# CS 760: Machine Learning
# **Reinforcement Learning II**


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# Announcements

- **Logistics**:
  - HW8 released tonight (last HW).

- Class roadmap:

| | |
|---|---|
| Thurs., Dec. 2 | RL II |
| Tues., Dec. 7 | RL III |
| Thurs., Dec 9 | Large Language Models |
| Tues., Dec 14 | Fairness & Ethics |

# Outline

- **Review: Intro to Reinforcement Learning**
  - Basic concepts, mathematical formulation, MDPs, policies
- **Valuing and Obtaining Policies**
  - Value functions, Bellman equation, value iteration, policy iteration
- **Q Learning**
  - Q function, Q-learning, SARSA, approximation

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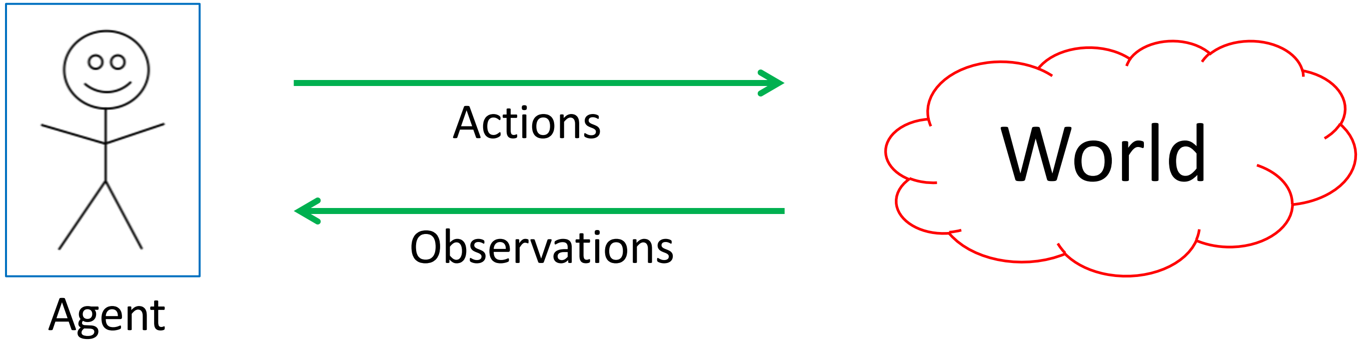
# Review: General Model

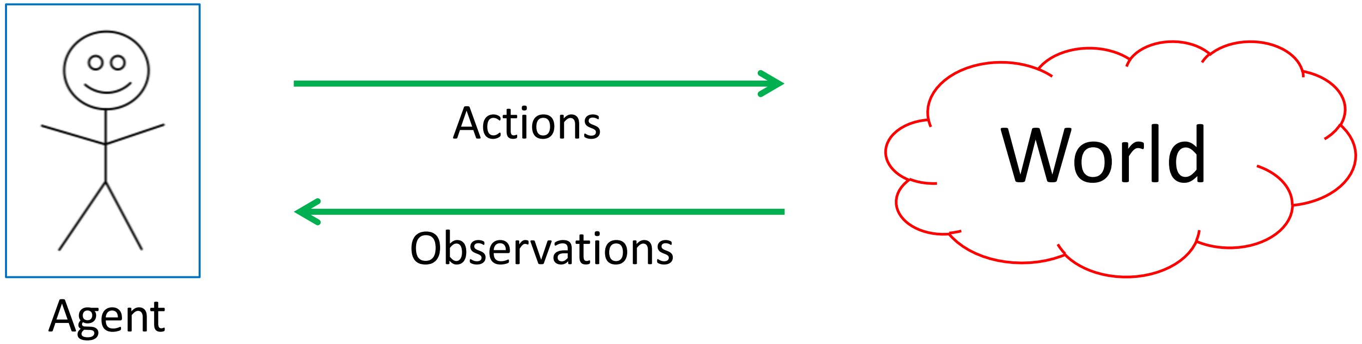
We have an **agent** **interacting** with the **world**




- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility **($$$)**
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning

# Building The Theoretical Model

Basic setup:




- Set of states, S
- Set of actions A
- Information: at time $t$, observe state $s_t \in$ S. Get reward $r_t$
- Agent makes choice $a_t \in$ A. State changes to $s_{t+1,}$ continue

Goal: find a map from **states to actions** maximize rewards.

A “policy”

# Markov Decision Process (MDP)

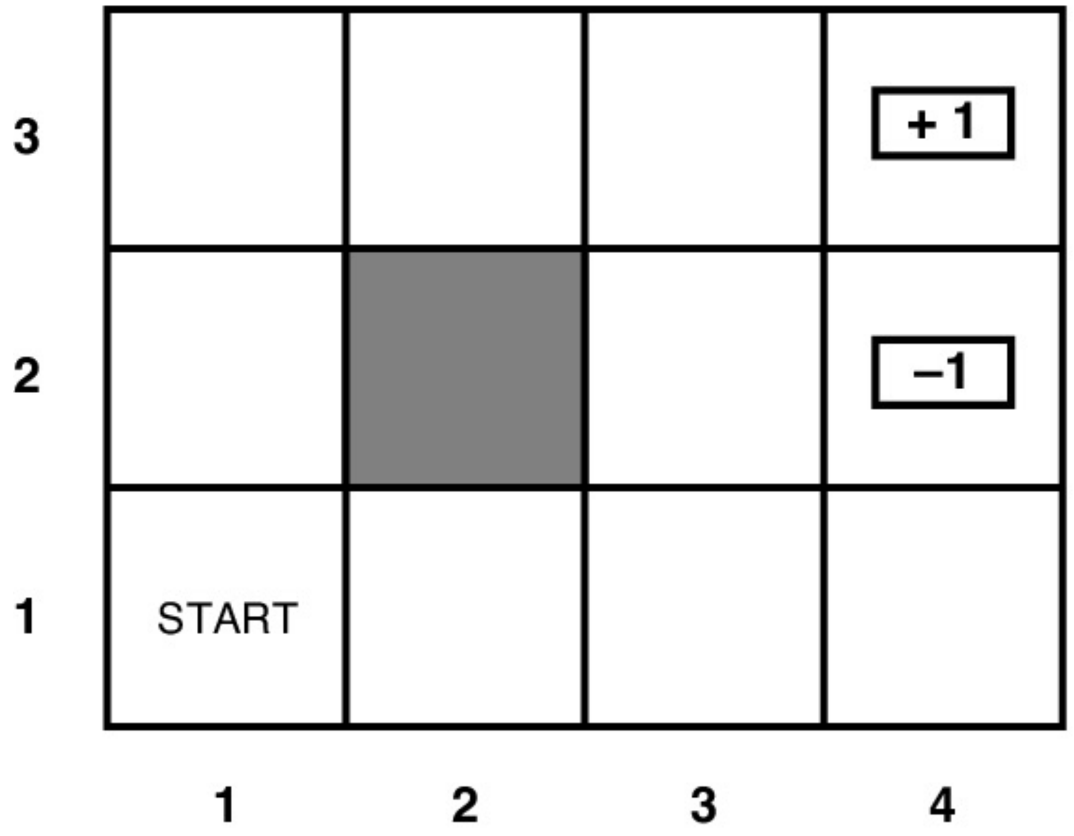
The formal mathematical model:

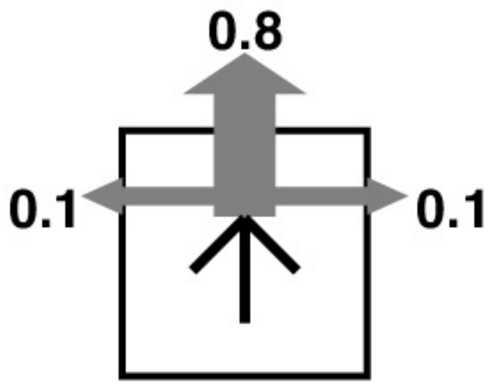
- **State set** S. Initial state $s_0$. **Action set** A
- **State transition model**: $P(s_{t+1} | s_t, a_t)$
  - Markov assumption: transition probability only depends on $s_t$ and $a_t$, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state.

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

# Grid World Abstraction

Note: (i) Robot is unreliable (ii) Reach target fast

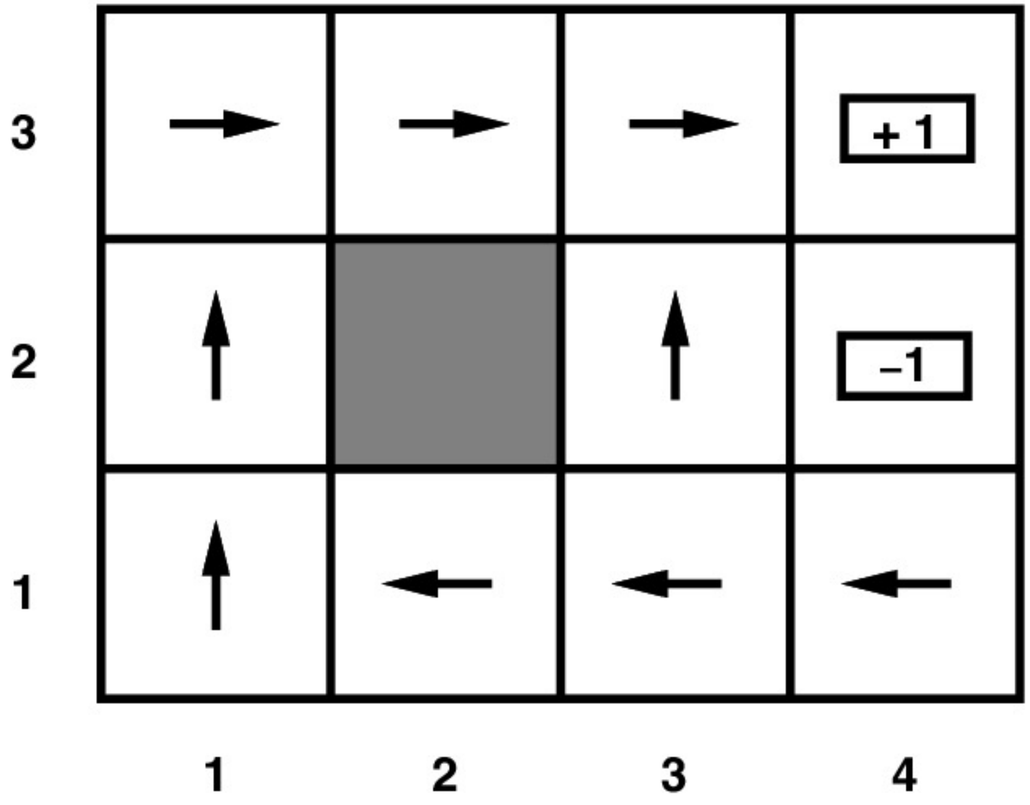


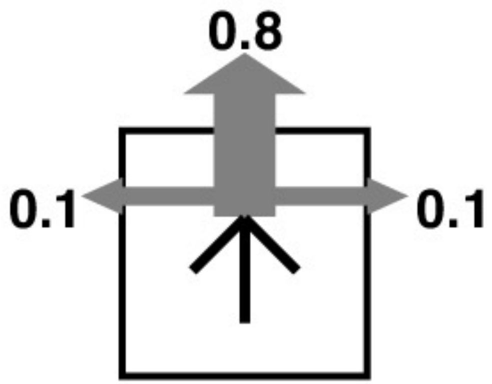




$r(s) = -0.04$ for every non-terminal state

# Grid World Optimal Policy

Note: (i) Robot is unreliable (ii) Reach target fast




$r(s) = -0.04$ for every non-terminal state

# Back to MDP Setup

The formal mathematical model:

- **State set** $S$. Initial state $s_{0.}$ **Action set** $A$
- **State transition model:** $P(s_{t+1}|s_t, a_t)$
  - Markov assumption: transition probability only depends on $s_t$ and $a_t$, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy:** $\pi(s) : S \rightarrow A$ action to take at a particular state.

**How do we find the best policy?**

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

# Break & Quiz

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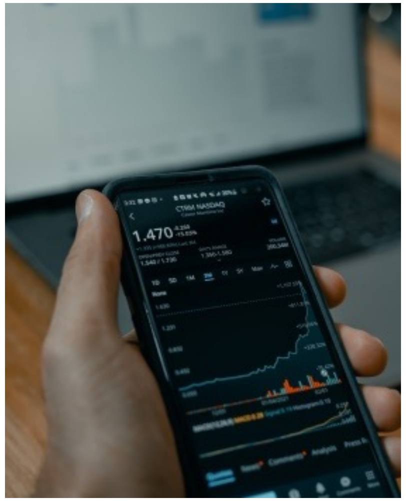
# Defining the Optimal Policy

For policy π, **expected utility** over all possible state sequences from $s_0$ produced by following that policy:

$$V^{\pi}(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$

Called the **value function** (for π, $s_0$)



# Discounting Rewards

One issue: these are infinite series. **Convergence**?

- Solution

$$U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is VS **future**
  - Note: has to be less than 1 for convergence

# From Value to Policy

Now that $V^{\pi}(s_0)$ is defined what *a* should we take?

- First, set V*(s) to be expected utility for **optimal** policy from s
- What's the expected utility of an action?
  - Specifically, action a in state s?

$$\sum_{s'} P(s'|s, a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

# Obtaining the Optimal Policy

We know the expected utility of an action.

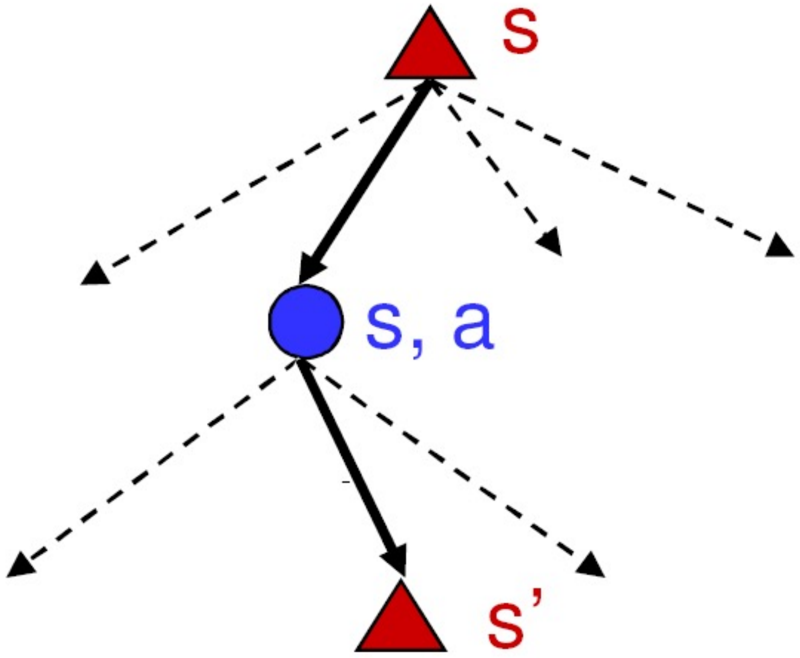
- So, to get the optimal policy, compute

$$\pi^*(s) = \mathrm{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards



Credit L. Lazbenik

# Slight Problem…

Now we can get the optimal policy by doing

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

- So we need to know $V^*(s)$.
  - But it was defined in terms of the optimal policy!
  - So we need some other approach to get $V^*(s)$.
  - Need some other **property** of the value function!

# Bellman Equation

Let's walk over one step for the value function:

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a)V^*(s')$$

Current state reward

Discounted expected future **rewards**

- Bellman: inventor of dynamic programming

# Value Iteration

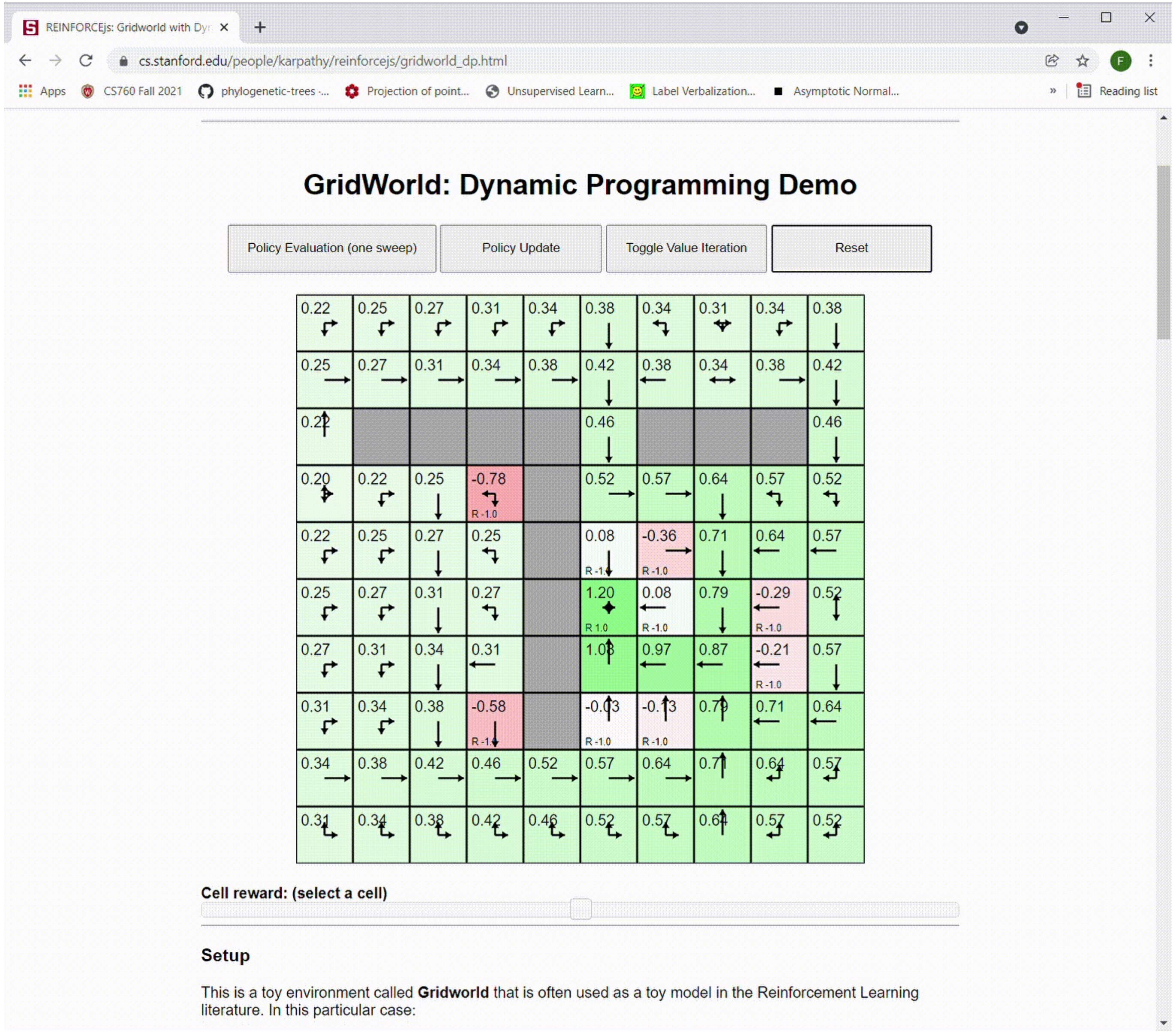
**Q**: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

**A**: Use the property. Start with $V_0(s)=0$. Then, update

$$V_{i+1}(s) = r(s) + \gamma \max_{a} \sum_{s'} P(s'|s,a) V_i(s')$$

# Value Iteration: Demo




Source: Karpathy

# **Policy** Iteration

With value iteration, we estimate V*

- Then get policy (i.e., indirect estimate of policy)

- Could also try to get policies directly

- This is **policy iteration.** Basic idea:
  - Start with random policy $\pi$
  - Use it to compute value function $V^{\pi}$ (for that policy)
  - Improve the policy: obtain $\pi'$

# **Policy** Iteration: Algorithm

**Policy iteration.** Algorithm

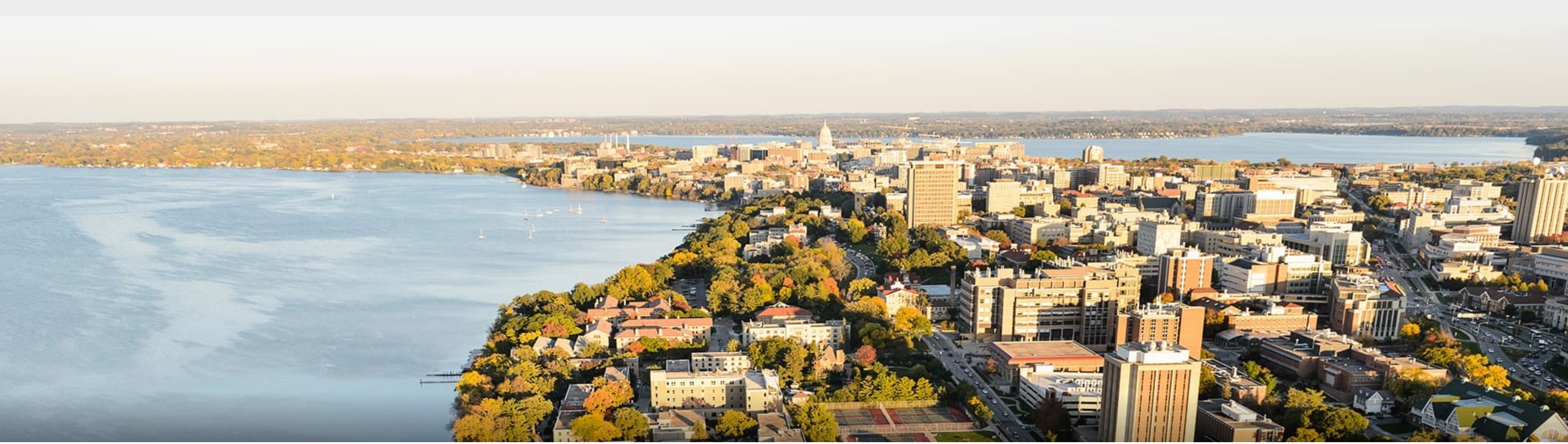
- Start with random policy $\pi$
- Use it to compute value function $V^\pi$ : a set of linear equations

$$V^\pi(s) = r(s) + \gamma \sum_{s'} P(s'|s,a) V^\pi(s')$$

- Improve the policy: obtain $\pi'$

$$\pi'(s) = \arg\max_a r(s) + \gamma \sum_{s'} P(s'|s,a) V^\pi(s')$$

- Repeat



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# Q-Learning

What if we don't know transition probability $P(s'|s,a)$?

- Need a way to learn to act without it.
- **Q-learning**: get an action-utility function $Q(s,a)$ that tells us the value of doing $a$ in state $s$
- Note: $V^*(s) = \max_a Q(s,a)$
- Now, we can just do $\pi^*(s) = \arg\max_a Q(s,a)$
  - But need to estimate $Q$!

# Q-Learning Iteration

How do we get Q(*s*,*a*)?

- Similar iterative procedure

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Learning rate

**Idea**: combine old value and new estimate of future value.

Note: We are using a policy to take actions; based on Q!

# Exploration Vs. Exploitation

General question!

- **Exploration:** take an action with unknown consequences
  - **Pros**:
    - Get a more accurate model of the environment
    - Discover higher-reward states than the ones found so far
  - **Cons**:
    - When exploring, not maximizing your utility
    - Something bad might happen
- **Exploitation:** go with the best strategy found so far
  - **Pros**:
    - Maximize reward as reflected in the current utility estimates
    - Avoid bad stuff
  - **Cons**:
    - Might also prevent you from discovering the true optimal strategy

# Q-Learning: Epsilon-Greedy Policy

## How to **explore**?

- With some 0<ε<1 probability, take a random action at each state, or else the action with highest Q(*s*,*a*) value.

$$a = \begin{cases} \operatorname{argmax}_{a \in A} Q(s, a) & \text{uniform}(0,1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$

# Q-Learning: SARSA

An alternative:

- Just use the next action, no max over actions:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

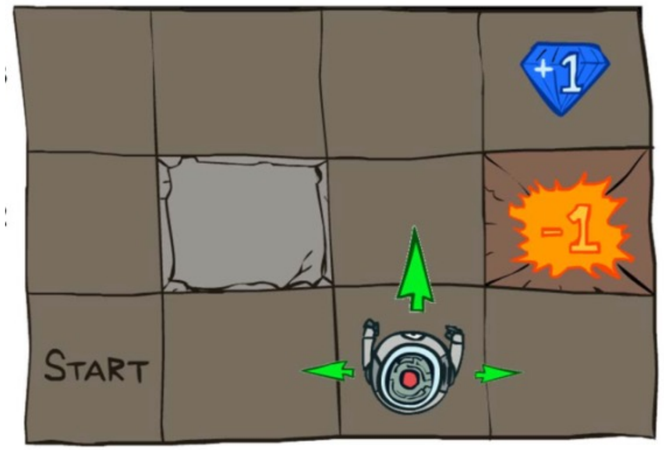
Learning rate

- Called state–action–reward–state–action (**SARSA**)
- Can use with epsilon-greedy policy

# Q-Learning Details

Note: if we have a **terminal** state, the process ends

- An **episode**: a sequence of states ending at a terminal state
- Want to run on many episodes
- Slightly different Q-update for terminal states

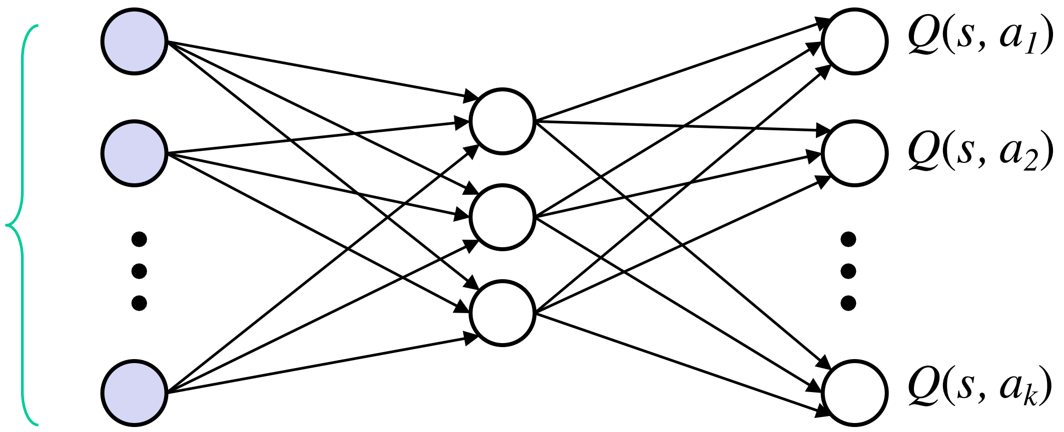


# Q-Learning – Compact Representations

## Q-table can be quite large… might not even fit memory

- Solution: use some other representation for a more compact version. Ex: neural networks.

encoding of
the state ($s$)

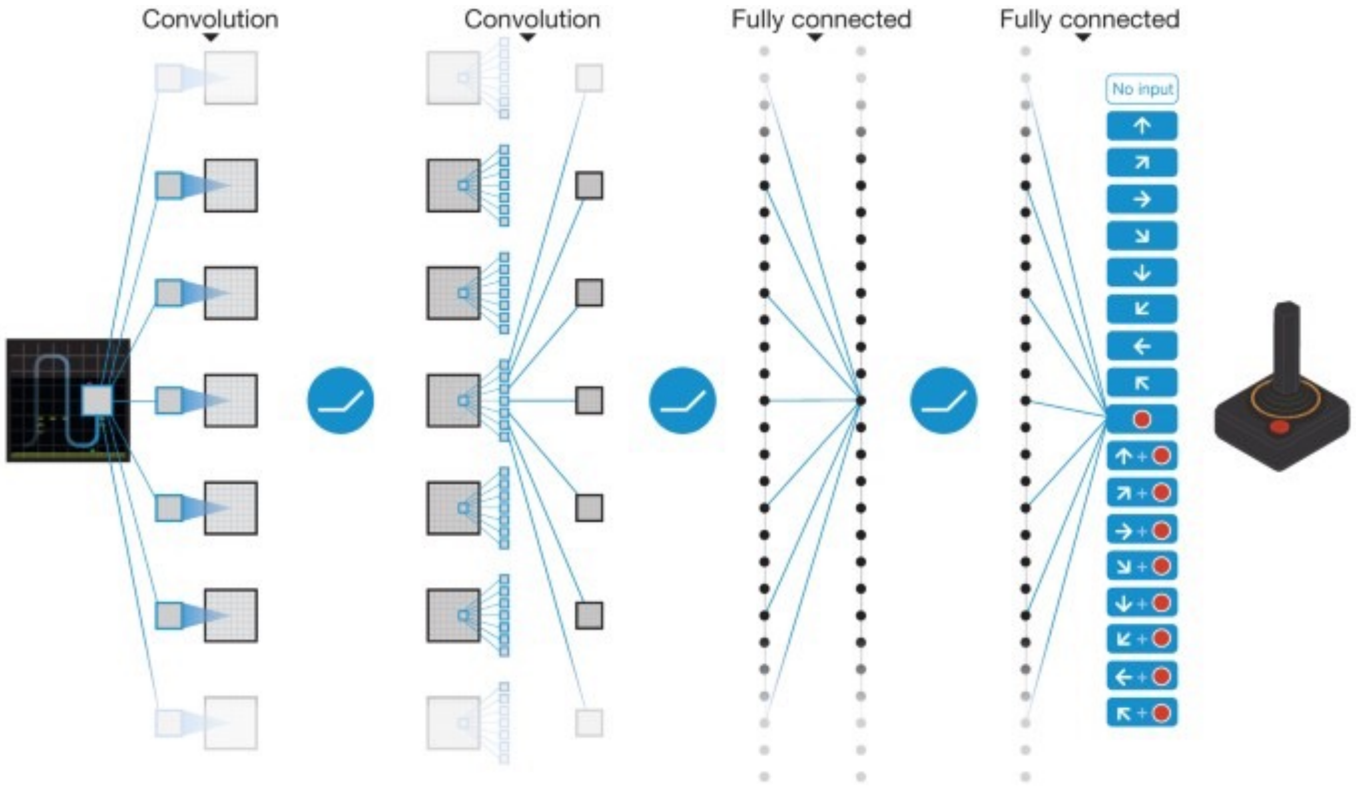



each input unit encodes a property of the state (e.g., a sensor value)

or could have one net for each possible action

# Deep Q-Learning

How do we get Q($s$,$a$)?




Mnih et al, "Human-level control through deep reinforcement learning"

# Thanks Everyone!

Some of the slides in these lectures have been adapted/borrowed from materials developed by Mark Craven, David Page, Jude Shavlik, Tom Mitchell, Nina Balcan, Elad Hazan, Tom Dietterich, Pedro Domingos, Jerry Zhu, Yingyu Liang, Volodymyr Kuleshov