CS 839: Foundation Models
Scaling & Scaling Laws

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Oct. 31, 2024

Announcements

- **Logistics:**
  - Note slight change: scaling today, diffusion models Tuesday
  - HW2 due today. HW3 due **Nov. 12**
  - Project. Dates: **Nov. 21**: proposal, **Dec. 13**: report
  - Presentation: **Nov:** 12,14,19,21,26 **Dec:** 3,5
    - Warning: will ask for volunteers for days with 4 groups to shift to Dec. 5
  - Presentation proposal due On **Nov. 7!**
- Class roadmap:

Thursday Oct. 31	Scaling & Scaling Laws
Tuesday Nov. 5	Diffusion Models
Thursday Nov. 7	Security, Privacy, Toxicity + Future Areas

Outline

- **Scaling Laws Intro**
  - What are laws and why, regimes, idealized versions, initial findings from Kaplan et al
- **Scaling Laws Revised**
  - Additional methods, new results, Chinchilla and related hypotheses
- **Beyond Scaling Laws**
  - Data pruning and others

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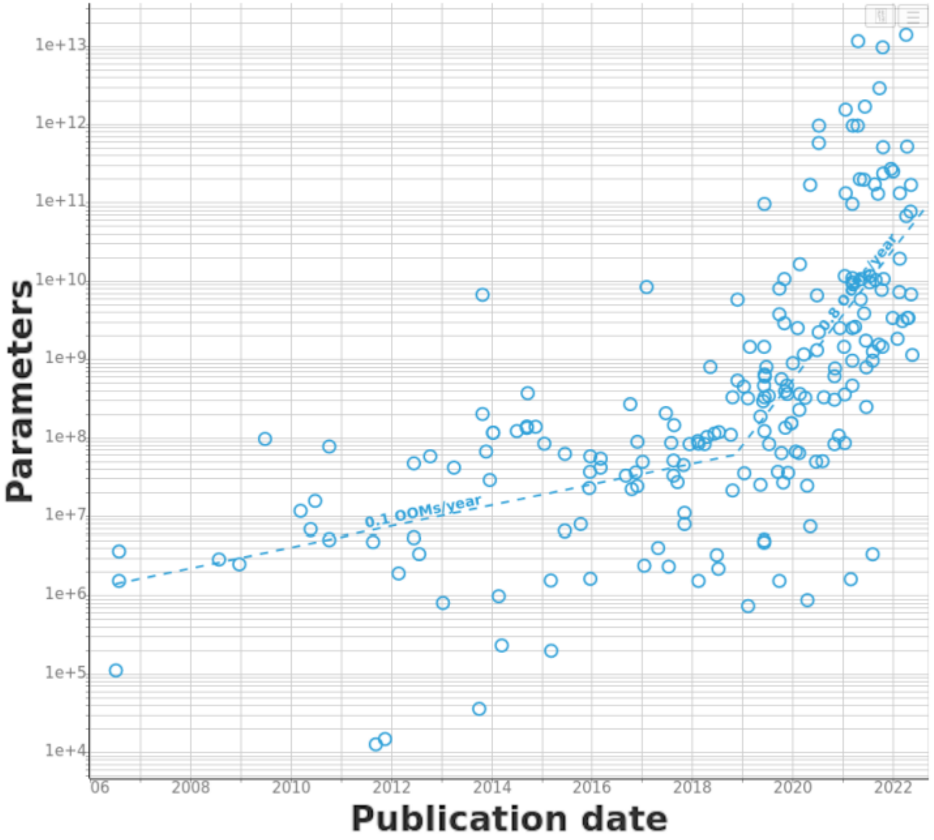
Trends: Models

Models have gotten bigger



Villalobos et al ‘22

Trends: **Compute**

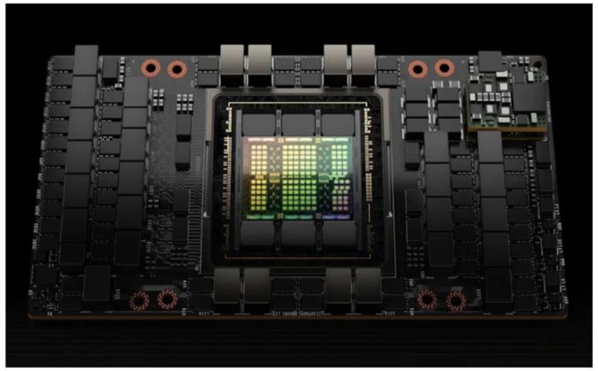
Compute has gotten bigger

Startup Builds Supercomputer with 22,000 Nvidia's H100 Compute GPUs

By Anton Shilov published July 05, 2023

The world's second highest performing supercomputer.

Comments (4)



(Image credit: Nvidia)

Inflection AI, a new startup found by the former head of deep mind and backed

https://www.tomshardware.com/news/startup-builds-supercomputer-with-22000-nvidias-h100-compute-gpus

Tesla's $300 Million AI Cluster Is Going Live Today

By Anton Shilov published August 28, 2023

Tesla is about to flip the switch on its new AI cluster, featuring 10,000 Nvidia H100 compute GPUs.

Comments (23)

(Image credit: Shutterstock)

https://www.tomshardware.com/news/teslas-dollar300-million-ai-cluster-is-going-live-today

Home › News › Components › Graphics Cards

Zuckerberg's Meta Is Spending Billions to Buy 350,000 Nvidia H100 GPUs

In total, Meta will have the compute power equivalent to 600,000 Nvidia H100 GPUs to help it develop next-generation AI, says CEO Mark Zuckerberg.

By Michael Kan January 18, 2024

(David Paul Morris/Bloomberg via Getty Images)

Mark Zuckerberg plans on acquiring 350,000 Nvidia H100 GPUs to help Meta build a next-generation AI that possesses human-like intelligence

https://www.pcmag.com/news/zuckerbergs-meta-is-spending-billions-to-buy-350000-nvidia-h100-gpus

Trends: **Data**

Datasets have gotten bigger

Dataset Name	Brief description	Preprocessing	Instances	Format	Default Task	Created (updated)
Statlog (Image Segmentation) Dataset	The instances were drawn randomly from a database of 7 outdoor images and hand-segmented to create a classification for every pixel.	Many features calculated.	2310	Text	Classification	1990
Caltech 101	Pictures of objects.	Detailed object outlines marked.	9146	Images	Classification, object recognition.	2003
LabelMe	Annotated pictures of scenes.	Objects outlined.	187,240	Images, text	Classification, object detection	2005
Caltech-256	Large dataset of images for object classification.	Images categorized and hand-sorted.	30,607	Images, Text	Classification, object detection	2007
ImageNet	Labeled object image database, used in the ImageNet Large Scale Visual Recognition Challenge	Labeled objects, bounding boxes, descriptive words, SIFT features	14,197,122	Images, text	Object recognition, scene recognition	2009 (2014)

wiki

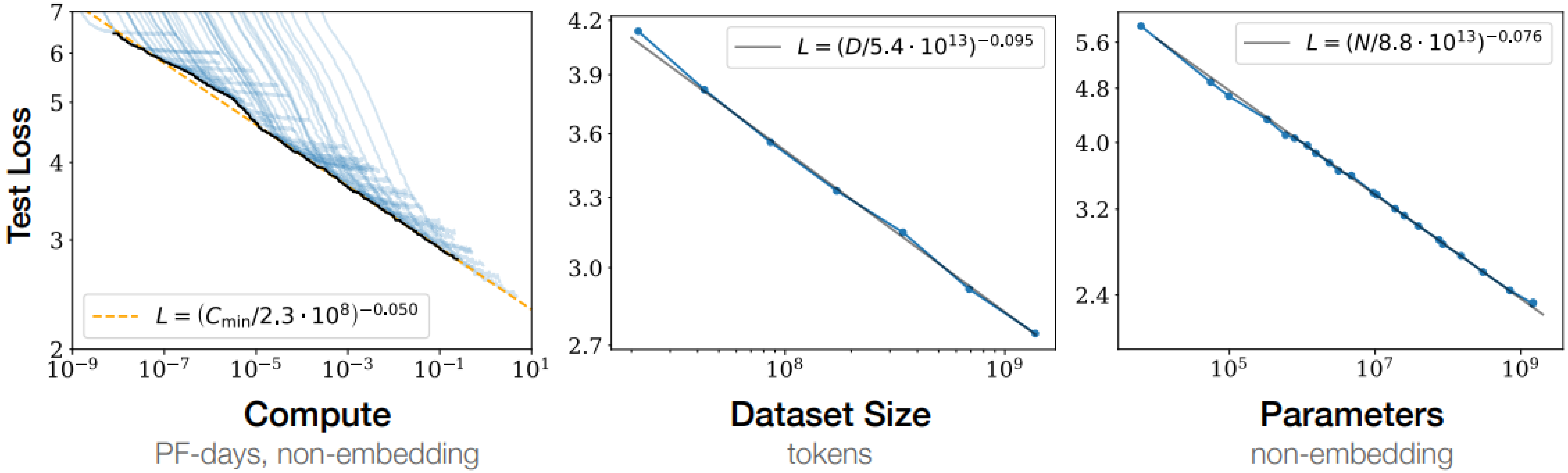
Model	Stock of data (#words)	Growth rate
Recorded speech	1.46e17 [3.41e16; 4.28e17]	5.2% [4.95%; 5.2%]
Internet users	2.01e15 [6.47e14; 6.28e15]	8.14% [7.89%; 8.14%]
Popular platforms	4.41e14 [1.21e14; 1.46e15]	8.14% [7.89%; 8.14%]
CommonCrawl	9.62e13 [4.45e13; 2.84e14]	16.68% [16.41%; 16.68%]
Indexed websites	2.21e14 [5.16e13; 6.53e15]	NA
Aggregated model	**7.41e14 [6.85e13; 7.13e16]**	**7.15% [6.41%; 17.49%]**

Villalobos et al, "Will we run out of data? An analysis of the limits of scaling datasets in Machine Learning"

Scaling Laws

We want to understand

- How performance scales with these quantities…
- And how they **interact!**



Kaplan et al '20

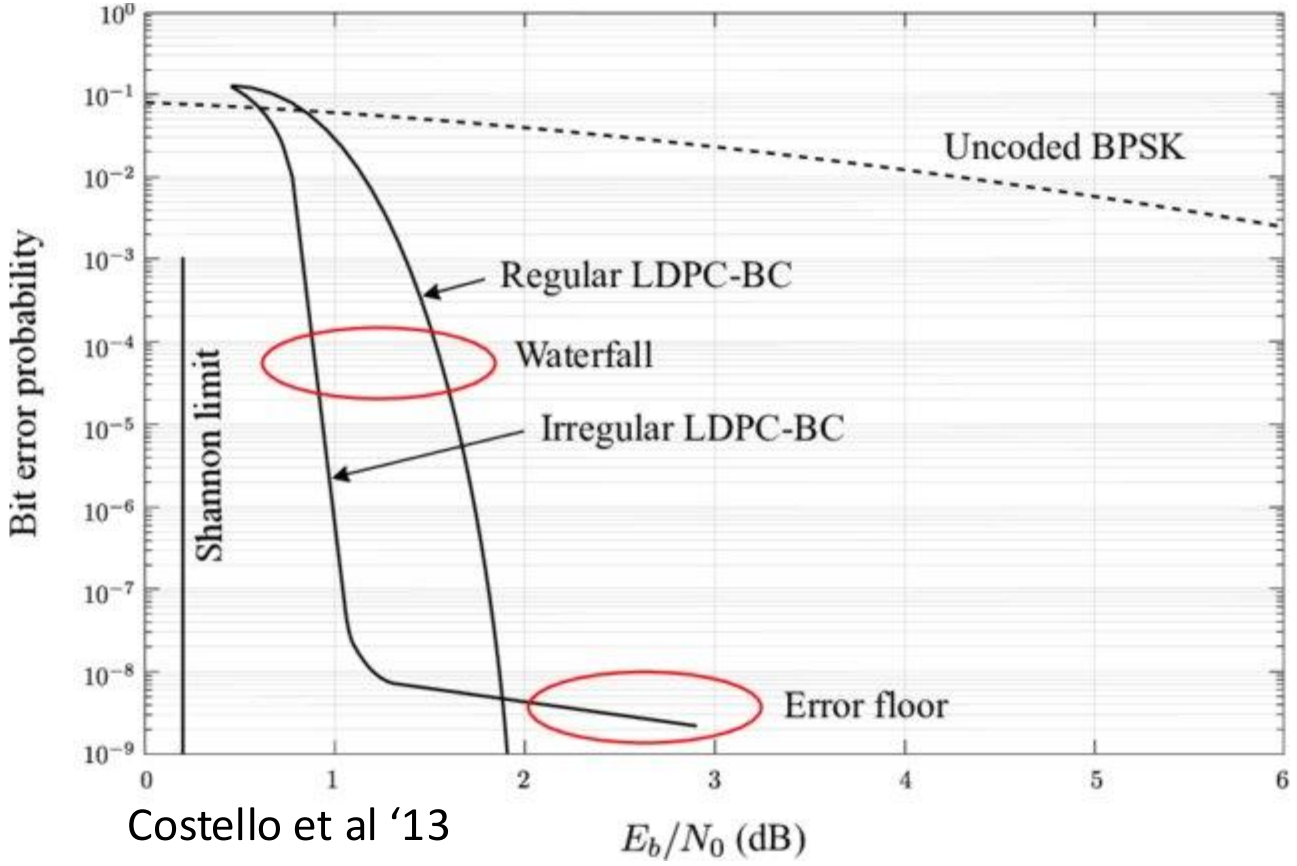
Scaling Laws

Not unique to machine learning models.

- **Note:** often have multiple "regimes"
- **Example:** LDPC and other codes

"Waterfall" regime,

"Error floor" regime



Costello et al '13

Scaling: **Setup**

Kaplan et al '20

Measurement units:

- **Compute**: FLOPs
- **Model size**: parameters
- **Data**: tokens

Scaling Laws for Neural Language Models

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- Ranges:
- **Model size** : 768 to 1.5B (non-embedding) parameters
- **Data**: 22M to 23B tokens

Compute: **FLOPS**

FLOPs: a measure of computing performance

- "**f**loating **p**oint **o**perations **p**er **s**econd"
- Our neural network operations involve adding and multiplying real numbers → flops
  - Note: standard approach 32 bit floating point
  - **Popular area of research**: smaller precision or mixed precision training, inference, or both

September 2022	$0.02	$0.02	RTX 4090	Nvidia's RTX 4090 is listed as having a peak performance of 82.6 TFLOPS (1.32 PFLOPS at 8-bit precision) at a retail price of $1599.[87]
May 2023	$0.01	$0.01	Radeon RX 7600	AMD's RX 7600 is listed as having a peak performance of 21.5 TFLOPS at a retail price of $269.[88]

Wiki

Scaling: **Power Laws**

How to model relationships measured?

- Power laws

$$f(x) = ax^{-k}$$

Coefficient Exponent

- In our case, for model size and training to convergence,

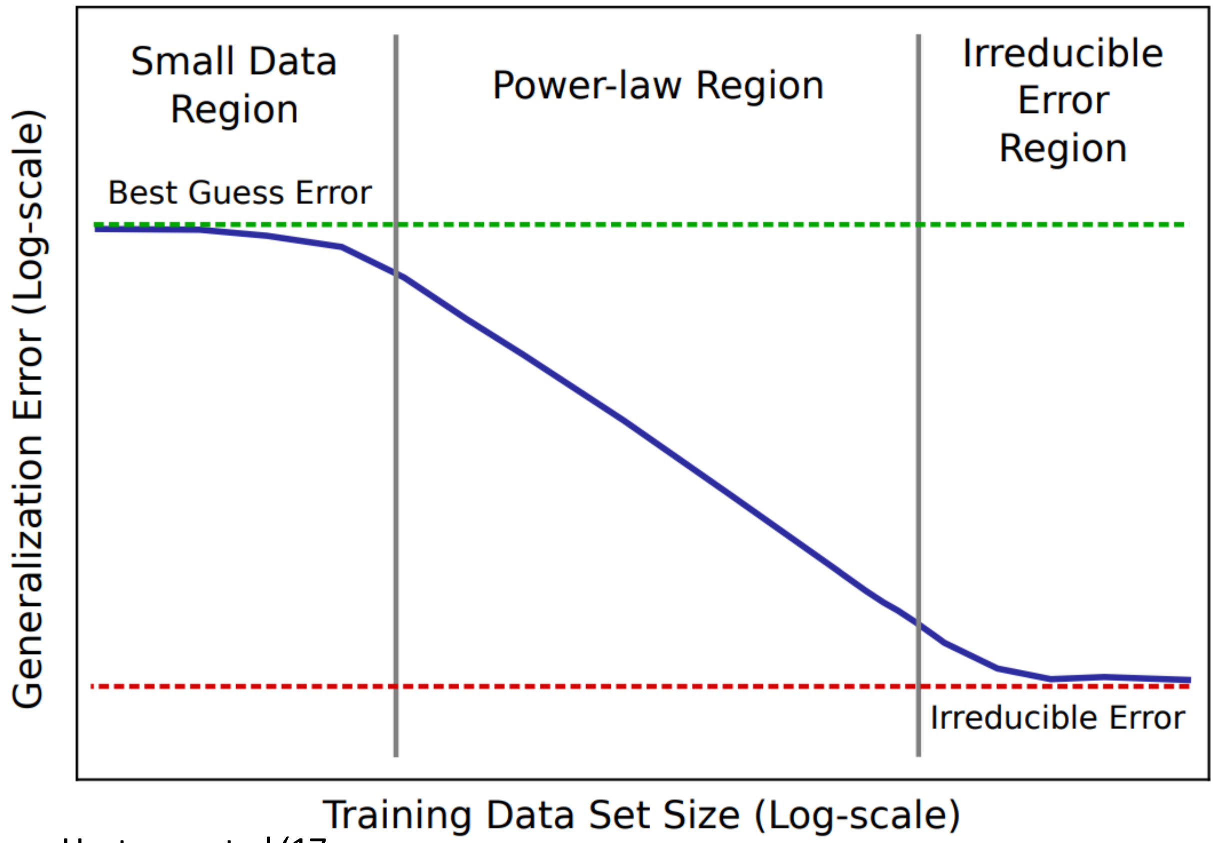
$$L(N) = (N_c/N)^{\alpha_N}; \quad \alpha_N \sim 0.076, \quad N_c \sim 8.8 \times 10^{13}$$

Coefficient Exponent

Scaling: Power Laws

Not a new idea. For data: hypothetical power-law like scaling

- **Note:** different regimes



Hestness et al ‘17

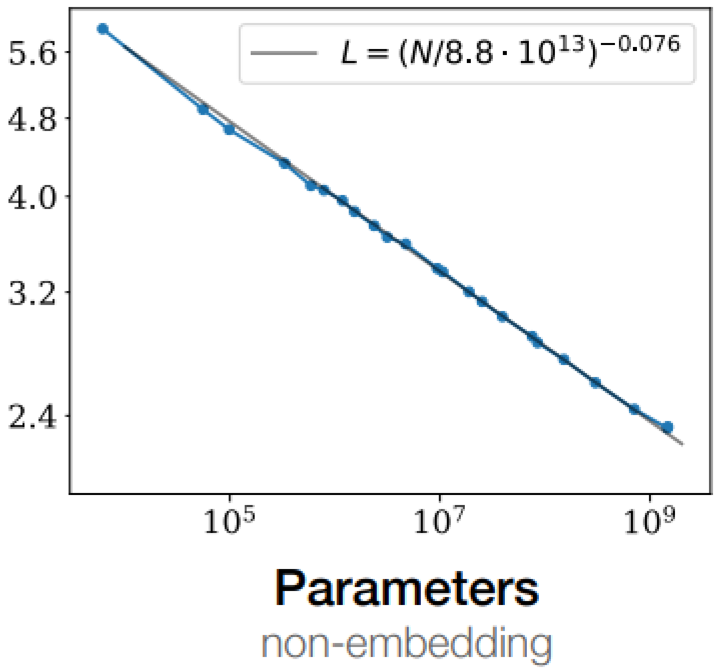
Scaling: **Varying the Model Size**

Let's see this in detail.

Kaplan et al '20. Fix the dataset (large).

- **Vary model size**: 769 to 1.5B
- Measure test loss

- Fit the curve as before:



$$L(N) = (N_c/N)^{\alpha_N}\ ; \ \ \alpha_N \sim 0.076, \quad N_c \sim 8.8 \times 10^{13}$$

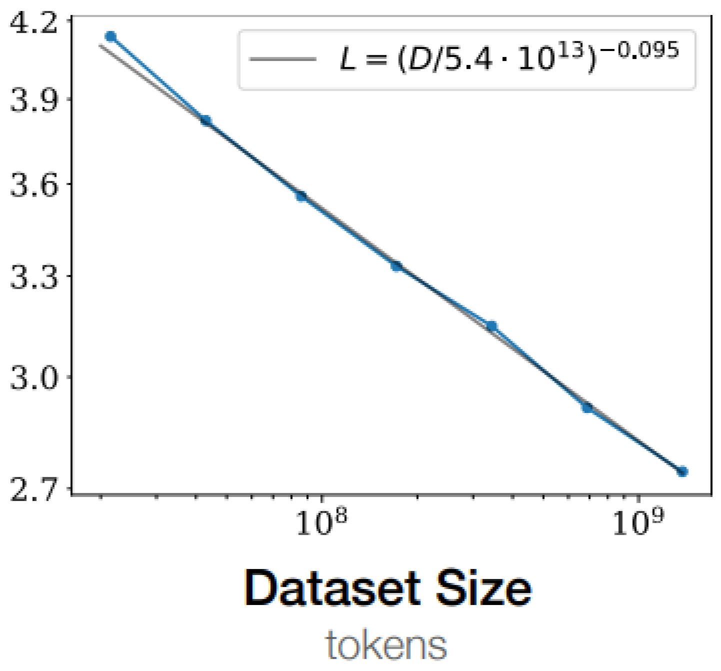
Scaling: **Varying the Dataset**

Same idea, but for data.

Fix the model size (large).

- **Vary Data**: 22M to 23B tokens

- Measure test loss
- Again fit a curve



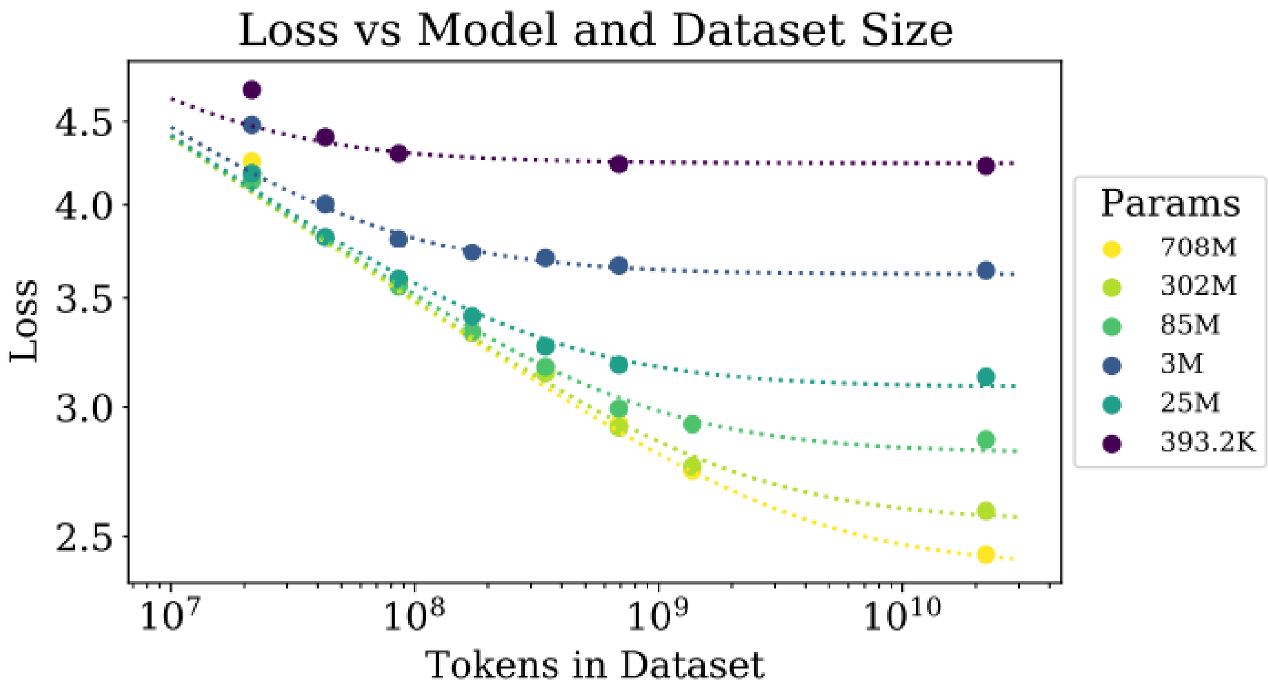
$$L(D) = \left(D_c/D\right)^{\alpha_D};\ \ \alpha_D \sim 0.095, \quad D_c \sim 5.4 \times 10^{13}\ \text{(tokens)}$$

Scaling: **Interactions**

What about the effect of both model size and data?

- **Why?** Need to figure out what to prioritize: get more data or increase the model size?
  - "as we increase the model size, we should increase the dataset size sublinearly according to $D \propto N^{\alpha\_N/\alpha\_D} \sim N^{0.74}$"

$$L(N, D) = \left[\left(\frac{N_c}{N} \right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D} \right]^{\alpha_D}$$



Scaling: **Compute**

How much compute do we need?

- **Note:** not independent of the data/model size!
- Rough equation: C = 6 N x B x S

 # Params **Batch Size** **Steps**

- C is a direct function of model size.
  - Batch size varies (existing heuristics for optimal batch size).
  - Steps depend on stopping rules

Scaling: **Compute**

What are the interactions?

- Using the **critical batch size** (optimizes the speed/efficiency tradeoff).

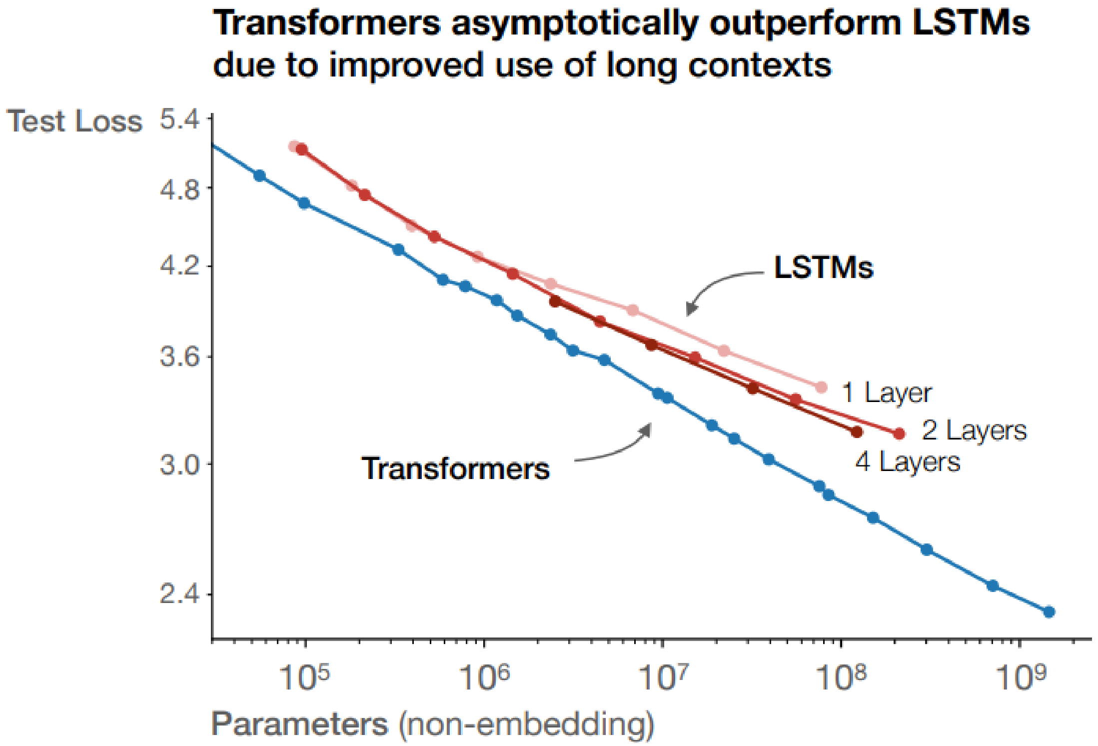
$$N \propto C^{\alpha_C^{\min}/\alpha_N}, \quad B \propto C^{\alpha_C^{\min}/\alpha_B}, \quad S \propto C^{\alpha_C^{\min}/\alpha_S}, \quad D = B \cdot S$$

- Empirically optimal results: $N \propto C^{0.73}$, $B \propto C^{0.24}$, and $S \propto C^{0.03}$
- "As the computational budget C increases, it should be spent primarily on larger models, without dramatic increases in training time or dataset size"

Scaling: **Architectures**

What about choosing various architectures?

- Compare **transformers** vs **LSTMs**
- Change parameter counts, #layers
  - Fixed dataset (WebText2)



- Transformers win here
  - Some recent work challenges this

Scaling: **Predicting**

All of this requires huge numbers of training runs…

- But, if the laws are reliable, can:

- Train smaller models,
- Obtain a scaling law,
- Make design decisions based on this law.

Break & Questions

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Scaling: **How Universal Is This?**

Kaplan et al made certain choices,

- Results used early stopping, etc.
- One particular learning rate schedule

- Scaling law results may change with different choices!

- Hoffman et al '22: another exploration with **different results**.

DeepMind

Training Compute-Optimal Large Language Models

Jordan Hoffmann*, Sebastian Borgeaud*, Arthur Mensch*, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals and Laurent Sifre*

*Equal contributions

We investigate the optimal model size and number of tokens for training a transformer language model under a given compute budget. We find that current large language models are significantly undertrained, a consequence of the recent focus on scaling language models whilst keeping the amount of training data constant. By training over 400 language models ranging from 70 million to over 16 billion parameters on 5 to 500 billion tokens, we find that for compute-optimal training, the model size and the number of training tokens should be scaled equally: for every doubling of model size the number of training tokens should also be doubled. We test this hypothesis by training a predicted compute-optimal model, *Chinchilla*, that uses the same compute budget as *Gopher* but with 70B parameters and 4× more more data. *Chinchilla* uniformly and significantly outperforms *Gopher* (280B), GPT-3 (175B), Jurassic-1 (178B), and Megatron-Turing NLG (530B) on a large range of downstream evaluation tasks. This also means that *Chinchilla* uses substantially less compute for fine-tuning and inference, greatly facilitating downstream usage. As a highlight, *Chinchilla* reaches a state-of-the-art average accuracy of 67.5% on the MMLU benchmark, greater than a 7% improvement over *Gopher*.

SL2: **Approach #1: Minimum Over Curves**

For each number of parameters (range: 70M to 10B),

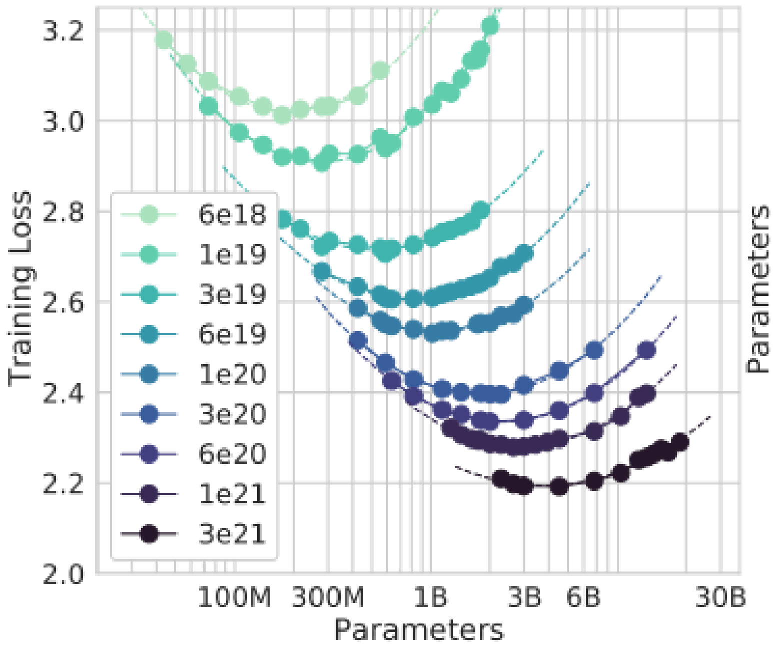
- Vary # of training steps,
- 4 training sequences, take overall minimum
- **Results**:

Approach	Coeff. a where $N_{opt} \propto C^a$	Coeff. b where $D_{opt} \propto C^b$
1. Minimum over training curves	$0.50\ (0.488, 0.502)$	$0.50\ (0.501, 0.512)$
Kaplan et al. (2020)	0.73	0.27

SL2: **Approach #2: IsoFLOP Profiles**

Vary model size for a fixed set of FLOP counts

- Obtain best performance for fixed FLOP at various models, use to obtain curve



Approach	Coeff. a where $N_{opt} \propto C^a$	Coeff. b where $D_{opt} \propto C^b$
1. Minimum over training curves	$0.50\ (0.488, 0.502)$	$0.50\ (0.501, 0.512)$
2. IsoFLOP profiles	$0.49\ (0.462, 0.534)$	$0.51\ (0.483, 0.529)$
Kaplan et al. (2020)	0.73	0.27

SL2: **Approach #3: Direct Fitting**

Fit the function (inspired by classical risk bounds)

$$\hat{L}(N, D) \triangleq E + \frac{A}{N^{\alpha}} + \frac{B}{D^{\beta}}$$

Results:

Approach	Coeff. a where $N_{opt} \propto C^a$	Coeff. b where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
2. IsoFLOP profiles	0.49 (0.462, 0.534)	0.51 (0.483, 0.529)
3. Parametric modelling of the loss	0.46 (0.454, 0.455)	0.54 (0.542, 0.543)
Kaplan et al. (2020)	0.73	0.27

SL2 **Conclusion**

Note all results fairly similar:

Approach	Coeff. a where $N_{opt} \propto C^a$	Coeff. b where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
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"All three approaches suggest that as compute budget increases, model size and the amount of training data should be increased in approximately equal proportions"

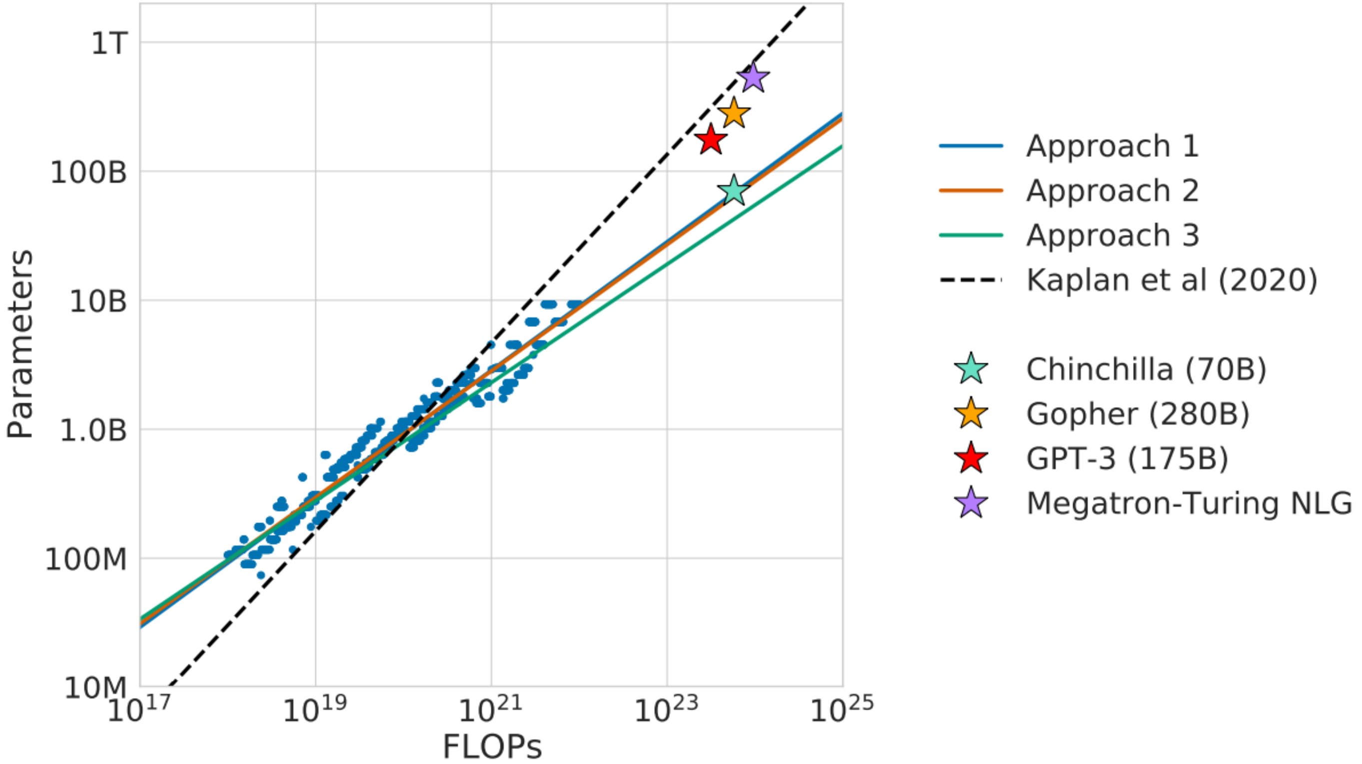
- Quite different from Kaplan et al!

SL2 **Chinchilla**

What are the implications?

- For a particular (large) compute budget, very massive models are not the way to go,
- "**Smaller**" is better.
- Chinchilla model: 70B parameters, 1.4T tokens
    - Comparison against Gopher: same compute in FLOPs, but much larger

Random	25.0%
Average human rater	34.5%
GPT-3 5-shot	43.9%
Gopher 5-shot	60.0%
***Chinchilla* 5-shot**	**67.6%**
Average human expert performance	*89.8%*



Reconciling Differences & Practical Use

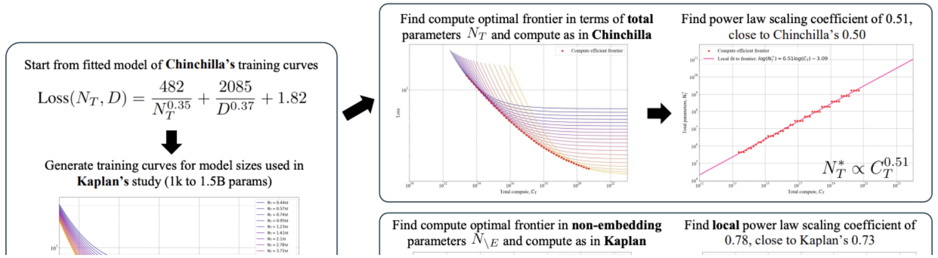
Reconciling Kaplan and Chinchilla Scaling Laws

Tim Pearce *Microsoft Research*

Jinyeop Song *MIT*

Abstract

Kaplan et al. (2020) ('Kaplan') and Hoffmann et al. (2022) ('Chinchilla') studied the scaling behavior of transformers trained on next-token language prediction. These studies produced different estimates for how the number of parameters (N) and training tokens (D) should be set to achieve the lowest possible loss for a given compute budget (C). Kaplan: $N_{\text{optimal}} \propto C^{0.73}$, Chinchilla: $N_{\text{optimal}} \propto C^{0.50}$. This paper finds that much of this discrepancy can be attributed to Kaplan counting non-embedding rather than total parameters, combined with their analysis being performed at small scale. Simulating the Chinchilla study under these conditions produces biased scaling coefficients close to Kaplan's. Hence, this paper reaffirms Chinchilla's scaling coefficients, by explaining the primary cause of Kaplan's original overestimation. As a second contribution, the paper explains differences in the reported relationships between loss and compute. These findings lead us to recommend that future scaling studies use total parameters and compute. [1]



Reproducing some scaling laws results from Chinchilla. Can't get the numbers to match exactly, but can still be used as a rough guide to help determine compute-optimal models. Also contains related utilities for calculating flops and param counts.

[1]:
```python
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
%matplotlib inline
```

params

First some parameter calculations:

[2]:
```python
def gpt_params(seq_len, vocab_size, d_model, num_heads, num_layers):
    """ Given GPT config calculate total number of parameters """
    ffw_size = 4*d_model # in GPT the number of intermediate features is always 4*d_model
    # token and position embeddings
    embeddings = d_model * vocab_size + d_model * seq_len
    # transformer blocks
    attention = 3*d_model**2 + 3*d_model # weights and biases
    attproj = d_model**2 + d_model
    ffw = d_model*(ffw_size) + ffw_size
    ffwproj = ffw_size*d_model + d_model
    layernorms = 2*2*d_model
    # dense
    ln_f = 2*d_model
    dense = d_model*vocab_size # note: no bias here
    # note: embeddings are not included in the param count!
    total_params = num_layers*(attention + attproj + ffw + ffwproj + layernorms) + ln_f + dense
    return total_params

gpt2 = dict(seq_len = 1024, vocab_size = 50257, d_model = 768, num_heads = 12, num_layers = 12)
gpt_params(**gpt2)/1e6
```

[2]: 123.653376

OpenAI reports gpt2 (small) as having 124M params, so this is a match. Also, loading the OpenAI weights into nanoGPT and then calling `model.parameters()` exactly matches the above number and verifies the implementation. Now Chinchilla parameters:

https://github.com/karpathy/nanoGPT/blob/master/scaling_laws.ipynb

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Back to Universality

Even if we could estimate these law parameters correctly, are we stuck with the implications?

- Maybe not!

- Better **data** via pruning

Beyond neural scaling laws: beating power law scaling via data pruning

Ben Sorscher[*1] **Robert Geirhos**[*2] **Shashank Shekhar**[3]

Surya Ganguli[1,3§] **Ari S. Morcos**[3§]

[*]equal contribution
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[2]University of Tübingen
[3]Meta AI (FAIR)
[§]Joint senior authors

Bibliography

- - Villalobos et al '22a: Pablo Villalobos, Jaime Sevilla, Tamay Besiroglu, Lennart Heim, Anson Ho, Marius Hobbhahn, "Machine Learning Model Sizes and the Parameter Gap" (https://arxiv.org/abs/2207.02852)
- - Villalobos et al '22b: Pablo Villalobos, Jaime Sevilla, Lennart Heim, Tamay Besiroglu, Marius Hobbhahn, Anson Ho, "Will we run out of data? An analysis of the limits of scaling datasets in Machine Learning" (https://arxiv.org/abs/2211.04325)
- - Kaplan et al '20: Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B. Brown, Benjamin Chess, Rewon Child, Scott Gray, Alec Radford, Jeffrey Wu, Dario Amodei, "Scaling Laws for Neural Language Models" (https://arxiv.org/abs/2001.08361)
- - Costello et al '13: Daniel J. Costello, Jr., Lara Dolecek, Thomas E. Fuja, Jörg Kliewer, David G. M. Mitchell, Roxana Smarandache, "Spatially Coupled Sparse Codes on Graphs - Theory and Practice" (https://ieeexplore.ieee.org/abstract/document/6852099)
- - Hestness et al '17: Joel Hestness, Sharan Narang, Newsha Ardalani, Gregory Diamos, Heewoo Jun, Hassan Kianinejad, Md. Mostofa Ali Patwary, Yang Yang, Yanqi Zhou, "Deep Learning Scaling is Predictable, Empirically" (https://arxiv.org/abs/1712.00409)
- - Hoffman et al '22: Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals and Laurent Sifre, "Training Compute-Optimal Large Language Models" (https://arxiv.org/pdf/2203.15556.pdf)
- - Sorscher et al '22: Ben Sorscher, Robert Geirhos, Shashank Shekhar, Surya Ganguli, Ari S. Morcos, "Beyond neural scaling laws: beating power law scaling via data pruning" (https://arxiv.org/abs/2206.14486)

Thank You!