



# CS 839: Foundation Models **Scaling & Scaling Laws**

Fred Sala

University of Wisconsin-Madison

**Oct. 31, 2023**

# Announcements

## •Logistics:

- HW2 due Nov. 9. One more (small) homework after
- Project info out. Dates: **Nov. 16**: proposal, **Dec. 19**
- Presentation dates: **Nov: 9,14,16,21,28,30 Dec: 5,7**
  - Two slots/date (worst-case three might be needed in some special cases)
- Due On **Nov. 2 (Thursday!)** --- presentation!

## •Class roadmap:

Tuesday Oct. 31	Scaling & Scaling Laws
Thursday Nov. 2	Security, Privacy, Toxicity
Tuesday Nov. 7	The Future

# Outline

- **Scaling Laws Intro**

- What are laws and why, regimes, idealized versions, initial findings from Kaplan et al

- **Scaling Laws Revised**

- Additional methods, new results, Chinchilla and related hypotheses

- **Beyond Scaling Laws**

- Data pruning and others

# Outline

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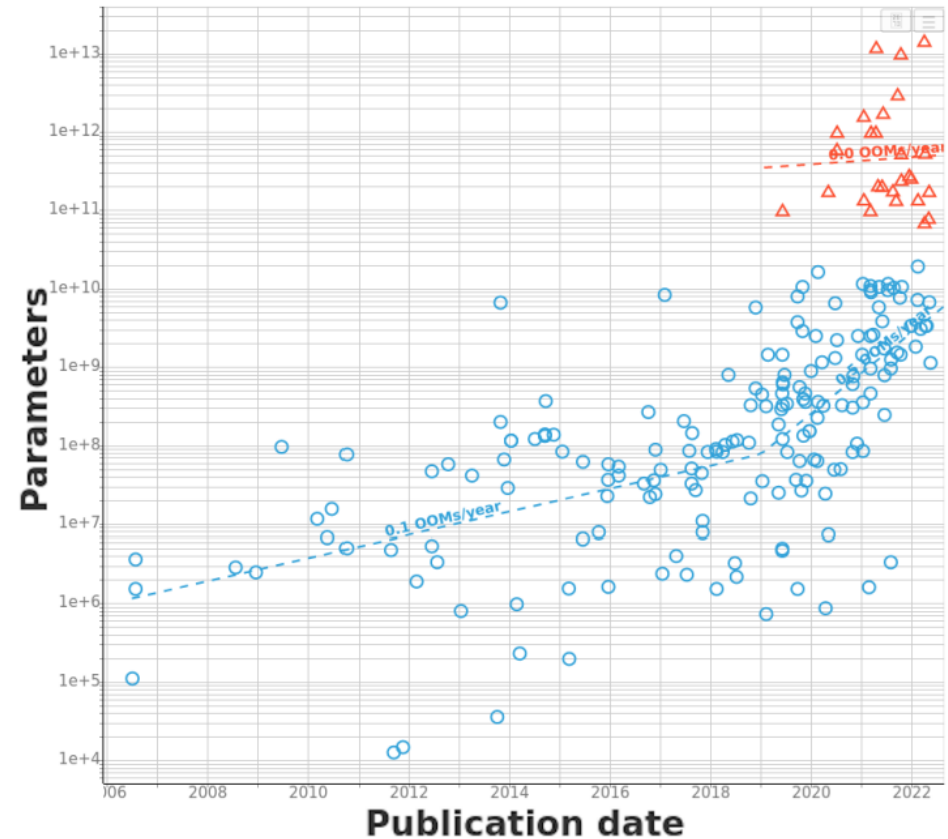
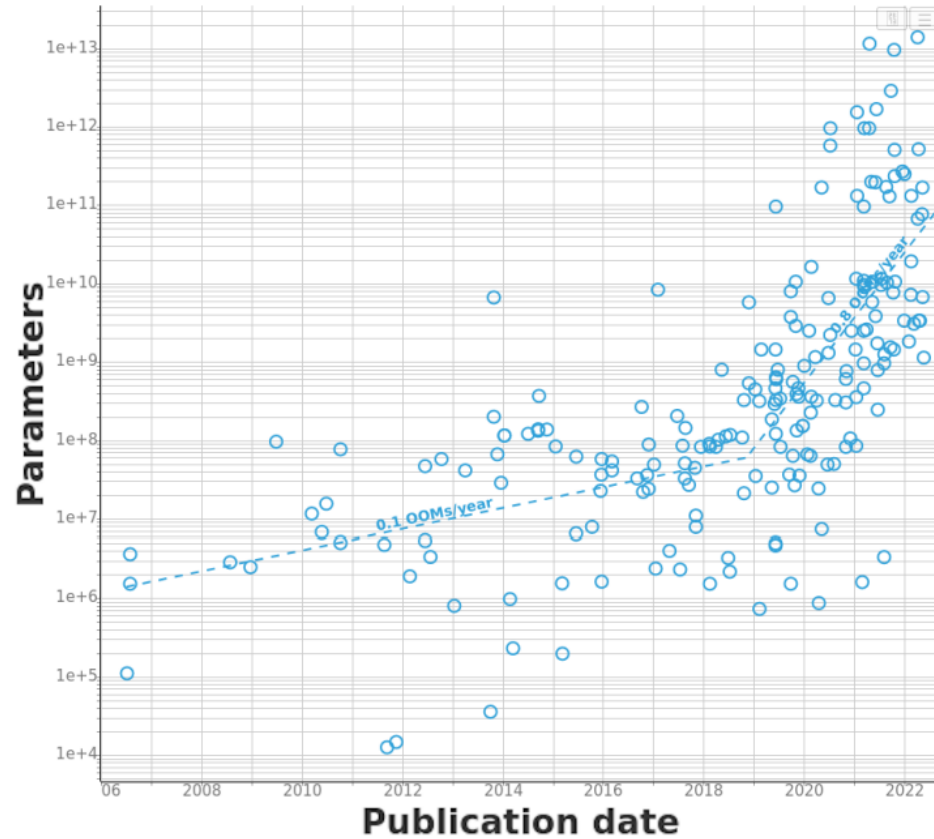
- Additional methods, new results, Chinchilla and related hypotheses

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# Trends: Models

Models have gotten bigger



Villalobos et al '22

# Trends: Compute

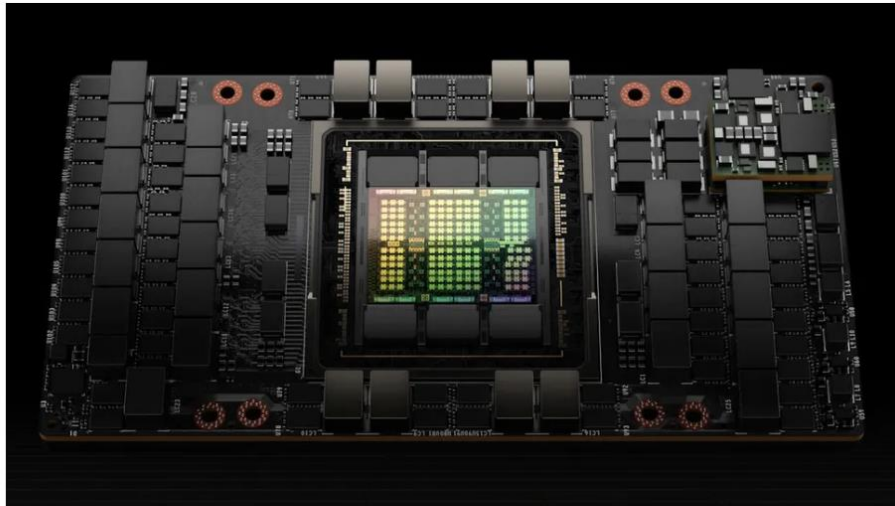
## Compute has gotten bigger

### Startup Builds Supercomputer with 22,000 Nvidia's H100 Compute GPUs

By Anton Shilov published July 05, 2023

The world's second highest performing supercomputer.

 Comments (4)



(Image credit: Nvidia)

Inflection AI, a new startup found by the former head of deep mind and backed  
<https://www.tomshardware.com/news/startup-builds-supercomputer-with-22000-nvidias-h100-compute-gpus>

### Tesla's \$300 Million AI Cluster Is Going Live Today

By Anton Shilov published August 28, 2023

Tesla is about to flip the switch on its new AI cluster, featuring 10,000 Nvidia H100 compute GPUs.

 Comments (23)



(Image credit: Shutterstock)

<https://www.tomshardware.com/news/teslas-dollar300-million-ai-cluster-is-going-live-today>

# Trends: Data

## Datasets have gotten bigger

Dataset Name	Brief description	Preprocessing	Instances	Format	Default Task	Created (updated)
Statlog (Image Segmentation) Dataset	The instances were drawn randomly from a database of 7 outdoor images and hand-segmented to create a classification for every pixel.	Many features calculated.	2310	Text	Classification	1990
Caltech 101	Pictures of objects.	Detailed object outlines marked.	9146	Images	Classification, object recognition.	2003
LabelMe	Annotated pictures of scenes.	Objects outlined.	187,240	Images, text	Classification, object detection	2005
Caltech-256	Large dataset of images for object classification.	Images categorized and hand-sorted.	30,607	Images, Text	Classification, object detection	2007
ImageNet	Labeled object image database, used in the <a href="#">ImageNet Large Scale Visual Recognition Challenge</a>	Labeled objects, bounding boxes, descriptive words, SIFT features	14,197,122	Images, text	Object recognition, scene recognition	2009 (2014)

wiki

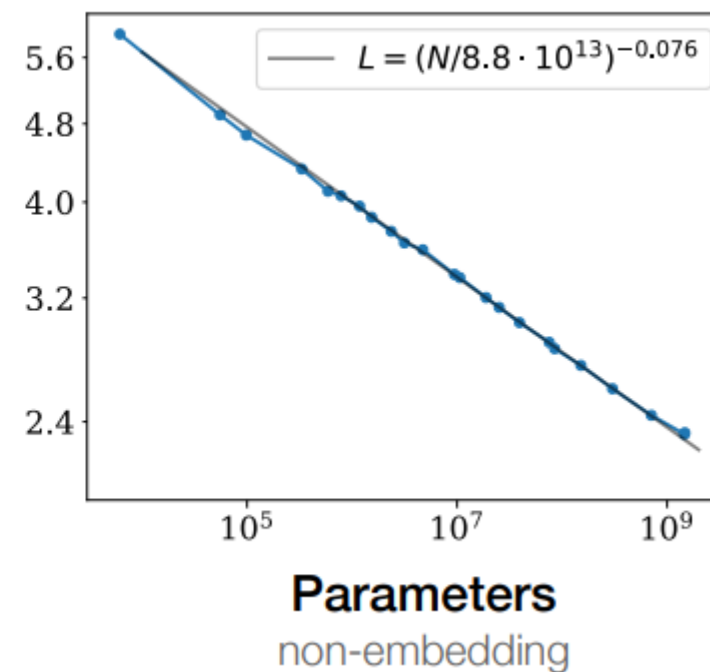
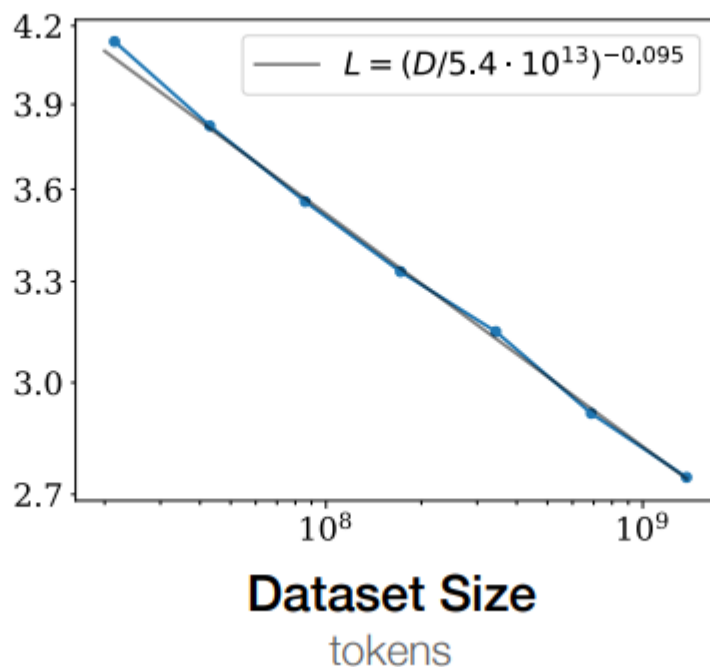
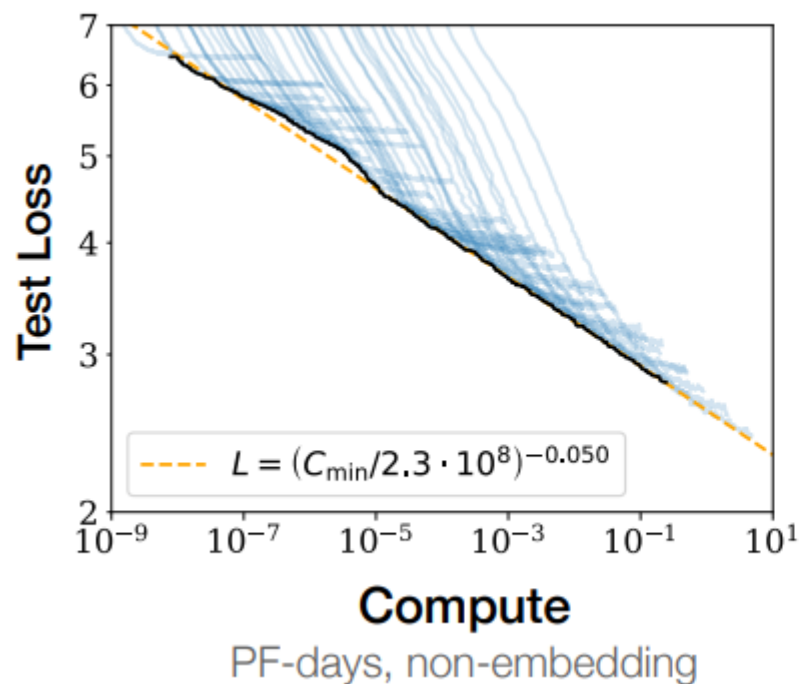
Model	Stock of data (#words)	Growth rate
Recorded speech	1.46e17 [3.41e16; 4.28e17]	5.2% [4.95%; 5.2%]
Internet users	2.01e15 [6.47e14; 6.28e15]	8.14% [7.89%; 8.14%]
Popular platforms	4.41e14 [1.21e14; 1.46e15]	8.14% [7.89%; 8.14%]
CommonCrawl	9.62e13 [4.45e13; 2.84e14]	16.68% [16.41%; 16.68%]
Indexed websites	2.21e14 [5.16e13; 6.53e15]	NA
<b>Aggregated model</b>	<b>7.41e14</b> <b>[6.85e13; 7.13e16]</b>	<b>7.15%</b> <b>[6.41%; 17.49%]</b>

Villalobos et al, “Will we run out of data? An analysis of the limits of scaling datasets in Machine Learning”

# Scaling Laws

We want to understand

- How performance scales with these quantities...
- And how they **interact!**



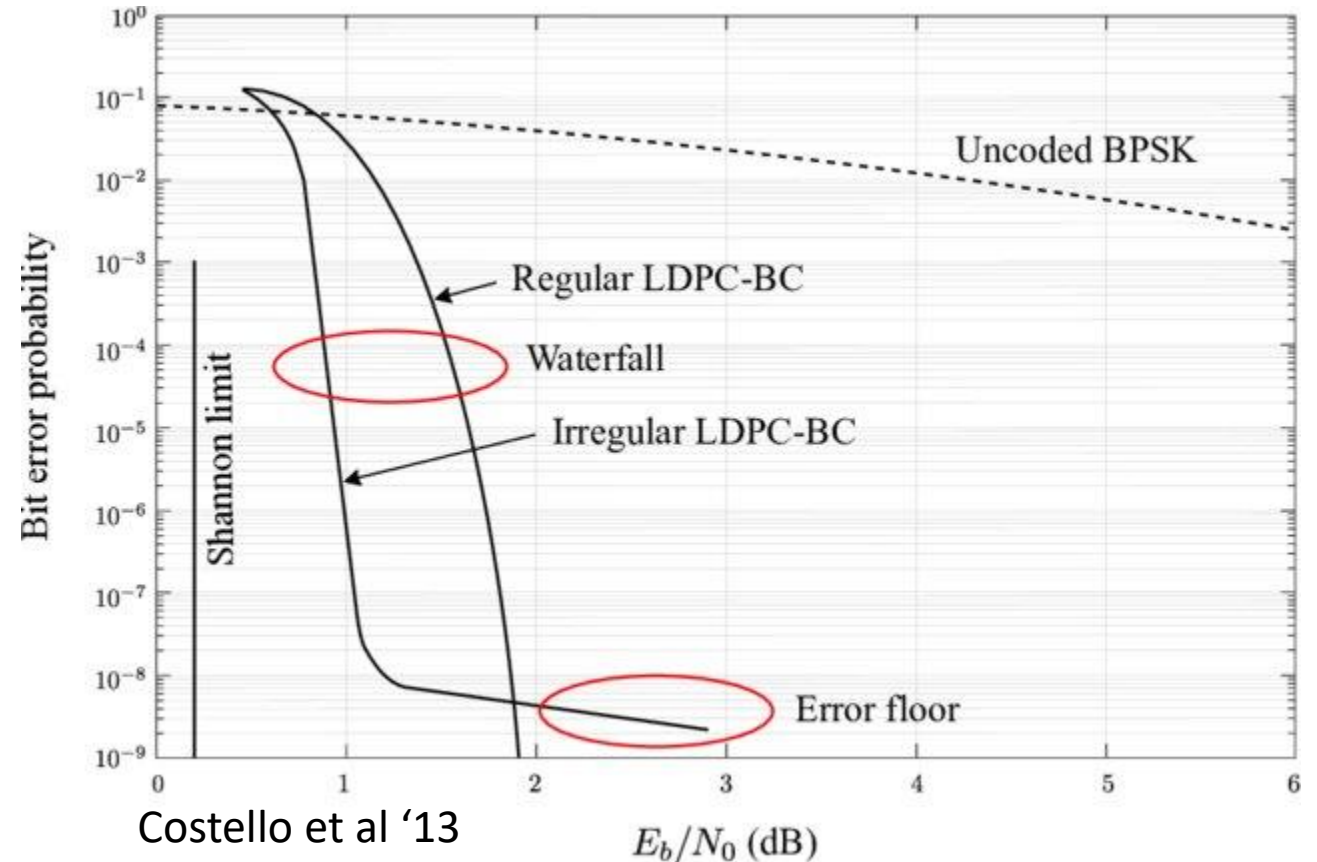


# Scaling Laws

Not unique to machine learning models.

- **Note:** often have multiple “regimes”
- **Example:** LDPC and other codes

“Waterfall” regime,  
“Error floor” regime



# Scaling: Setup

Kaplan et al '20

Measurement units:

- **Compute:** FLOPs
- **Model size:** parameters
- **Data:** tokens
  
- Ranges:
- **Model size :** 768 to 1.5B (non-embedding) parameters
- **Data:** 22M to 23B tokens

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## Scaling Laws for Neural Language Models

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<b>Jared Kaplan *</b> Johns Hopkins University, OpenAI jaredk@jhu.edu		<b>Sam McCandlish*</b> OpenAI sam@openai.com	
<b>Tom Henighan</b> OpenAI henighan@openai.com	<b>Tom B. Brown</b> OpenAI tom@openai.com	<b>Benjamin Chess</b> OpenAI bchess@openai.com	<b>Rewon Child</b> OpenAI rewon@openai.com
<b>Scott Gray</b> OpenAI scott@openai.com	<b>Alec Radford</b> OpenAI alec@openai.com	<b>Jeffrey Wu</b> OpenAI jeffwu@openai.com	<b>Dario Amodei</b> OpenAI damodei@openai.com

# Compute: FLOPS

FLOPs: a measure of computing performance

- “floating point operations per second”
- Our neural network operations involve adding and multiplying real numbers → flops
  - Note: standard approach 32 bit floating point
  - **Popular area of research:** smaller precision or mixed precision training, inference, or both

September 2022	\$0.02	\$0.02	<a href="#">RTX 4090</a>	Nvidia's <a href="#">RTX 4090</a> is listed as having a peak performance of 82.6 TFLOPS (1.32 PFLOPS at 8-bit precision) at a retail price of \$1599. <sup>[87]</sup>
May 2023	\$0.01	\$0.01	<a href="#">Radeon RX 7600</a>	AMD's <a href="#">RX 7600</a> is listed as having a peak performance of 21.5 TFLOPS at a retail price of \$269. <sup>[88]</sup>

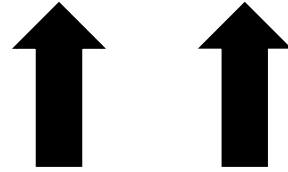
Wiki

# Scaling: Power Laws

How to model relationships measured?

- Power laws

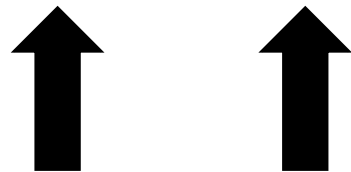
$$f(x) = ax^{-k}$$



**Coefficient** **Exponent**

- In our case, for model size and training to convergence,

$$L(N) = (N_c/N)^{\alpha_N}; \quad \alpha_N \sim 0.076, \quad N_c \sim 8.8 \times 10^{13}$$

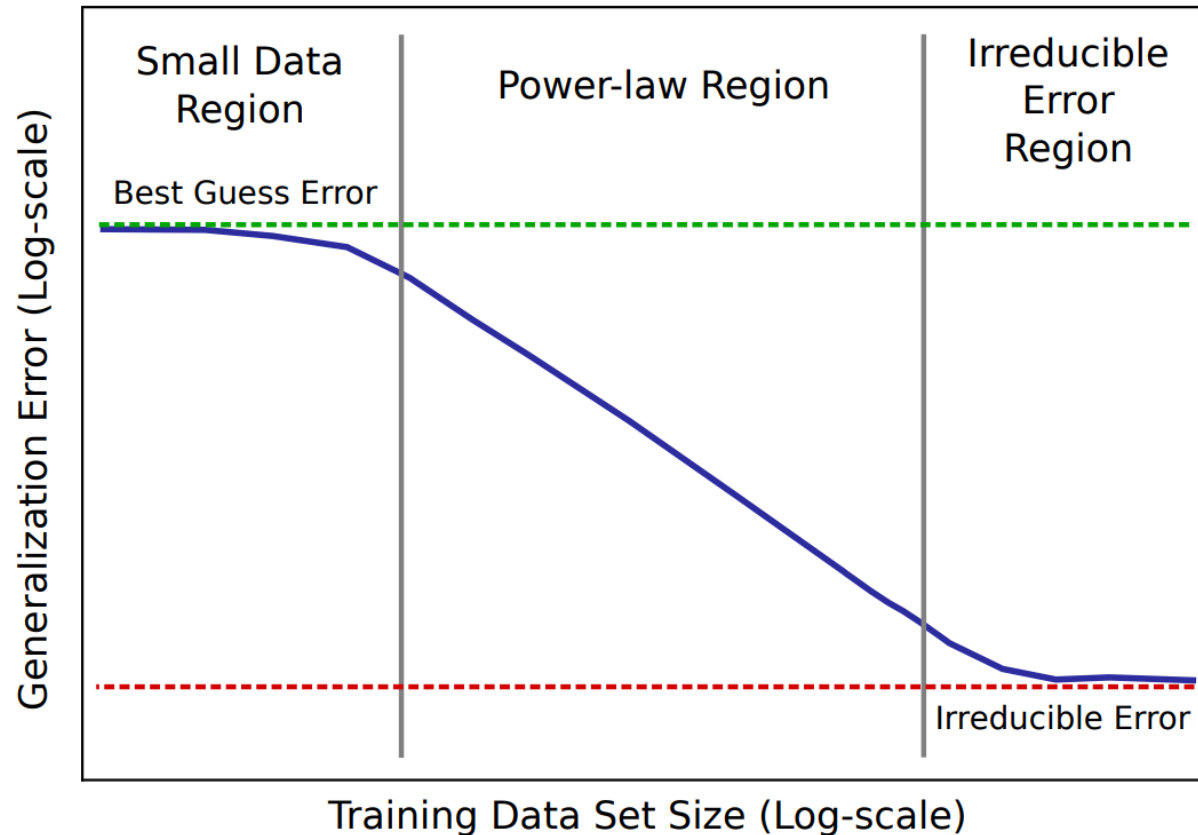


**Coefficient** **Exponent**

# Scaling: Power Laws

Not a new idea. For data: hypothetical power-law like scaling

- **Note:** different regimes



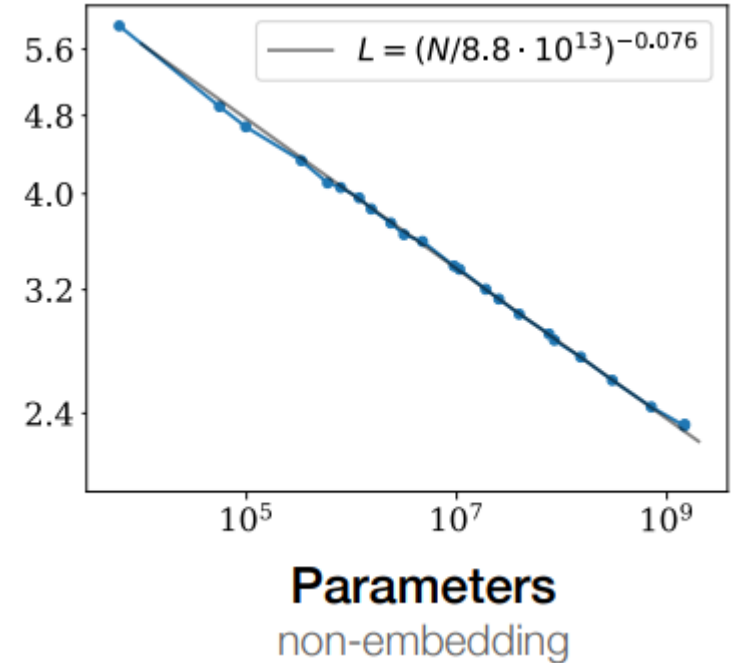
# Scaling: Varying the Model Size

Let's see this in detail.

Kaplan et al '20. Fix the dataset (large).

- **Vary model size:** 22M to 23B tokens
- Measure test loss
- Fit the curve as before:

$$L(N) = (N_c/N)^{\alpha_N} ; \quad \alpha_N \sim 0.076, \quad N_c \sim 8.8 \times 10^{13}$$

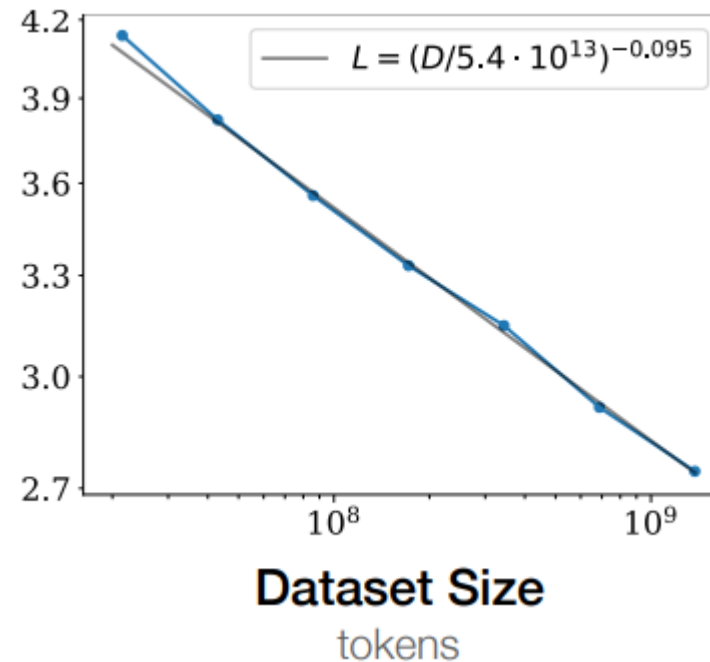


# Scaling: Varying the Dataset

Same idea, but for data.

Fix the model size (large).

- **Vary Data:** 22M to 23B tokens
- Measure test loss
- Again fit a curve



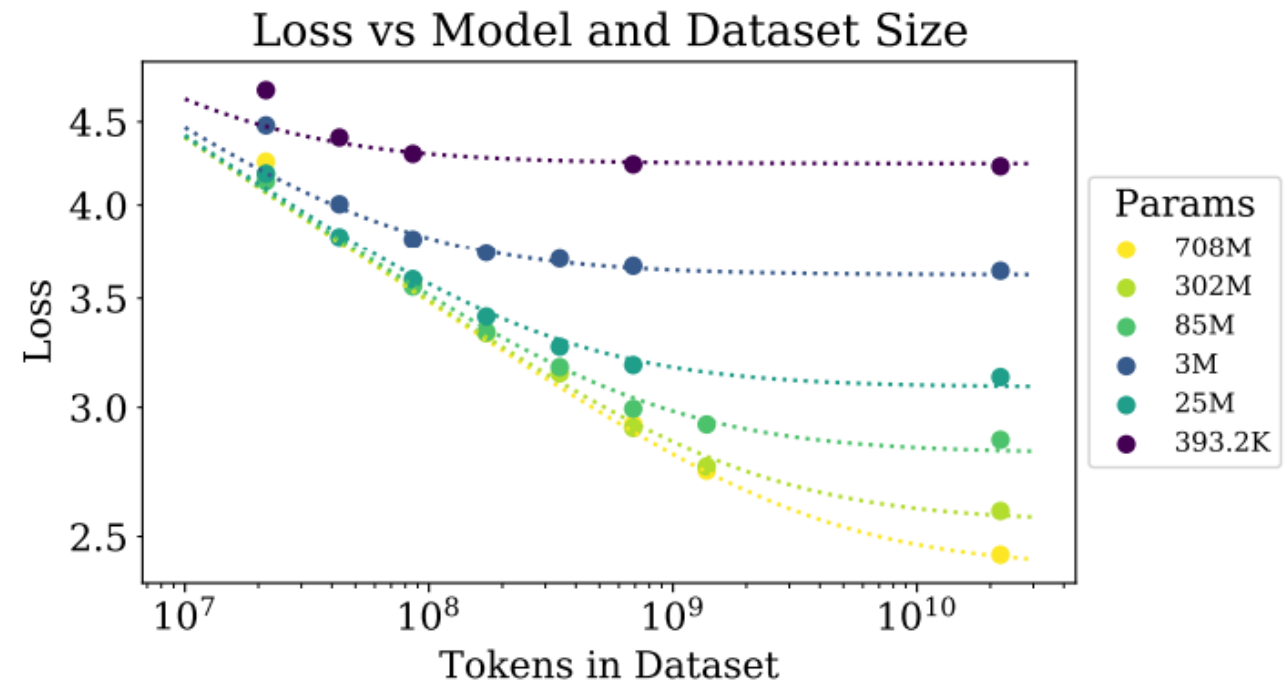
$$L(D) = (D_c/D)^{\alpha_D}; \quad \alpha_D \sim 0.095, \quad D_c \sim 5.4 \times 10^{13} \text{ (tokens)}$$

# Scaling: Interactions

What about the effect of both model size and data?

- **Why?** Need to figure out what to prioritize: get more data or increase the model size?
  - “as we increase the model size, we should increase the dataset size sublinearly according to  $D \propto N^{\alpha_N/\alpha_D} \sim N^{0.74}$ ”

$$L(N, D) = \left[ \left( \frac{N_c}{N} \right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D} \right]^{\alpha_D}$$

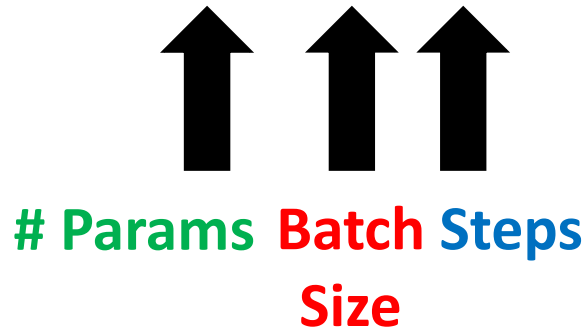




# Scaling: Compute

How much compute do we need?

- **Note:** not independent of the data/model size!
- Rough equation:  $C = 6 N \times B \times S$



- C is a direct function of model size.
  - Batch size varies (existing heuristics for optimal batch size).
  - Steps depend on stopping rules

# Scaling: Compute

What are the interactions?

- Using the **critical batch size** (optimizes the speed/efficiency tradeoff).

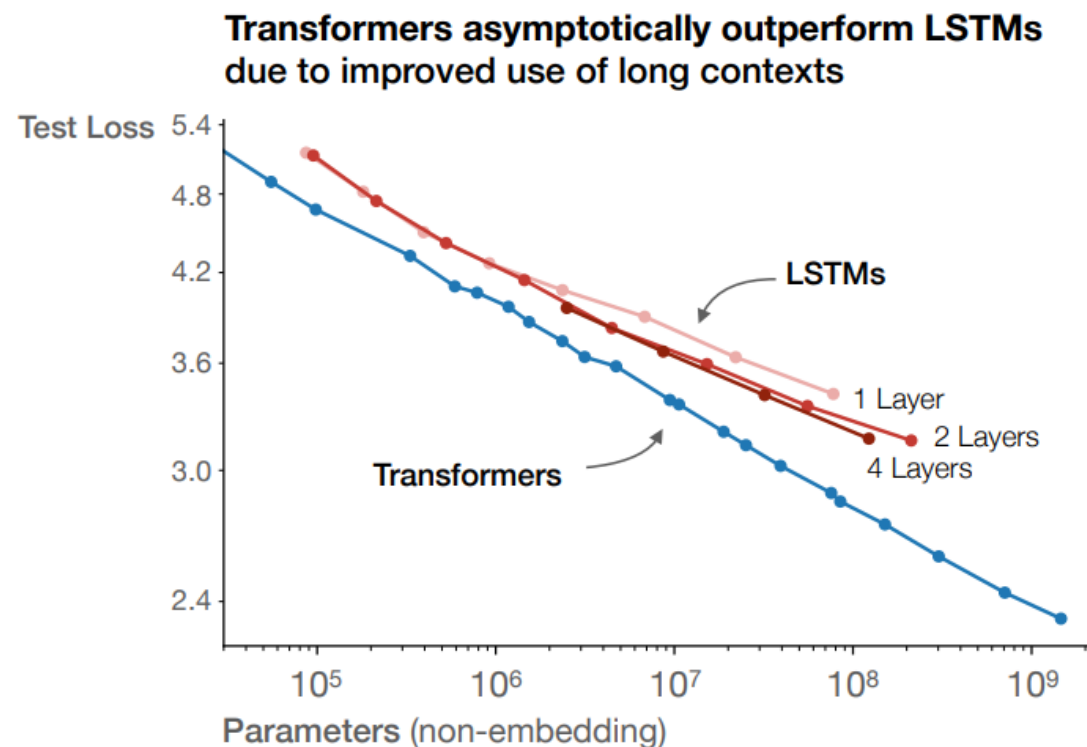
$$N \propto C^{\alpha_C^{\min}/\alpha_N}, \quad B \propto C^{\alpha_C^{\min}/\alpha_B}, \quad S \propto C^{\alpha_C^{\min}/\alpha_S}, \quad D = B \cdot S$$

- Empirically optimal results:  $N \propto C^{0.73}$ ,  $B \propto C^{0.24}$ , and  $S \propto C^{0.03}$
- “As the computational budget  $C$  increases, it should be spent primarily on larger models, without dramatic increases in training time or dataset size”

# Scaling: Architectures

What about choosing various architectures?

- Compare **transformers** vs **LSTMs**
- Change parameter counts, #layers
  - Fixed dataset (WebText2)
- Transformers win here
  - Some recent work challenges this



# Scaling: Predicting

All of this requires huge numbers of training runs...

- But, if the laws are reliable, can:
- Train smaller models,
- Obtain a scaling law,
- Make design decisions based on this law.





# Break & Questions

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- What are laws and why, regimes, idealized versions, initial findings from Kaplan et al

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# Scaling: How Universal Is This?

Kaplan et al made certain choices,

- Results used early stopping, etc.
- One particular learning rate schedule
- Scaling law results may change with different choices!
- Hoffman et al '22: another exploration with **different results**.



## Training Compute-Optimal Large Language Models

Jordan Hoffmann\*, Sebastian Borgeaud\*, Arthur Mensch\*, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals and Laurent Sifre\*

\*Equal contributions

We investigate the optimal model size and number of tokens for training a transformer language model under a given compute budget. We find that current large language models are significantly under-trained, a consequence of the recent focus on scaling language models whilst keeping the amount of training data constant. By training over 400 language models ranging from 70 million to over 16 billion parameters on 5 to 500 billion tokens, we find that for compute-optimal training, the model size and the number of training tokens should be scaled equally: for every doubling of model size the number of training tokens should also be doubled. We test this hypothesis by training a predicted compute-optimal model, *Chinchilla*, that uses the same compute budget as *Gopher* but with 70B parameters and 4× more data. *Chinchilla* uniformly and significantly outperforms *Gopher* (280B), GPT-3 (175B), Jurassic-1 (178B), and Megatron-Turing NLG (530B) on a large range of downstream evaluation tasks. This also means that *Chinchilla* uses substantially less compute for fine-tuning and inference, greatly facilitating downstream usage. As a highlight, *Chinchilla* reaches a state-of-the-art average accuracy of 67.5% on the MMLU benchmark, greater than a 7% improvement over *Gopher*.

# SL2: Approach #1: Minimum Over Curves

For each number of parameters (range: 70M to 10B),

- Vary # of training steps,
- 4 training sequences, take overall minimum
- **Results:**

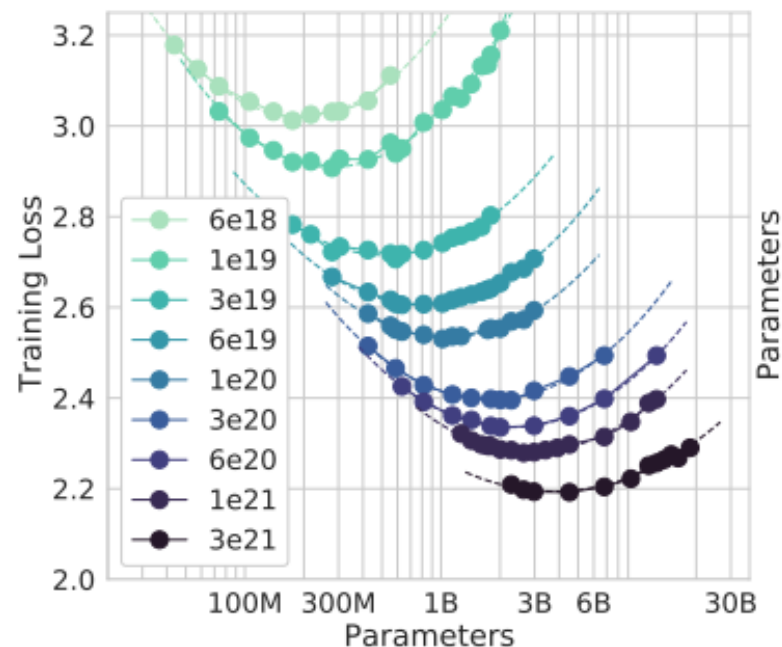
Approach	Coeff. $a$ where $N_{opt} \propto C^a$	Coeff. $b$ where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
<a href="#">Kaplan et al. (2020)</a>	0.73	0.27



# SL2: Approach #2: IsoFLOP Profiles

Vary model size for a fixed set of FLOP counts

- Obtain best performance for fixed FLOP at various models, use to obtain curve



Approach	Coeff. $a$ where $N_{opt} \propto C^a$	Coeff. $b$ where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
2. IsoFLOP profiles	0.49 (0.462, 0.534)	0.51 (0.483, 0.529)
<a href="#">Kaplan et al. (2020)</a>	0.73	0.27

# SL2: Approach #3: Direct Fitting

Fit the function (inspired by classical risk bounds)

$$\hat{L}(N, D) \triangleq E + \frac{A}{N^\alpha} + \frac{B}{D^\beta}$$

## Results:

Approach	Coeff. $a$ where $N_{opt} \propto C^a$	Coeff. $b$ where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
2. IsoFLOP profiles	0.49 (0.462, 0.534)	0.51 (0.483, 0.529)
3. Parametric modelling of the loss	0.46 (0.454, 0.455)	0.54 (0.542, 0.543)
<a href="#">Kaplan et al. (2020)</a>	0.73	0.27

# SL2 Conclusion

Note all results fairly similar:

Approach	Coeff. $a$ where $N_{opt} \propto C^a$	Coeff. $b$ where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
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<a href="#">Kaplan et al. (2020)</a>	0.73	0.27

“All three approaches suggest that as compute budget increases, model size and the amount of training data should be increased in approximately equal proportions”

- Quite different from Kaplan et al!

# SL2 Chinchilla

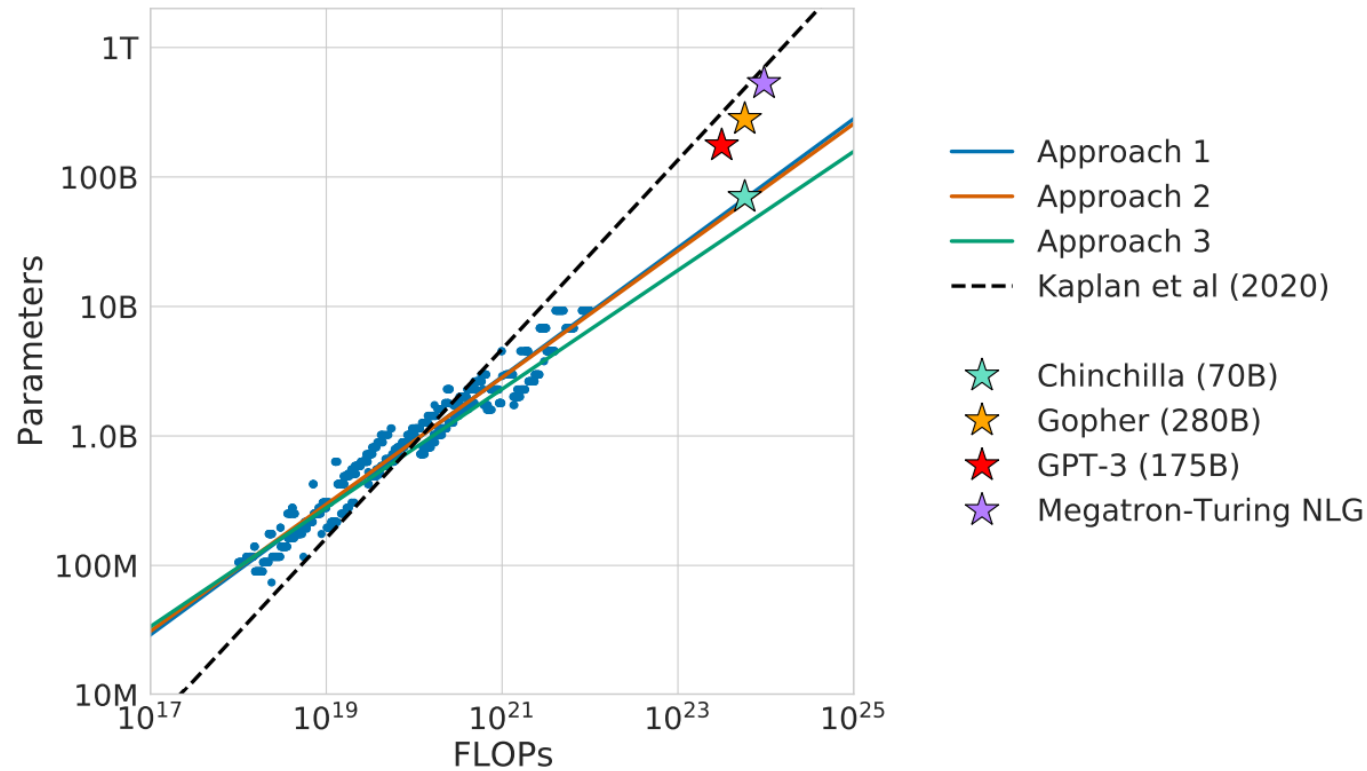
What are the implications?

- For a particular (large) compute budget, very massive models are not the way to go,
- “**Smaller**” is better.
- Chinchilla model: 70B parameters, 1.4T tokens
  - Comparison against Gopher: same compute in FLOPs, but much larger

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Random	25.0%
Average human rater	34.5%
GPT-3 5-shot	43.9%
<i>Gopher</i> 5-shot	60.0%
<b><i>Chinchilla</i> 5-shot</b>	<b>67.6%</b>
Average human expert performance	89.8%

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# Back to Universality

Even if we could estimate these law parameters correctly, are we stuck with the implications?

- Maybe not!

- Better **data** via pruning

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**Beyond neural scaling laws:  
beating power law scaling via data pruning**

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Ben Sorscher<sup>\*1</sup>

Robert Geirhos<sup>\*2</sup>

Shashank Shekhar<sup>3</sup>

Surya Ganguli<sup>1,3§</sup>

Ari S. Morcos<sup>3§</sup>

<sup>\*</sup>equal contribution

<sup>1</sup>Department of Applied Physics, Stanford University

<sup>2</sup>University of Tübingen

<sup>3</sup>Meta AI (FAIR)

<sup>§</sup>Joint senior authors

# Bibliography

- - Villalobos et al '22a: Pablo Villalobos, Jaime Sevilla, Tamay Besiroglu, Lennart Heim, Anson Ho, Marius Hobbhahn, "Machine Learning Model Sizes and the Parameter Gap" (<https://arxiv.org/abs/2207.02852>)
- - Villalobos et al '22b: Pablo Villalobos, Jaime Sevilla, Lennart Heim, Tamay Besiroglu, Marius Hobbhahn, Anson Ho, "Will we run out of data? An analysis of the limits of scaling datasets in Machine Learning" (<https://arxiv.org/abs/2211.04325>)
- - Kaplan et al '20: Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B. Brown, Benjamin Chess, Rewon Child, Scott Gray, Alec Radford, Jeffrey Wu, Dario Amodei, "Scaling Laws for Neural Language Models" (<https://arxiv.org/abs/2001.08361>)
- - Costello et al '13: Daniel J. Costello, Jr., Lara Dolecek, Thomas E. Fuja, Jörg Kliewer, David G. M. Mitchell, Roxana Smarandache, "Spatially Coupled Sparse Codes on Graphs - Theory and Practice" (<https://ieeexplore.ieee.org/abstract/document/6852099>)
- - Hestness et al '17: Joel Hestness, Sharan Narang, Newsha Ardalani, Gregory Diamos, Heewoo Jun, Hassan Kianinejad, Md. Mostofa Ali Patwary, Yang Yang, Yanqi Zhou, "Deep Learning Scaling is Predictable, Empirically" (<https://arxiv.org/abs/1712.00409>)
- - Hoffman et al '22: Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals and Laurent Sifre, "Training Compute-Optimal Large Language Models" (<https://arxiv.org/pdf/2203.15556.pdf>)
- - Sorscher et al '22: Ben Sorscher, Robert Geirhos, Shashank Shekhar, Surya Ganguli, Ari S. Morcos, "Beyond neural scaling laws: beating power law scaling via data pruning" (<https://arxiv.org/abs/2206.14486>)





**Thank You!**