Static Single Assignment Form

Many of the complexities of optimization and code generation arise from the fact that a given variable may be assigned to in *many* different places.

Thus reaching definition analysis gives us the *set* of assignments that *may* reach a given use of a variable.

Live range analysis must track *all* assignments that may reach a use of a variable and merge them into the same live range.

Available expression analysis must look at *all* places a variable may be assigned to and decide if any kill an already computed expression.

What If

each variable is assigned to in only one place?
(Much like a named constant).

Then for a given use, we can find a single *unique* definition point.

But this seems *impossible* for most programs—or is it?

In *Static Single Assignment* (SSA) Form each assignment to a variable, v, is changed into a unique assignment to new variable, v_i.

If variable v has n assignments to it throughout the program, then (at least) n new variables, v_1 to v_n, are created to replace v. All uses of v are replaced by a use of some v_i.

Phi Functions

Control flow can't be predicted in advance, so we can't always know which definition of a variable reached a particular use.

To handle this uncertainty, we create *phi functions*.

As illustrated below, if v_i and v_j both reach the top of the same block, we add the assignment

$$v_k \leftarrow \phi(v_i, v_j)$$

to the top of the block.

Within the block, all uses of v become uses of v_k (until the next assignment to v).

What does $\phi(v_i, v_j)$ Mean?

One way to read $\phi(v_i, v_j)$ is that if control reaches the phi function via the path on which v_i is defined, ϕ "selects" v_i; otherwise it "selects" v_j.
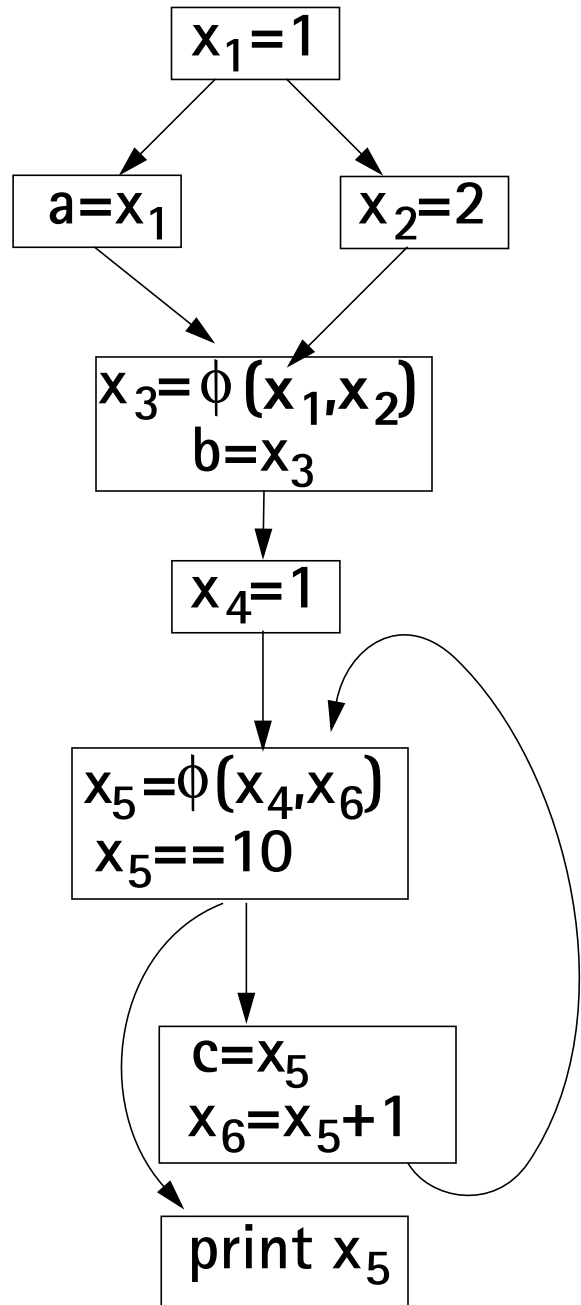
Phi functions may take more than 2 arguments if more than 2 definitions might reach the same block.

Through phi functions we have simple links to all the places where v receives a value, directly or indirectly.

Example
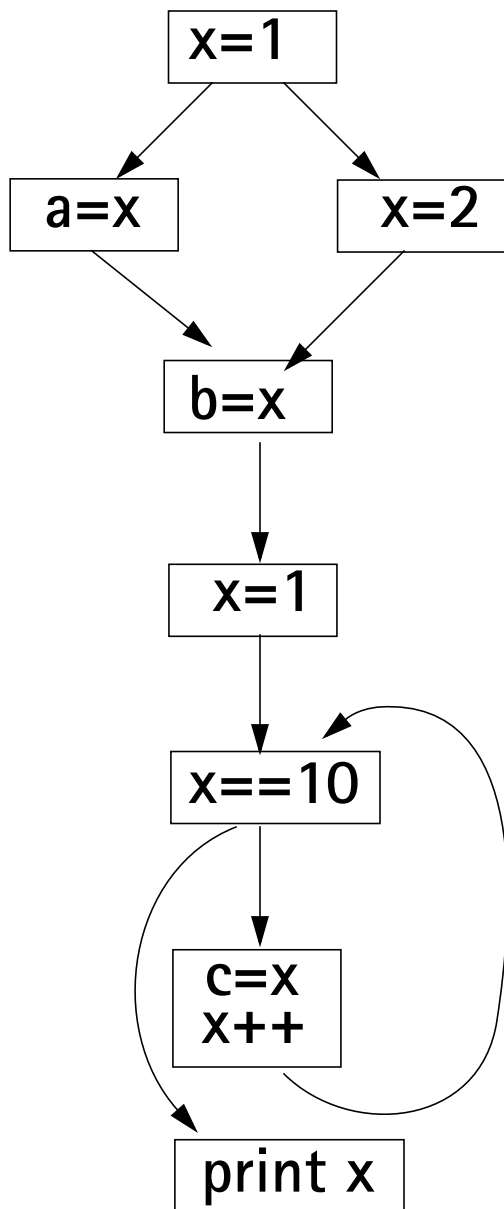
Original CFG

CFG in SSA Form

In SSA form computing live ranges is almost trivial. For each x_i include all x_j variables involved in phi functions that define x_i.

Initially, assume x_1 to x_6 (in our example) are independent. We then union into equivalence classes x_i values involved in the same phi function or assignment.

Thus x_1 to x_3 are unioned together (forming a live range). Similarly, x_4 to x_6 are unioned to form a live range.

Constant Propagation in SSA

In SSA form, constant propagation is simplified since values flow directly from assignments to uses, and phi functions represent natural "meet points" where values are combined (into a constant or $\perp$).

Even conditional constant propagation fits in. As long as a path is considered unreachable, it variables are set to T (and therefore ignored at phi functions, which meet values together).

Example

```
i=6
j=1
k=1
repeat
   if (i==6)
        k=0
   else
        i=i+1
   i=i+k
   j=j+1
until (i==j)
```

$i_1=6$
$j_1=1$
$k_1=1$
```
repeat
```
$i_2=\phi(i_1,i_5)$
$j_2=\phi(j_1,j_3)$
$k_2=\phi(k_1,k_4)$
```
   if (i₂==6)
```
$k_3=0$
```
   else
```
$i_3=i_2+1$
$i_4=\phi(i_2,i_3)$
$k_4=\phi(k_3,k_2)$
$i_5=i_4+k_4$
$j_3=j_2+1$
```
until (i₅==j₃)
```

	i_1	j_1	k_1	i_2	j_2	k_2	k_3	i_3	i_4	k_4	i_5	j_3
Pass1	6	1	1	$6\wedge T$	$1\wedge T$	$1\wedge T$	0	T	$6\wedge T$	0	6	2
Pass2	6	1	1	$6\wedge6$	$\perp$	$\perp$	0	T	6	0	6	$\perp$

We have determined that i=6 everywhere.

Putting Programs into SSA Form

Assume we have the CFG for a program, which we want to put into SSA form. We must:

- Rename all definitions and uses of variables

- Decide where to add phi functions

Renaming variable definitions is trivial—each assignment is to a new, unique variable.

After phi functions are added (at the heads of selected basic blocks), only one variable definition (the most recent in the block) can reach any use. Thus renaming uses of variables is easy.

Placing Phi Functions

Let b be a block with a definition to some variable, v. If b contains more than one definition to v, the last (or most recent) applies.

What is the first basic block following b where some other definition to v *as well as* b's definition can reach?

In blocks dominated by b, b's definition *must* have been executed, though other later definitions may have overwritten b's definition.

Domination Frontiers (Again)

Recall that the Domination Frontier of a block b, is defined as

$DF(N) =$
  $\{Z \mid M{\to}Z$ & $(N$ dom $M)$ &
    $\neg(N$ sdom $Z)\}$

The Dominance Frontier of a basic block N, DF(N), is the set of all blocks that are immediate successors to blocks dominated by N, but which aren't themselves strictly dominated by N.

Assume that an initial assignment to all variables occurs in b_0 (possibly of some special "uninitialized value.")

We will need to place a phi function at the start of all blocks in b's Domination Frontier.

The phi functions will join the definition to v that occurred in b (or in a block dominated by b) with definitions occurring on paths that don't include b.

After phi functions are added to blocks in DF(b), the domination frontier of blocks with newly added phi's will need to be computed (since phi functions imply assignment to a new v_i variable).

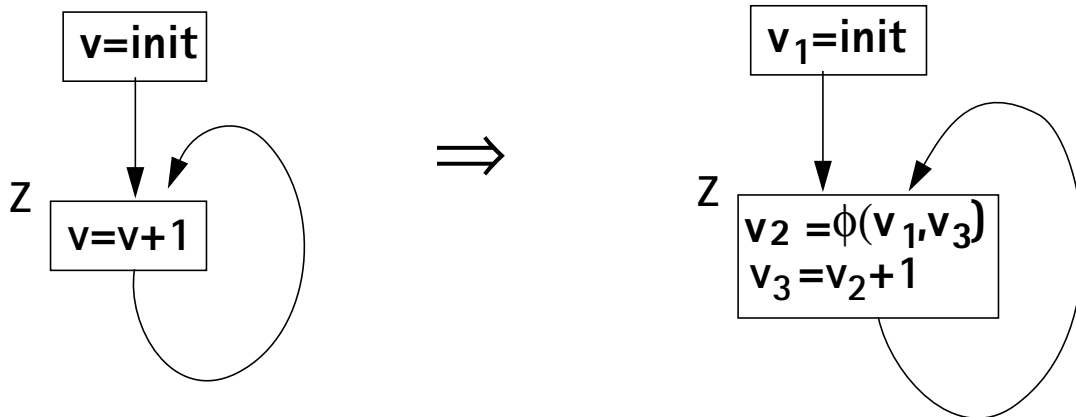# Examples of How Domination Frontiers Guide Phi Placement

$DF(N) =$
$\{Z \mid M \rightarrow Z$ & $(N$ dom $M)$ &
$\neg(N$ sdom $Z)\}$

Simple Case:
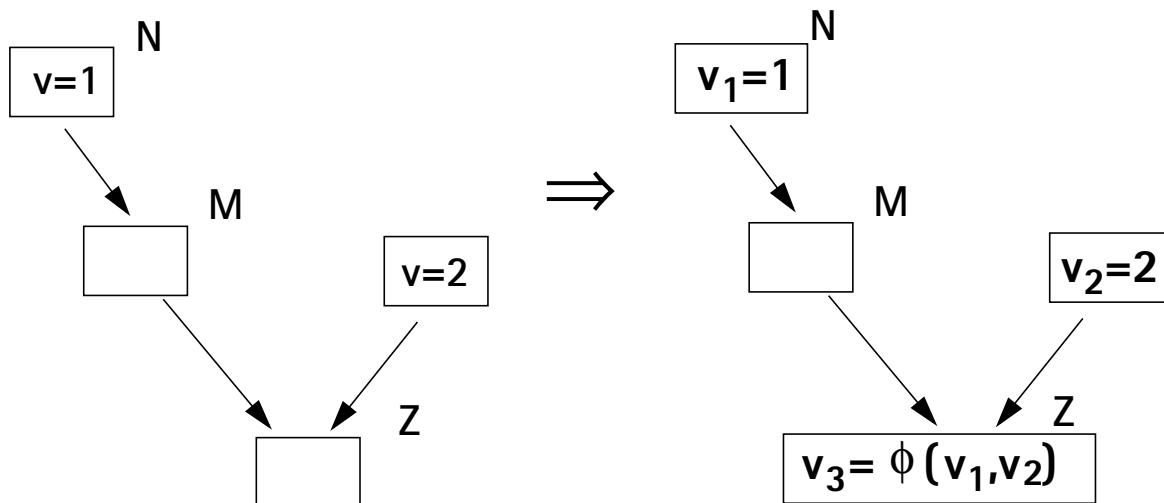
Here, $(N$ dom $M)$ but $\neg(N$ sdom $Z)$,
so a phi function is needed in Z.

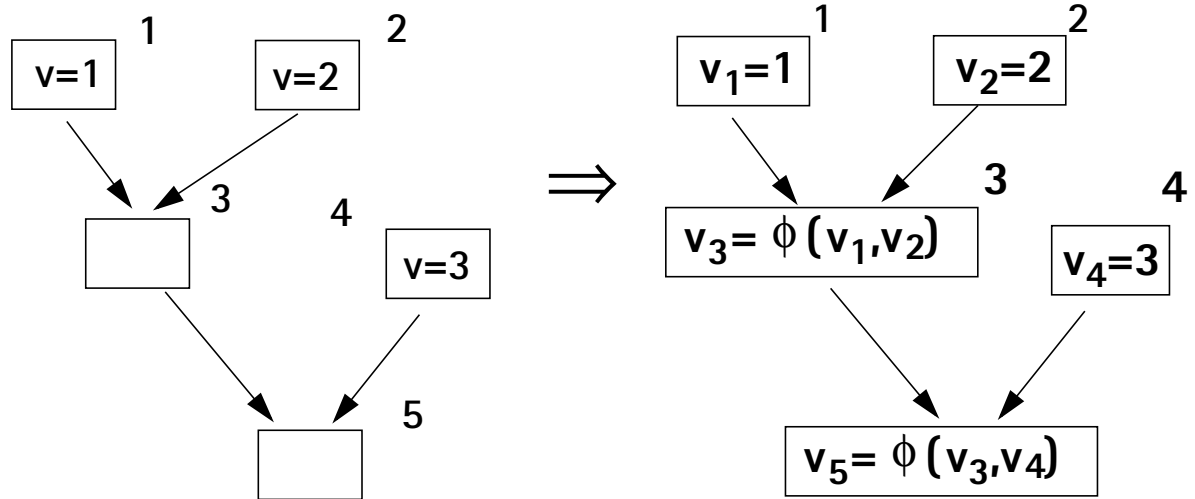# Loop:

Here, let M = Z = N. M$\rightarrow$Z,

(N dom M) but $\neg$(N sdom Z),

so a phi function *is* needed in Z.

DF(N) =
 {Z | M$\rightarrow$Z & (N dom M) &
  $\neg$(N sdom Z)}

Sometimes Phi's must be Placed Iteratively

Now, DF(b1) = {b3}, so we add a phi function in b3. This adds an assignment into b3. We then look at DF(b3) = {b5}, so another phi function must be added to b5.

Phi Placement Algorithm

To decide what blocks require a phi function to join a definition to a variable v in block b:

1. Compute $D_1 = DF(b)$.

   Place Phi functions at the head of all members of D_1.

2. Compute $D_2 = DF(D_1)$.

   Place Phi functions at the head of all members of D_2-D_1.

3. Compute $D_3 = DF(D_2)$.

   Place Phi functions at the head of all members of $D_3-D_2-D_1$.

4. Repeat until no additional Phi functions can be added.

```
PlacePhi{
  For (each variable v ∈ program) {
      For (each block b ∈ CFG ){
          PhiInserted(b) = false
          Added(b) = false }
      List = φ
      For (each b ∈ CFG that assigns to V ){
          Added(b) = true
          List = List U {b}    }
      While (List ≠ φ) {
          Remove any b from List
          For (each d ∈ DF(b)){
              If (! PhiInserted(d)) {
                  Add a Phi Function to d
                  PhiInserted(d) = true
                  If (! Added(d)) {
                      Added(d) = true
                      List = List U {d}
                  }
              }
          }
      }
  }
}
```
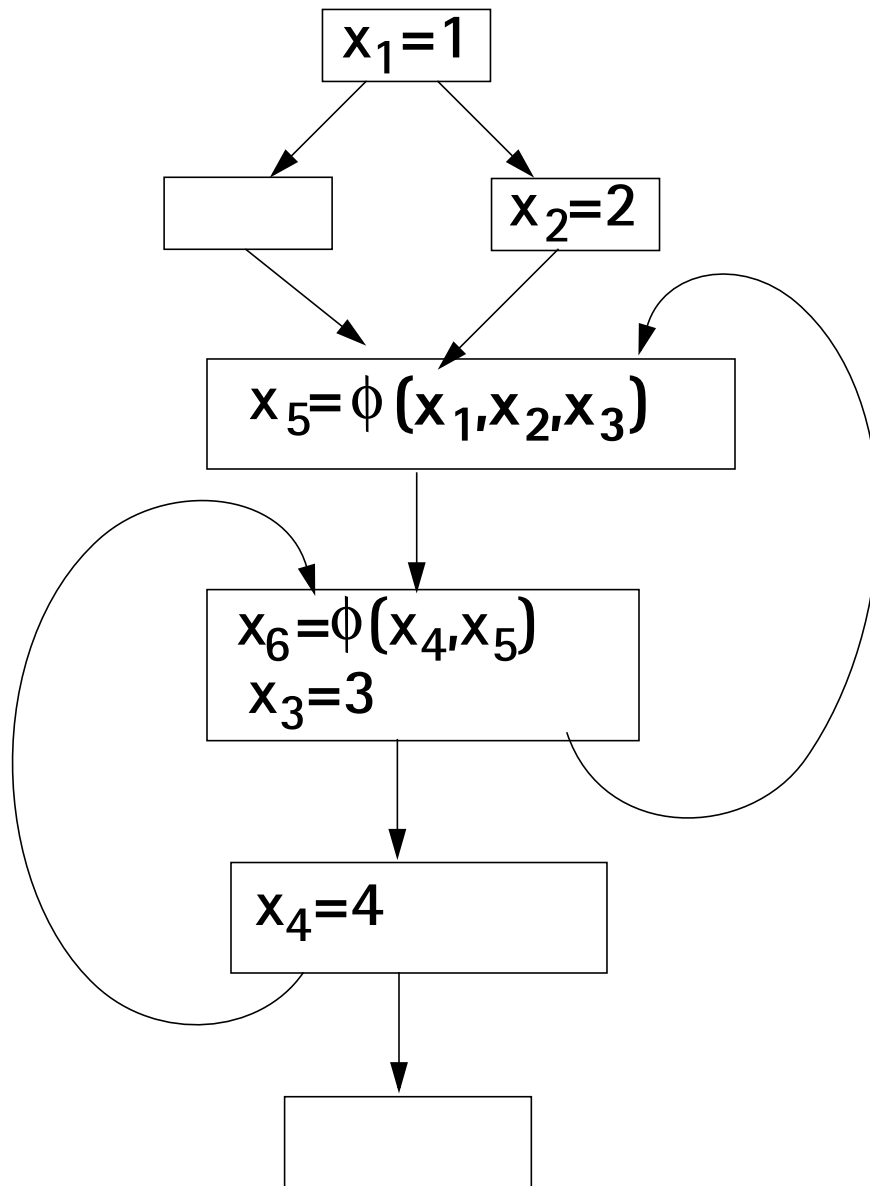
Example
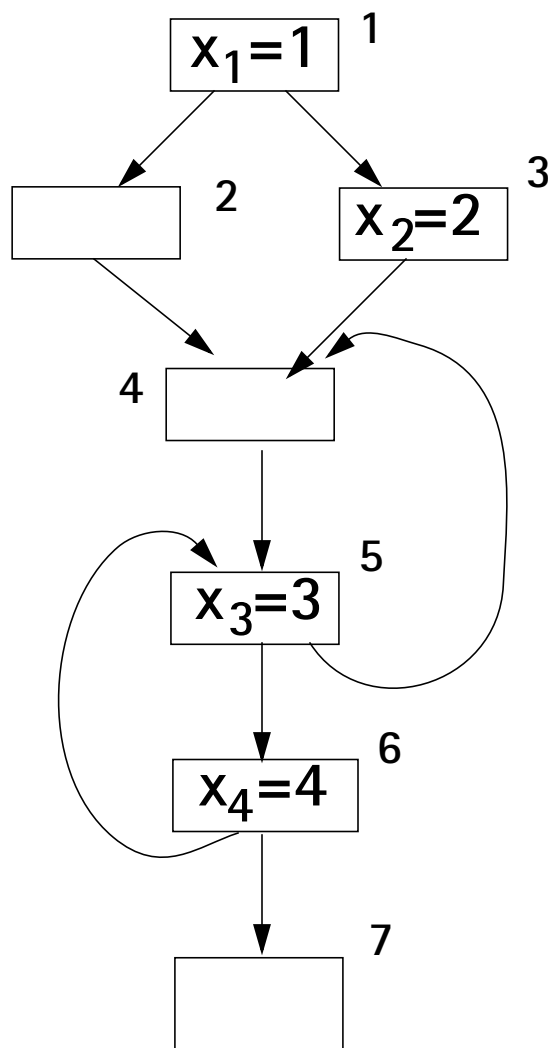
Initially, List={1,3,5,6}

Process 1: DF(1) = ϕ

Process 3: DF(3) = 4, so add 4 to List and add phi fct to 4.

Process 5: DF(5)={4,5} so add phi fct to 5.

Process 5: DF(6) = {5}

Process 4: DF(4) = {4}

We will add Phi's into blocks 4 and 5. The arity of each phi is the number of in-arcs to its block. To find the args to a phi, follow each arc "backwards" to the sole reaching def on that path.

SSA and Value Numbering

We already know how to do available expression analysis to determine if a previous computation of an expression can be reused.

A limitation of this analysis is that it can't recognize that two expressions that aren't syntactically identical may actually still be equivalent.

For example, given

```
t1 = a + b
c = a
t2 = c + b
```

Available expression analysis won't recognize that `t1` and `t2` must be equivalent, since it doesn't track the fact that `a = c` at `t2`.

Value Numbering

An early expression analysis technique called *value numbering* worked only at the level of basic blocks. The analysis was in terms of "values" rather than variable or temporary names.

Each non-trivial (non-copy) computation is given a number, called its *value number*.

Two expressions, using the same operators and operands with the same value numbers, must be equivalent.

For example,

```
t1 = a + b
c = a
t2 = c + b
```

is analyzed as

```
v1 = a
v2 = b
t1 = v1 + v2
c = v1
t2 = v1 + v2
```

Clearly `t2` is equivalent to `t1` (and hence need not be computed).

In contrast, given

```
t1 = a + b
a = 2
t2 = a + b
```

the analysis creates

```
v1 = a
v2 = b
t1 = v1 + v2
v3 = 2
t2 = v3 + v2
```

Clearly t2 is not equivalent to t1 (and hence will need to be recomputed).

Extending Value Numbering to Entire CFGs

The problem with a global version of value numbering is how to reconcile values produced on different flow paths. But this is exactly what SSA is designed to do!

In particular, we know that an ordinary assignment

```
x = y
```

does *not* imply that all references to x can be replaced by y after the assignment. That is, an assignment *is not* an assertion of value equivalence.

But,

in SSA form

$$x_i = y_j$$

does mean the two values are *always* equivalent after the assignment. If y_j reaches a use of x_i, that use of x_i *can* be replaced with y_j.

Thus in SSA form, an assignment *is* an assertion of value equivalence.

We will assume that simple variable to variable copies are removed by substituting equivalent SSA names.

This alone is enough to recognize some simple value equivalences.

As we saw,

$$t_1 = a_1 + b_1$$
$$c_1 = a_1$$
$$t_2 = c_1 + b_1$$

becomes

$$t_1 = a_1 + b_1$$
$$t_2 = a_1 + b_1$$

Partitioning SSA Variables

Initially, all SSA variables will be partitioned by the *form* of the expression assigned to them.

Expressions involving different constants or operators won't (in general) be equivalent, even if their operands happen to be equivalent.

Thus

$$v_1 \; = \; 2 \;\; \text{and} \; w_1 \; = \; a_2 \; + \; 1$$

are always considered inequivalent.

But,

$$v_3 \; = \; a_1 \; + \; b_2 \; \text{and} \; w_1 \; = \; d_1 \; + \; e_2$$

may *possibly* be equivalent since both involve the same operator.

Phi functions are potentially equivalent only if they are in the same basic block.

All variables are initially considered equivalent (since they all initially are considered uninitialized until explicit initialization).

After SSA variables are grouped by assignment form, groups are split.

If a_i op b_y and c_k op d_l
are in the same group (because they both have the same operator, op)
and $a_i \not\equiv c_k$ or $b_j \not\equiv d_l$
then we split the two expressions apart into different groups.

We continue splitting based on operand inequivalence, until no more splits are possible. Values still grouped are equivalent.

Example

```
if (...) {
   a₁=0
   if (...)
      b₁=0
   else {
      a₂=x₀
      b₂=x₀ }
   a₃=φ(a₁,a₂)
   b₃=φ(b₁,b₂)
   c₂=*a₃
   d₂=*b₃ }
else {
   b₄=10 }
a₅=φ(a₀,a₃)
b₅=φ(b₃,b₄)
c₃=*a₅
d₃=*b₅
e₃=*a₅
```

Initial Groupings:

$G_1=[a_0,b_0,c_0,d_0,e_0,x_0]$

$G_2=[a_1=0, b_1=0]$

$G_3=[a_2=x_0, b_2=x_0]$

$G_4=[b_4=10]$

$G_5=[a_3=\phi(a_1,a_2),$
$\qquad b_3=\phi(b_1,b_2)]$

$G_6=[a_5=\phi(a_0,a_3),$
$\qquad b_5=\phi(b_3,b_4)]$

$G_7=[c_2=*a_3,$
$\qquad d_2=*b_3,$
$\qquad d_3=*b_5,$
$\qquad c_3=*a_5,$
$\qquad e_3=*a_5]$

Now b_4 isn't equivalent to anything, so split a_5 and b_5. In G_7 split operands b_3, a_5 and b_5. We now have

```
if (...) {
   a_1=0
   if (...)
      b_1=0
   else {
      a_2=x_0
      b_2=x_0 }
   a_3=φ(a_1,a_2)
   b_3=φ(b_1,b_2)
   c_2=*a_3
   d_2=*b_3 }
else {
   b_4=10 }
a_5=φ(a_0,a_3)
b_5=φ(b_3,b_4)
c_3=*a_5
d_3=*b_5
e_3=*a_5
```

Final Groupings:

$G_1 = [a_0, b_0, c_0, d_0, e_0, x_0]$

$G_2 = [a_1=0, b_1=0]$

$G_3 = [a_2=x_0, b_2=x_0]$

$G_4 = [b_4=10]$

$G_5 = [a_3=\phi(a_1,a_2),$
$\quad b_3=\phi(b_1,b_2)]$

$G_{6a} = [a_5=\phi(a_0,a_3)]$

$G_{6b} = [b_5=\phi(b_3,b_4)]$

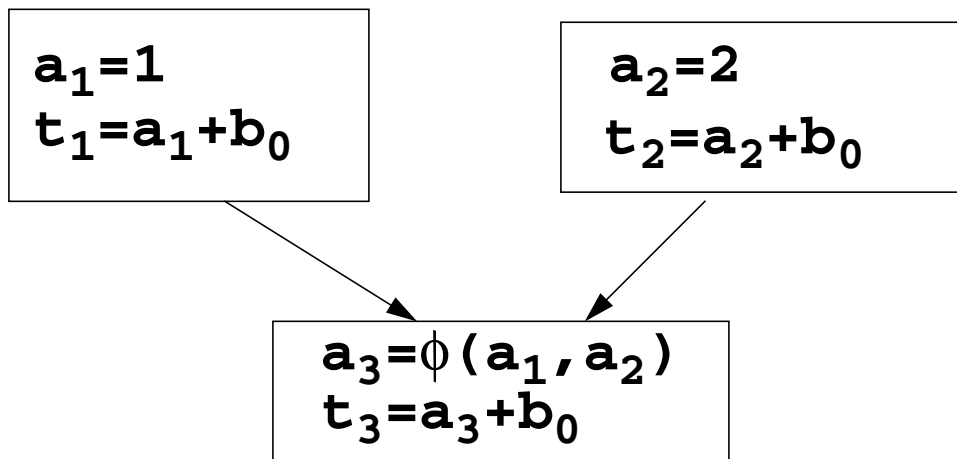$G_{7a} = [c_2=*a_3,$
$\quad d_2=*b_3]$

$G_{7b} = [d_3=*b_5]$

$G_{7c} = [c_3=*a_5,$
$\quad e_3=*a_5]$

Variable e_3 can use c_3's value and d_2 can use c_2's value.
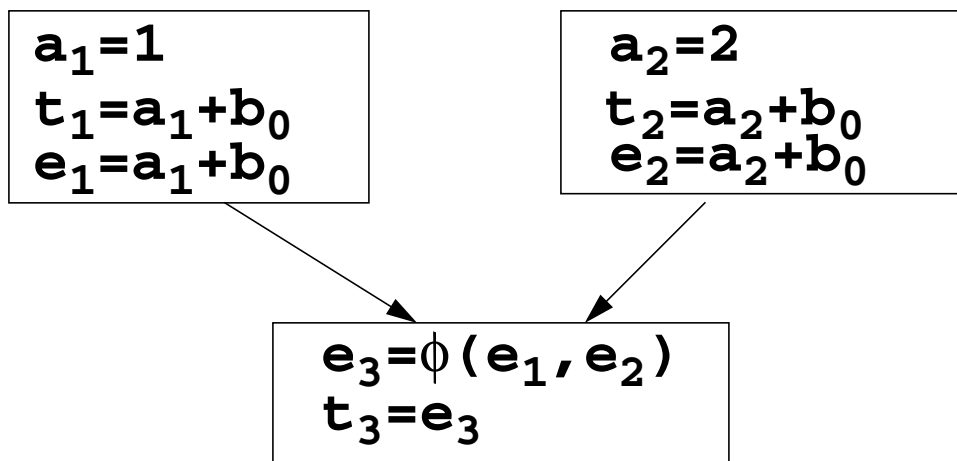
Limitations of Global Value Numbering

As presented, our global value numbering technique doesn't recognize (or handle) computations of the same expression that produce different values along different paths.

Thus in

$$a_1=1$$
$$t_1=a_1+b_0$$

$$a_2=2$$
$$t_2=a_2+b_0$$

$$a_3=\phi(a_1,a_2)$$
$$t_3=a_3+b_0$$

variable a_3 isn't equivalent to either a_1 or a_2.

But,
we can still remove a redundant computation of **a+b** by moving the computation of t_3 to each of its predecessors:

$$a_1 = 1$$
$$t_1 = a_1 + b_0$$
$$e_1 = a_1 + b_0$$

$$a_2 = 2$$
$$t_2 = a_2 + b_0$$
$$e_2 = a_2 + b_0$$
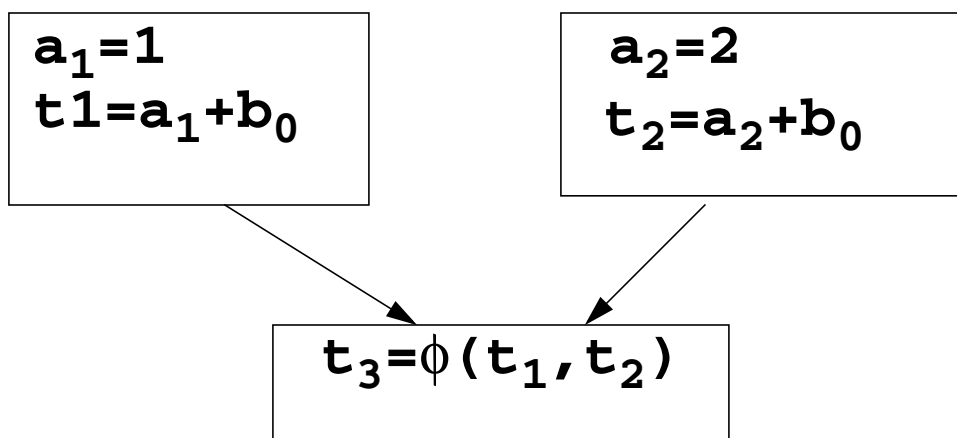
$$e_3 = \phi(e_1, e_2)$$
$$t_3 = e_3$$

Now a redundant computation of **a+b** is evident in each predecessor block. Note too that this has a nice register targeting effect—e_1, e_2 and e_3 can be readily mapped to the same live range.

The notion of moving expression computations above phi functions also meshes nicely with notion of partial redundancy elimination. Given

```
a₁=1
t₁=a₁+b₀
```

```
a₂=2
```

```
a₃=ɸ(a₁,a₂)
t₃=a₃+b₀
```
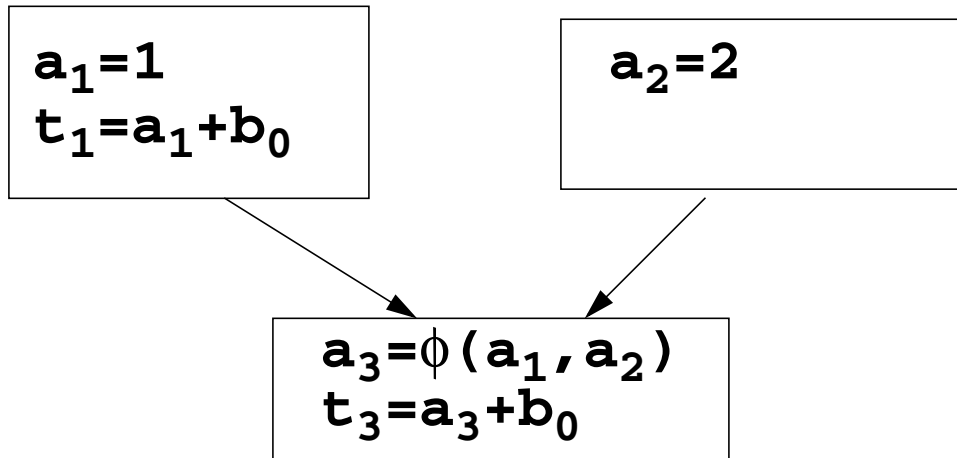
moving **a+b** above the phi produces

```
a₁=1
t1=a₁+b₀
```

```
a₂=2
t₂=a₂+b₀
```

```
t₃=ɸ(t₁,t₂)
```

Now **a+b** is computed only once on each path, an improvement.