coloring Problem on the Interference Graph

We wish to use the fewest possible colors (registers) subject to the rule that two connected nodes can't share the same color.

Optimal Graph Coloring is NP-Complete

Reference:

> "Computers and Intractability," M. Garey and D. Johnson, W.H. Freeman, 1979.

We'll use a Heuristic Algorithm originally suggested by Chaitin et. al. and improved by Briggs et. al.

References:

> "Register Allocation Via Coloring," G. Chaitin et. al., Computer Languages, 1981.
>
> "Improvement to Graph Coloring Register Allocation," P. Briggs et. al., PLDI, 1989.

Coloring Heuristic

To R-Color a Graph (where R is the number of registers available)

1. While any node, n, has < R neighbors:
   Remove n from the Graph.
   Push n onto a Stack.
2. If the remaining Graph is non-empty:
   Compute the Cost of each node.
   The Cost of a Node (a Live Range) is the number of extra instructions needed if the Node isn't assigned a register, scaled by 10^{loop_depth}.
   Let NB(n) =
      Number of Neighbors of n.
   Remove that node n that has the smallest Cost(n)/NB(n) value.
   Push n onto a Stack.
   Return to Step 1.

3. While Stack is non-empty:

Pop n from the Stack.

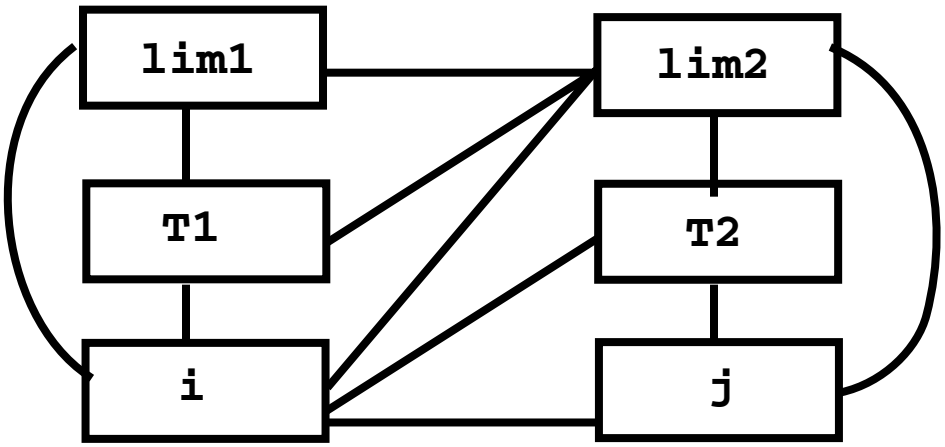
If n's neighbors are assigned fewer than R colors

Then assign n any unassigned color

Else leave n uncolored.

Example

```
int p(int lim1, int lim2) {
  int *T1 = &A[0];
  for (i=0; i<lim1 && *(T1+i)>0;i++){}
  int *T2 = &B[0];
  for (j=0; j<lim2 && *(T2+j)>0;j++){}
  return i+j;
}
```



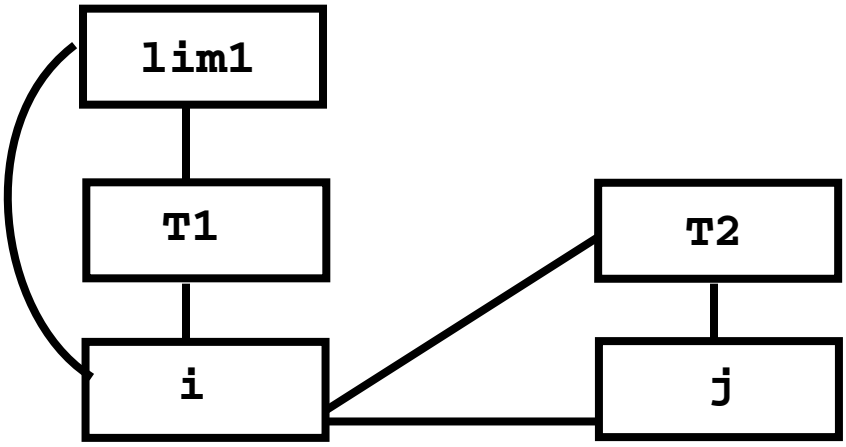
	lim1	lim2	T1	T2	i	j
Cost	11	11	11	11	42	42
Cost/ Neighbors	11/3	11/5	11/3	11/3	42/5	42/3

Do a 3 coloring

Since no node has fewer than 3 neighbors, we remove a node based on the minimum Cost/Neighbors value.

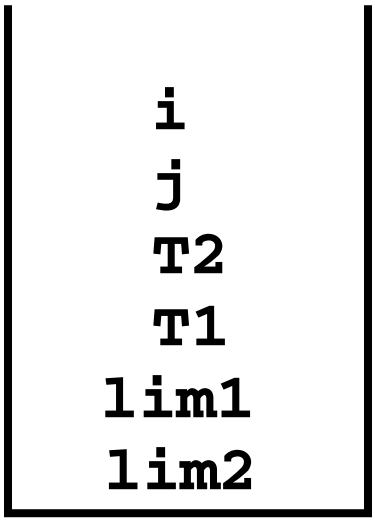
`lim2` is chosen.

We now have:



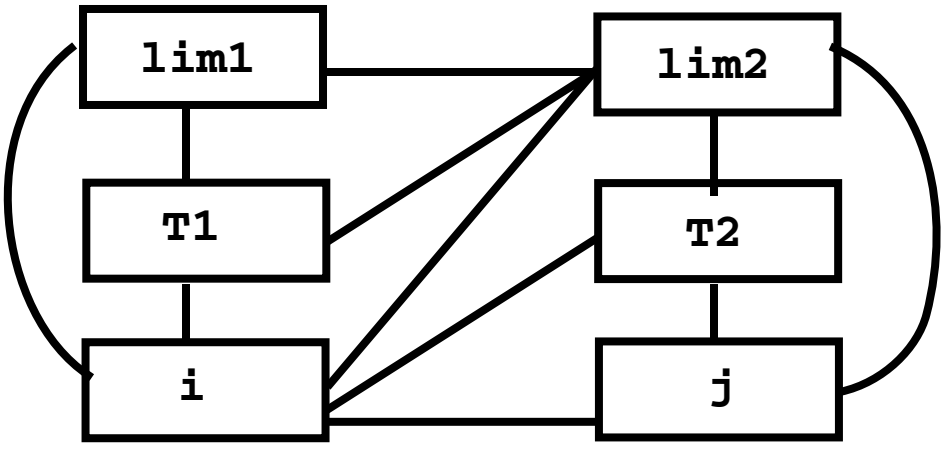
Remove (say) **`lim1`**, then **`T1`**, **`T2`**, **`j`** and **`i`** (order is arbitrary).

The Stack is:



Assuming the colors we have are R1, R2 and R3, the register assignment we choose is

`i`:R1, `j`:R2, `T2`:R3, `T1`:R2, `lim1`:R3, `lim2`:spill



Color Preferences

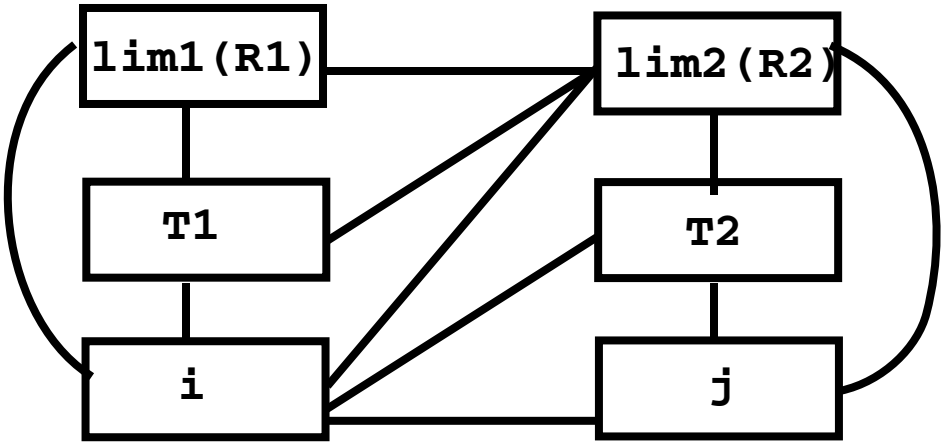
Sometimes we wish to assign a particular register (color) to a selected Live Range (e.g., a parameter or return value) *if possible*.

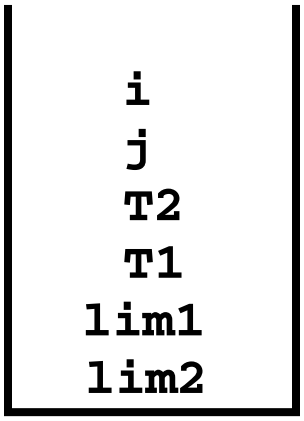
We can mark a node in the Interference Graph with a *Color Preference*.

When we unstack nodes and assign colors, we will avoid choosing color c if an uncolored neighbor has indicted a preference for it. If only color c is left, we take it (and ignore the preference).

Example

Assume in our previous example that lim1 has requested register R1 and lim2 has requested register R2 (because these are the registers the parameters are passed in).





Now when `i`, `j` and `T1` are unstacked, they respect `lim1`'s and `lim2`'s preferences:

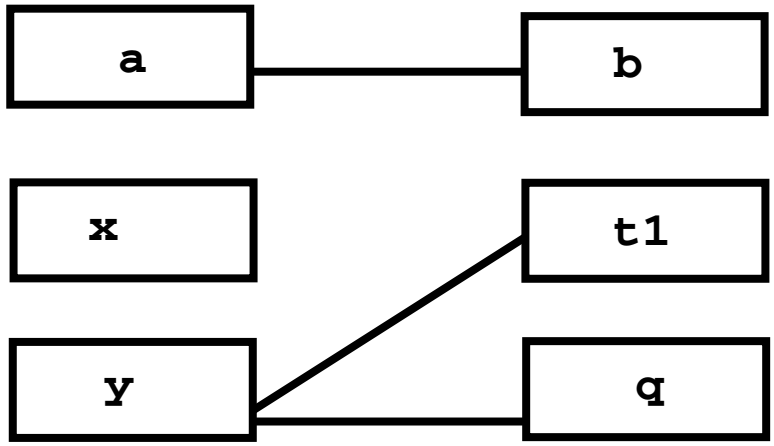
`i`:R3, `j`:R1, `T2`:R2, `T1`:R2, `lim1`:R1, `lim2`:spill

Using Coloring to Optimize Register Moves

A nice "fringe benefit" of allocating registers via coloring is that we can often *optimize away* register to register moves by giving the source and target the *same color*.

Consider

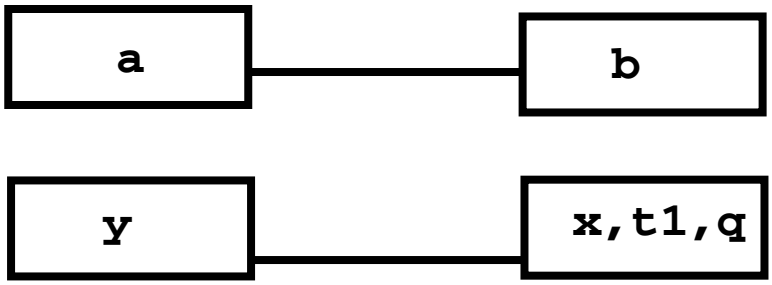
```
Live in: a,b

t1 = a + b

x = t1

y = x + 1

q = t1

Live out: y,q
```



We'd like `x`, `t1` and `q` to get the same color. How do we "force" this?

We can "merge" **x**, **t1** and **q** together:

```
Live in: a,b

t1 = a + b

x = t1

y = x + 1

q = t1

Live out: y,q
```



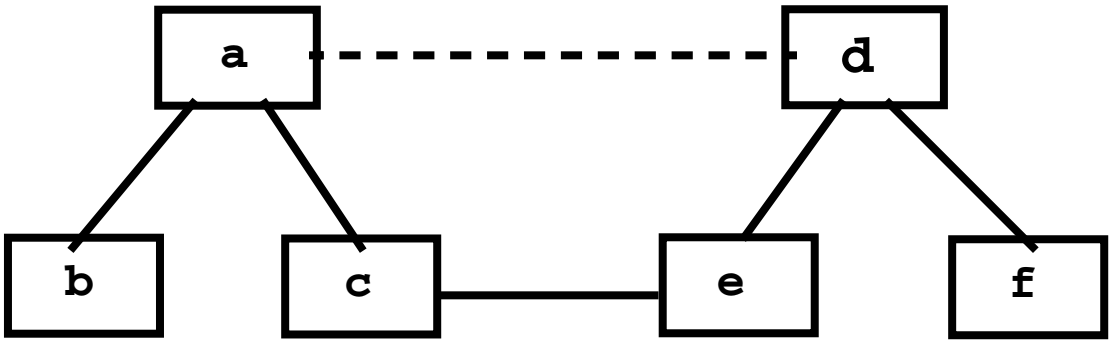
Now a 2-coloring that optimizes away both register to register moves is trivial.

Reckless Coalescing

Originally, Chaitin suggested merging *all* move-related nodes that don't interfere.

This is *reckless*—the merged node may not be colorable!

(Is it worth a spill to save a move??)



This Graph is 2-colorable before the reckless merge, but *not* after.

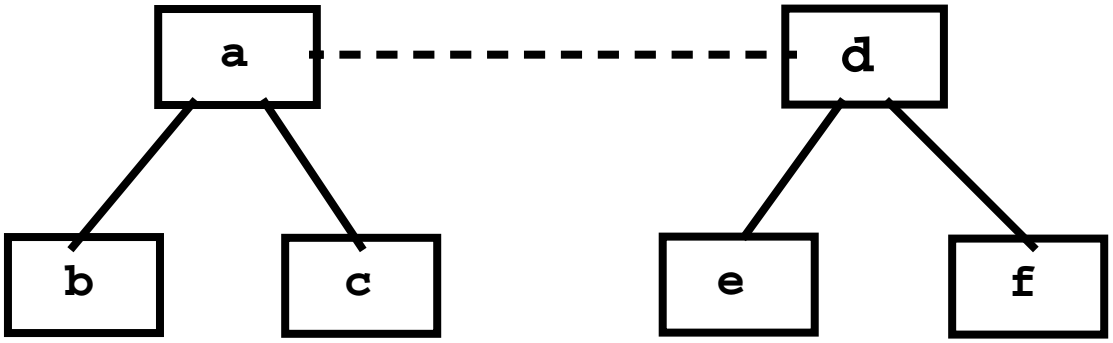
Conservative Coalescing

In response to Chaitin's reckless coalescing approach, Briggs suggested a *more conservative* approach.

See "Improvement to Graph Coloring Register Allocation," P. Briggs et. al., ACM Toplas, May 1994.

Briggs suggested that two move-related nodes should be merged *only if* the combined source and target node has fewer than R neighbors.

This *guarantees* that the combined node will be colorable, but may miss some optimization opportunities.



After a merge of nodes **a** and **d**, there will be four neighbors, but a 2-coloring is still possible.

Iterated Coalescing

This is an intermediate approach, that seeks to be safer than reckless coalescing and more effective than conservative coalescing. It was proposed by George and Appel.

1. Build:

   Create an Interference Graph, as usual. Mark source-target pairs with a special move-related arc (denoted as a dashed line).

2. Simplify:

   Remove and stack non-move-related nodes with < R neighbors.

3. Coalesce:

   Combine move-related pairs that will have < R neighbors after coalescing.

Repeat steps 2 and 3 until only nodes with R or more neighbors or move-related nodes remain or the graph is empty.

4. Freeze:

If the Interference Graph is non-empty:

Then If there exists a move-related node with < R neighbors

Then: "Freeze in" the move and make the node non-move-related.

Return to Steps 2 and 3.

Else: Use Chaitin's Cost/Neighbors criterion to remove and stack a node.

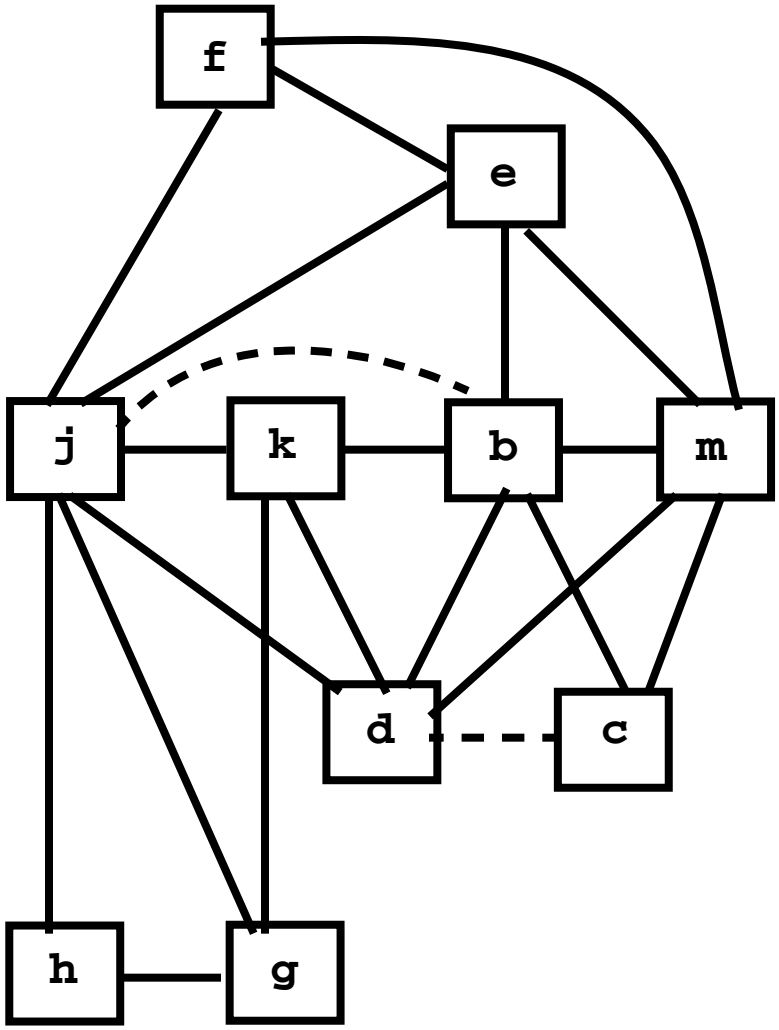
Return to Steps 2 and 3.

5. Unstack:

Color nodes as they are unstacked as per Chaitin and Briggs.

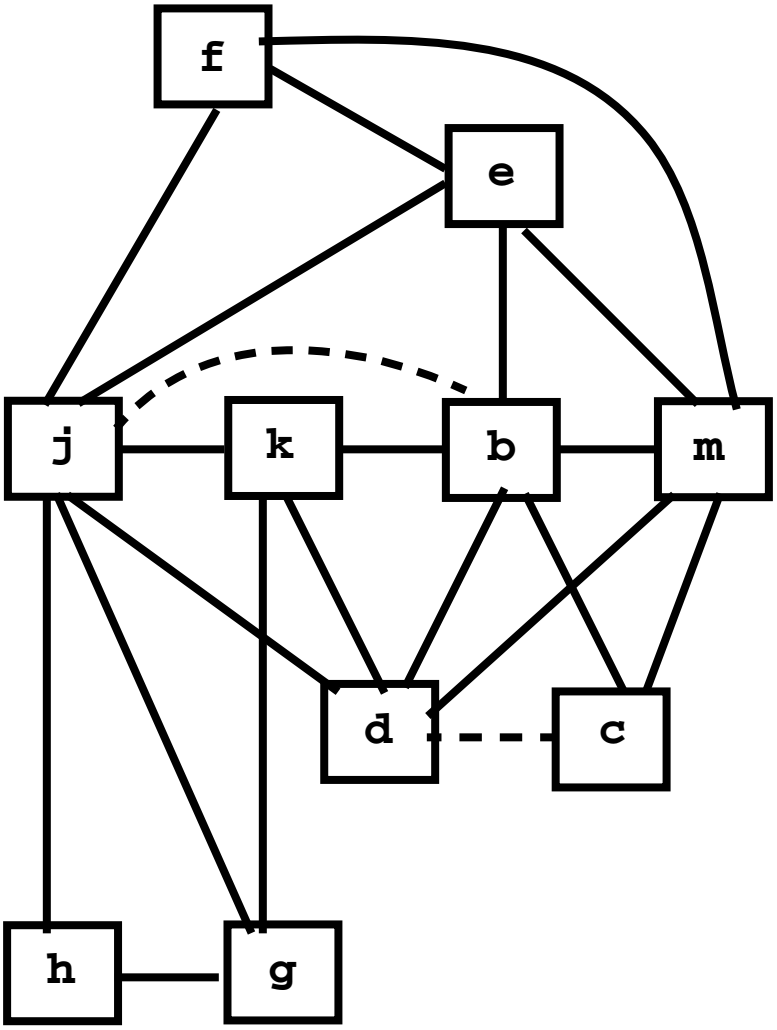
Example

```
Live in: k,j
g = mem[j+12]
h = k-1
f = g*h
e = mem[j+8]
m = mem[j+16]
b = mem[f]
c = e+8
d = c
k = m+4
j = b
goto d
Live out: d,k,j
```



Assume we want a 4-coloring.

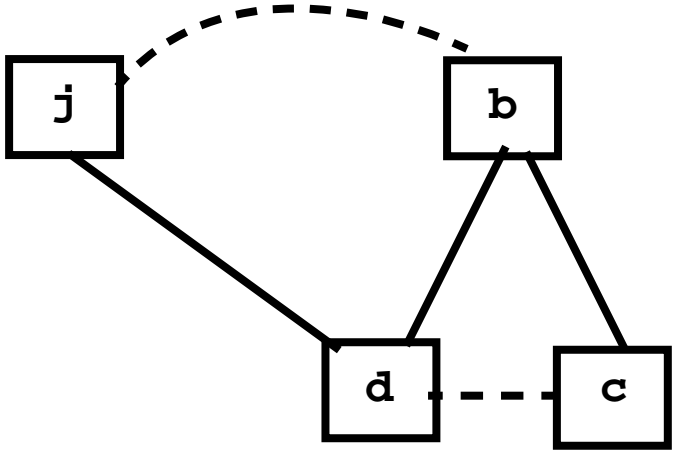
Note that neither **`j&b`** nor **`d&c`** can be conservatively colored.



We simplify by removing nodes with fewer than 4 neighbors.

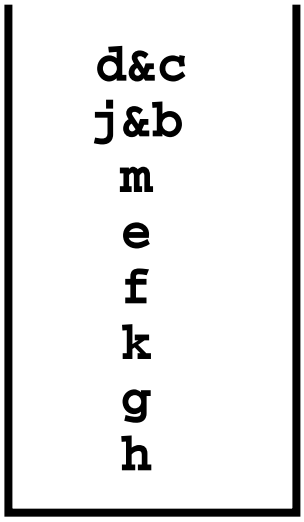
We remove and stack: **g, h, k, f, e, m**

The remaining Interference Graph is



We can now conservatively coalesce the move-related pairs to obtain

These remaining nodes can now be removed and stacked.



We can now unstack and color:

d&c:R1, **j&b**:R2, **m**:R3, **e**:R4, **f**:R1, **k**:R3, **h**:R1, **g**:R4

No spills were required and both moves were optimized away.